Stark broadening of the lower Rydberg n_α, n_β and n_γ lines of atomic hydrogen

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Abstract. This paper deals with the calculation of line widths and lineshapes of the lower Rydberg lines of atomic hydrogen ($n \sim 5 - 20$) in the conditions of stellar envelopes and atmospheres - electron densities scaling from 10^9 to 10^{16} cm^{-3} and temperatures varying between 5000 and 40000 K. For these lines, the traditional semi-classical method breaks down, especially at low densities. Another or other theoretical approaches are needed for the determination of linewidths. An alternative tool is proposed in terms of collision rate methods. Data are provided for n_α, n_β and n_γ lines.

Key words: atomic data — line profiles — plasmas

1. Introduction

Hydrogen far infrared, submillimetric and radiofrequency lines have been the object of a large number of works during the last three decades. The importance attributed to such studies lies in the fact that they may provide valuable diagnostics for the thermal structure of the solar atmosphere (Hoang Binh 1982; Brault & Noyes 1983; Boreiko & Clark 1986; Hoang Binh et al. 1987; Carlsson & Rutten 1992), and also of some stellar atmospheres (see Waters & Marlborough 1992); they may also help in the understanding of the proton-electron recombination process in the H II regions. In their paper published in 1987, Hoang Binh et al. have given theoretical data for the calculation of hydrogen solar line broadening in the far infrared and submillimetric spectra ($n \sim 5 - 20$), within the impact approximation for ions. We aim to study the linewidths and lineshapes of the same lines for a wide range of densities, scaling from low to mean densities - for which the impact approximation breaks down for ions, but is still valid for electrons. However, the densities must be chosen below

the Inglis-Teller limit, such as to avoid or reduce the line dissolution (or overlap) under the broadening process.

In fact, according to Hoang Binh (1982), most of the solar infrared hydrogen lines are emitted in the chromosphere at heights between 750 and 1600 km above the photosphere, where the electron density N_e and ion density N_p vary between 6 10^{10} and 10^{11} cm^{-3}, and where the temperature is of the order of 6 10^3 K (Vernazza et al. 1981). In such conditions, the impact approximation is valid or almost valid for ions, and - of course for electrons - for the n_α lines and most of the n_β lines ($5 \leq n \leq 20$). For the lines emitted in the neighbourhood of the phosphere - $N_e \sim N_p \sim 10^{14}$ cm^{-3} and $T \sim 6500$ K - the impact approximation is not valid for ions, even for $n = 5$, but is still valid for electrons.

In a previous paper (Motapon et al. 1997), we used a quantum formalism to calculate the linewidths and lineshapes of the lower infrared hydrogen lines broadened by electrons, ions and both electrons and ions. This formulation, using the no-quenching approximation, has been found to be not suitable for the description of electron broadening of lines emitted from transitions between atomic levels above $n = 5$, since in such cases, electron broadening is mainly due to inelastic collisions. On the other hand, it is expected to apply conveniently to these lines, broadened by impact with ions since this last process is dominated by elastic ion-atom collisions. However, quantum calculations are long for ions and may be costly.

There is a need for simple and efficient approaches for the determination of linewidths. Even the semi-classical calculations may be doubtful if an adequate choice of the cut-offs is not made in the integration over the impact parameters. In this work, a collision rate approach is used to describe electron impact widths. Electronic linewidths are determined from the collision rates via the electron-atom excitation cross sections. In a few cases, the traditional semi-classical theory is used to calculate ion and electron linewidths for comparison with the collision rate method,

and for quantitative comparison of the electron and ion contributions to the total impact linewidths.

In the second section of this paper, the semi-classical method is briefly described. The linewidth calculation is presented, and the limits of the approach are discussed. The third section deals with the collision rate methods. The comparison of the collision rate approach and the semi-classical method in the fourth section is followed by the conclusion. Values of collision rates are provided in Tables 4 to 6, for further calculations of electron impact widths and approximate profiles.

2. The semi-classical method

The traditional semi-classical treatment of line broadening depends on two main hypothesis:

(i) the classical path approximation for the perturber, i.e. the perturber is supposed to follow a well known path, which is a straight line for the broadening of neutral hydrogen;

(ii) the impact approximation, the validity conditions of which have been largely treated in the literature (Griem 1974; Sobel'man 1981).

Furthermore, we make the dipole approximation for the interaction potential.

During the transition from level m to level n, the electron collision operator is given (Sobel'man 1981; Lisitsa 1977; Griem 1974) by:

$$\Phi_{mn} = N_e \int_0^\infty v f(v) \int_0^\infty 2\pi\rho d\rho \left\{1 - S_m^\dagger S_n\right\} \qquad (1)$$

where $\{...\}$ represents the mean over the angles of the perturber, and $f(v)$ is Maxwell's distribution. S_m (resp. S_n) is the collision matrix for level m (resp. n) and $S_m^\dagger$ is the hermitian conjugate of S_m. S_m reads in terms of $\tilde{V}_m(t) = \exp(-iH_0t/\hbar) V \exp(iH_0t/\hbar)$, the potential in the interaction representation:

$$S_m = \exp\left[-\frac{i}{\hbar}\int_{-\infty}^\infty \tilde{V}_m(t)\,dt\right] \qquad (2)$$

V being the perturber-radiator interaction potential.

After the average over the angles has been carried out as well as the two integrations in (1), the collision operator is approximated by (Lisitsa 1977):

$$\Phi_{mn} = \frac{16}{3} N_e \frac{e^4}{\hbar^2} \left\langle \frac{1}{v} \right\rangle [\mathbf{r}_m.\mathbf{r}_m + \mathbf{r}_n.\mathbf{r}_n - 2\mathbf{r}_m.\mathbf{r}_n]$$
$$\times \left[0.215 + \ln\frac{\rho_{\max}}{\rho_{\min}}\right] \qquad (3)$$

where $\mathbf{r}_m$ and $\mathbf{r}_n$ are the position operators in levels m and n, $\rho_{\max}$ and $\rho_{\min}$ being the maximum and minimum impact parameters. The linewidths are given by the real and the imaginary parts of the matrix elements of Φ_{mn}. In the dipole approximation, the shifts vanish since the collision operator is real. In Eq. (3), 0.215 is the contribution of strong collision while $\ln\frac{\rho_{\max}}{\rho_{\min}}$, which is that of

weak collisions, is supposed - by hypothesis - to be the major contribution to line broadening. $\mathbf{r}_m.\mathbf{r}_m$ and $\mathbf{r}_n.\mathbf{r}_n$ are the broadening terms for both upper and lower levels, and $-2\mathbf{r}_m.\mathbf{r}_n$ is the interference term.

2.1. Ion impact widths

For ions, the quantity $v_i/\rho_{\min}^i$ is of the order of the angular frequency separating two close levels (Griem 1967). Hence, only collisions with impact parameters ρ less than $\rho_{\min}^i$ can lead to states of different principal quantum numbers, i.e. non-adiabatic transitions, and those with ρ greater than $\rho_{\min}^i$ are adiabatic. This explains why, in ion broadening, within the impact approximation, the inelastic contribution which is not significant in the dominant term - that of weak collisions - is neglected.

Let us consider the transition $m = n + \Delta n \to n$. According to Griem the contribution of weak collisions to the ion impact width is:

$$\Delta\omega_i \approx \frac{4}{3}\pi \left\langle \frac{1}{v_i} \right\rangle \left(\frac{\hbar}{m_e}\right) N_e \ln\frac{\rho_{\max}}{\rho_{\min}} G(n, \Delta n) \qquad (4)$$

where m_e is the electron mass. Owing to the major contribution of elastic collisions to broadening, $G(n, \Delta n)$ corresponds to the F-function of Hoang Binh et al.:

$$F(n, \Delta n) = \sum_{l,ml',m'} |\langle n + \Delta n, l+1, m \mid Z \mid n, l, m\rangle|^2$$
$$\times f(n, l, \Delta n) \qquad (5)$$

where

$$\sum_{l,ml',m'} |\langle n + \Delta n, l+1, m \mid Z \mid n, l, m\rangle|^2 \qquad (6)$$

is normalized to unity, with

$$f(n, l, \Delta n) = \frac{9}{4}n^2(n^2 - l^2 - l - 1)$$
$$+ \frac{9}{4}(n + \Delta n)^2(n^2 - l^2 + 2n\Delta n - 3l - 3 + \Delta n^2)$$
$$- \left\{\frac{2l}{2l+1}R_{nl}^{nl-1}R_{n+\Delta n,l}^{n+\Delta n,l+1}R_{n+\Delta n,l}^{nl-1}\right.$$
$$+ \left.\frac{2l+4}{2l+3}R_{nl}^{nl+1}R_{n+\Delta n,l+1}^{n+\Delta n,l+2}R_{n+\Delta n,l+2}^{nl+1}\right\} / R_{n+\Delta n,l+1}^{nl} \qquad (7)$$

where the $R_{nl}^{n'l''}$'s are the the dipole radial integrals.

2.2. Electron impact widths

In the electron case, the velocities are greater. The inelastic contribution to the linewidth becomes important, and dominates the elastic one in most cases. The contribution of weak collisions to the electronic width is:

$$\Delta\omega_i \approx \frac{4}{3}\pi \frac{1}{v_e} \left(\frac{\hbar}{m_e}\right)^2 N_e \ln\frac{\rho_{\max}}{\rho_{\min}} G(n, \Delta n) \qquad (8)$$

where $G(n, \Delta n)$ is a function containing the elastic and inelastic contributions to broadening. To take these two contributions into account, let us introduce the intermediate states in the summation over the matrix elements of the operators $\mathbf{r}.\mathbf{r}$. $G(n, \Delta n)$ is:

$$G(n, \Delta n) = \sum_{l,m l',m'} \mid \langle n + \Delta n, l + 1, m \mid Z \mid n, l, m \rangle \mid^2$$
$$\times g(n, l, \Delta n) \qquad (9)$$

where

$$g(n, l, \Delta n) = g_{\text{elast}}(n, l, \Delta n) + g_{\text{inel}}(n, l, \Delta n) \qquad (10)$$

and

$$g_{\text{elast}}(n, l, \Delta n) = f(n, l, \Delta n). \qquad (11)$$

One then has:

$$g(n, \Delta n) = F(n, \Delta n) + H(n, \Delta n) \qquad (12)$$

where $F(n, \Delta n)$ accounts for the elastic part, given by Eq. (5), and $H(n, \Delta n)$ for the inelastic part:

$$H(n, \Delta n) = \sum_{l,m} \mid \langle n + \Delta n, l + 1, m \mid Z \mid n, l, m \rangle \mid^2$$
$$\times h(n, l, \Delta n) \qquad (13)$$

with

$$h(n, l, \Delta n) = \sum_{l',m'} g_{\text{inel}}(n, l, \Delta n) \qquad (14)$$

$$= \sum_{l',m'} \sum_{n' \neq n} \sum_{n'' \neq n + \Delta n} \{ \langle nlm \mid r \mid n'l'm' \rangle^2$$
$$+ \langle n + \Delta n, l + 1, m \mid r \mid n'', l' + 1, m' \rangle^2 \} . \qquad (15)$$

On the other hand,

$$\sum_{l',m'} \langle nlm \mid r \mid n'l'm' \rangle^2$$
$$= \sum_{m'} \{ \langle n, l, m \mid r \mid n', l - 1, m' \rangle^2$$
$$+ \langle n, l, m \mid r \mid n', l + 1, m' \rangle^2 \}$$
$$= \frac{l}{2l+1} \left(R_{nl}^{n'l-1} \right)^2 + \frac{l+1}{2l+1} \left(R_{nl}^{n'l+1} \right)^2 \qquad (16)$$

and

$$\sum_{l',m'} \langle n + \Delta n, l + 1, m \mid r \mid n'', l' + 1, m' \rangle^2$$
$$= \frac{l+1}{2l+3} \left(R_{n+\Delta n,l+1}^{n''l} \right)^2 + \frac{l+2}{2l+3} \left(R_{n+\Delta n,l+1}^{n'l+2} \right)^2 \qquad (17)$$

so

$$h(n, l, \Delta n) = \sum_{n' \neq n} \left\{ \frac{l}{2l+1} \left(R_{nl}^{n'l-1} \right)^2 \right.$$
$$+ \frac{l+1}{2l+3} \left(R_{nl}^{n'l+1} \right)^2 \right\}$$
$$+ \sum_{n'' \neq n + \Delta n} \left\{ \frac{l+1}{2l+3} \left(R_{n+\Delta n,l+1}^{n''l} \right)^2 \right.$$
$$\left. + \frac{l+2}{2l+3} \left(R_{n+\Delta n,l+1}^{n'l+2} \right)^2 \right\} . \qquad (18)$$

Using sum rules for dipole radial integrals (Bethe & Salpeter 1957) one finds:

$$h(n, l, \Delta n) = \frac{n^2}{4} \left\{ n^2 + 3l^2 + 3l + 1 \right\}$$
$$+ \frac{(n + \Delta n)^2}{4} \left\{ (n + \Delta n)^2 + 3l^2 + 9l + 17 \right\} . \qquad (19)$$

The details for the calculation of $F(n, \Delta n)$ and $H(n, \Delta n)$ are given in the Appendix. This calculation, for a few transitions, has led to three main observations:

i) For n greater than 5, the inelastic contribution is dominant for the first transitions ($n_\alpha, n_\beta,$ and n_γ).
ii) For large values of n, $H(n, \Delta n)$ is practically independent of n.
iii) As Δn increases at low values of n, $F(n, \Delta n)$ becomes greater than $H(n, \Delta n)$, and elastic collisions become a significant contribution.

Finally, it turns out that inelastic electron-atom collisions constitute the dominant factor of electron broadening for the lines we are interested in. Therefore, the inelastic width may be a satisfactory approximation to the total linewidth in most cases.

2.3. Limitation of the semi-classical method - Need for other approaches for linewidths determination

In the impact approximation, it is supposed that the mean effect of the collision is weak. Thus, the strong collision contribution (Eq. 3) is small compared to that of weak collisions $\left(\ln \frac{\rho_{\max}}{\rho_{\min}} \right)$. For Rydberg lines, the linewidth from Eq. (7) may be "uncertain" at some values of the electron densities. In fact $\rho_{\max}$ is generally taken to be the Debye radius $\left(\sqrt{\frac{kT}{4\pi N_e e^2}} \right)$ or the Lewis cut-off while $\rho_{\min}$ can be chosen similarly to the lower cut-off of Hoang-Binh et al., i.e.:

$$\rho_{\min} = \left(\frac{2}{3} \right)^{1/2} \frac{\hbar}{m v_e} \left[G(n, \Delta n) \right]^{1/2} . \qquad (20)$$

However, at typical densities, for the lines we are interested in, $\rho_{\min}$ and $\rho_{\max}$ are sometimes too close,

providing a weak contribution comparable to that of strong collisions. Such a situation is non sensical, owing to the basic hypothesis of the semi-classical method. A higher order in Born series is needed or, at least, close coupling calculations would be required. In this case, particular attention must be paid to the description of the strong collision's contribution; a well adapted cut-off must be chosen. Another or other methods are needed for comparison.

3. The collision rate methods

Quantum calculations if they apply, provide good results, but they do not constitute a rapid way to obtain the linewidths, especially for ion broadening. On the other hand, it is known (Sobel'man et al. 1981) that for highly excited states ($n, n' \gg 1$ and $\mid n - n' \mid \ll n$), the electron impact widths can be expressed approximately in terms of the total transition rates of the level n and n'.

3.1. Electron broadening

For highly excited states, the electron-produced width can be approximated by:

$$\gamma_e \sim N_e \left(\langle v\sigma(n) \rangle + \langle v\sigma(n') \rangle \right) \tag{21}$$

where $\langle v\sigma(n) \rangle$ is the total transition rate for level n, and $\sigma(n)$ the total transition cross section for the same level, given by:

$$\sigma(n) = \sum_{k=1-n}^{\infty} \sigma(n, n+k) \tag{22}$$

$n + k$ varies from 1 to ∞ so that k varies from $1 - n$ to ∞.

In (21), the interference term is not taken into account. Its contribution, which is more important for n_α transitions is considered to be cancelled by the adiabatic contribution, since the major contribution to electron broadening comes from inelastic collisions. Thus, the summation in Eq. (22) must be taken over $k \neq 0$.

Gee et al. (1976) have proposed a semi-empirical formula for the calculation of $e^- -$ atom excitation cross section ($\sigma(n, n+k), k > 1$) - de-excitation cross section being obtained via the detailed balance equation - for electrons energies obeying:

$$\frac{4}{n^2} \leq \frac{\varepsilon}{R_y} \ll (137)^2 . \tag{23}$$

They have also proposed a formula for the calculation of transition rates $\alpha(n, n+k)$ for temperatures and energy levels satisfying:

$$\left(\frac{10^6}{n^2} \leq \frac{T}{K} \ll 3 \ 10^9 \right) . \tag{24}$$

This last condition does not permit us to determine the total transition rates for all the the lines and temperatures we are interested in $\left(n \sim 5 - 20, \ T \sim 10^3 - 10^5 \right)$.

Thus, using the cross sections, we have defined a numerical method for the calculation of the total transition rates, based on a Gauss-Laguerre integration. The merit of this approach is that it can provide transition rates out of the validity domain of Gee et al.

In fact, putting $x = \frac{\varepsilon}{kT}$ where $\varepsilon = \frac{1}{2}mv^2$ is the perturber's kinetic energy (here the electron), one easily obtains:

$$\langle v\sigma(n) \rangle = \int_0^\infty v\sigma(n, v) f(v)$$

$$= \left(\frac{8kT}{\pi m} \right)^{1/2} G(T) \tag{25}$$

where

$$G(T) = \int_0^\infty x\sigma \left(n, \frac{kTx}{R_y} \right) \exp(-x) \, dx. \tag{26}$$

can be calculated by Gauss-Laguerre with good precision.

There are three collision rate methods. The first (Method A) consists of determining total transition rates by summation of transition rates provided by the semi-em-pirical formula of Gee et al. (1976). The second (Method B) is based on the semi-empirical formula of Sobel'man et al. The third (Method C) is the numerical method developed in this work, which uses accurate transition cross sections available in the literature.

Using these methods, we have calculated the total transition rates for a few hydrogen lines. The results obtained were compared in Motapon & Tran Minh (1995). Good agreement was found between methods A and C using the conditions of Gee et al.'s semi-empirical formula. A good agreement was also found between methods B and C, except for high n (about 100) at high energies ($T > 25000$ K).

3.2. Ion broadening

In the atomic energy level range we are concerned with, elastic collisions dominate ion broadening. Thus, the collision rate method for ions requires cross sections for elastic scatering. On the other hand, the approximation of complete degeneracy yields divergent cross sections for the process $nl \longrightarrow nl \pm 1$ (Toshima 1977a,b); Pengelly & Seaton (1964), so that this degeneracy must be removed by the fine structure and the Lamb shift, the hyperfine structure being neglected here. The total cross section $\sigma(n)$ is given by:

$$\sigma(n) = \frac{1}{n^2} \sum_{l,l'} \sigma(nl \longrightarrow nl') . \tag{27}$$

As

$$\sigma(nl \longrightarrow nl') = \frac{1}{2(2l+1)} \sum_{j=l-1/2}^{l+1/2} \sum_{j'=l'-1/2}^{l'+1/2} (2j+1)$$
$$\times \sigma(nlj \longrightarrow nl'j') \tag{28}$$

one has:

$$\sigma\left(n\right) = \frac{1}{2n^2} \sum_{l=0}^{n-1} \sum_{l'=0}^{n'-1} \sum_{j=l-1/2}^{l+1/2} \sum_{j'=l'-1/2}^{l'+1/2} \left(2j+1\right)$$
$$\times \sigma\left(nlj \longrightarrow nl'j'\right) \qquad (29)$$

where $\sigma\left(nlj \longrightarrow nl'j'\right)$ is the proton-hydrogen atom excitation cross section from state $\mid i\rangle \equiv \mid nlj\rangle$ to state $\mid f\rangle \equiv \mid n'l'j'\rangle$, given, for large impact parameters by Seaton (1962), Pengelly & Seaton (1963)

$$\sigma\left(nlj \longrightarrow nl'j'\right) = \frac{8\pi Z^2 e^2}{3\hbar^2} \frac{S}{v^2} \left\{\ln\left(\frac{R_c}{R_1}\right) + \frac{1}{2}\right\} \qquad (30)$$

where R_c is the smallest value of $0.72\tau v$, $1.12\hbar v/\Delta E$ and R_D the Debye radius. R_1 is given by

$$R_1^2 = \frac{8Z^2 e^2 S}{3\hbar^2 v^2} \qquad (31)$$

where $S = e^2 \mid \langle nlj \mid \mathbf{r} \mid nl'j'\rangle \mid^2$, $\langle nlj \mid \mathbf{r} \mid nl'j'\rangle$ being a dipole matrix element in the j-representation (Condon & Shortley 1959).

$\Delta E = \Delta E\left(nlj \longrightarrow nl'j'\right)$ is the energy splitting between $\mid nlj\rangle$ and $\mid nl'j'\rangle$.

Transition rates can be carried out in a similar way as in the electronic case.

From a general point of view, estimations give $1.12\frac{\hbar v}{\Delta E}$ to be less than $0.72\tau v$ for the atomic levels involved in the transitions we are interested in. On the other hand, in the ordinary conditions ($T \sim 5000$ to 40000 K) the Debye radius R_D $(= 6.90(T/N_e)^{1/2}$ in centimetres) is greater than $1.12\frac{\hbar v}{\Delta E}$ unless the density exceeds 10^9 cm^{-3}. Hence, the density range for application of the collision rate method to ions is reduced since the impact approximation for ions impose $N_e < 10^{11}$ cm^{-3}. Furthermore, there is a formal analogy between this description and the traditional semi-classical one. It finally comes out that one can merely apply the formulae (5) to (8) of Hoang Binh et al. (1987) to the calculation of $\Delta\omega_i$, especially because in this density range, the semi-classical description (given above) does not yet break down.

4. Comparison of both approaches and discussion

Before comparing the widths provided by the semi-classical method and the collision rate method (respectively hereafter referred to as SCM and CRM), we must first determine the limits in density of the impact approximation, that is the densities above which the impact approximation does not apply. It is well known (Sobel'man et al. 1981) that the impact approximation is valid for the level n if the dimensionless quantity $h_e = N_e\left(n(n-1)\frac{\hbar}{m_e}\right)^3$ is very small compared to unity. This gives the "limiting" values mentioned in Table 1 below.

Table 1. Limiting densities of the impact approximation for electron broadening

n	N_e (cm^{-3})
5	2.5E+19
10	2.7E+17
15	2.1E+16
16	1.4E+16
17	9.8E+15
18	6.9E+15
19	4.9E+15
20	3.6E+15

Further, the density must be taken below the Inglis-Teller limit given in Griem (1974) by:

$$N_{IT} \approx \left(Z^{9/2}/120a_0^3 Z_p^{3/2}\right)\left(1/n^2 - 1/(n+1)^2\right)^{3/2}$$
$$\times \left(m^2 - n^2\right)^{-3/2} \qquad (32)$$

where $Z_p e$ is the perturber's charge, Ze the radiator's charge: m is the upper atomic level for the transition and n the lower level.

The cut-offs on the impact parameters are:

$$\rho_{max} \approx \min\left(\rho_D, \lambda/2\pi\right) \qquad (33)$$

and, for $\Delta\omega$ greater than the ion plasma frequency ω_{p_i} the Lewis cut-off at ρ_{max} $(\approx v/\mid \Delta\omega \mid)$ is more appropriate.

$$\rho_{min} \approx \left(\frac{2}{3}\right)^{1/2} \frac{\hbar}{mv_e} \left[G\left(n, \Delta n\right)\right]^{1/2} \qquad (34)$$

where $G\left(n, \Delta n\right)$ is given by Eq. (9).

The other characteristic radii are:

i) ρ_{eff}, the radius above which the collision is complete, i.e. $\rho_{eff}^2 = \sigma\left(n\right)/\pi$. ρ_{eff} could be a possible lower cut-off on the impact parameters; but n scaling from 5 to 20 is of the order of ρ_{min} - as chosen in (34) - which is about two to five times the atomic mean radius $\left(n^2 a_0\right)$;

ii) the mean radius r_0 of the sphere occupied by one perturber, which is given by $\frac{4}{3}\pi r_0^3 N_e = 1$. r_0 is large, as compared to the lower cut-off ρ_{min} at low densities, but gets closer to ρ_{min} when the density increases. In some critical cases, the perturber is too close to the Rydberg atom or even inside it. In such cases, the dipole approximation as well as the impact approximation break down, and the SCM and CMR are problematical. At higher densities (above the impact approximation limit and the Inglis-Teller limit), there may be many perturbers in the Rydberg atom; line dissolution is important; the problem becomes more complicated and deserves particular attention.

Following the two approaches described in the previous sections, we have calculated the electron impact linewidths of a few n_γ lines at densities varying from 10^9 to 10^{17} cm^{-3} and $T = 10^4$ K. The results are given in Table 2 below.

Table 2. Comparison of the linewidths of n_γ transitions. CRM is referred to the collision method and SCM to the semi-classical

n	$N_e\,(\mathrm{cm}^{-3})$	CRM	SCM
	10^9	1.55E+05	4.84E+06
	10^{12}	1.55E+08	3.16E+09
5	10^{15}	1.55E+11	1.47E+12
	10^{17}	-	-
	10^9	2.15E+06	2.39E+07
	10^{12}	2.15E+09	1.48E+10
10	10^{15}	2.15E+12	5.72E+12
	10^{17}	-	-
	10^9	1.19E+07	7.57E+07
	10^{12}	1.19E+10	4.48E+10
15	10^{15}	1.19E+13	1.40E+13
	10^{17}	-	-
	10^9	4.17E+07	1.86E+08
	10^{12}	4.17E+10	1.05E+11
20	10^{15}	4.17E+13	5.72E+12
	10^{17}	-	-

Table 2 shows discrepancies between CRM and SCM at nearly all densities, especially at low densities which correspond to large values of the Debye radius. Analogous results are found for n_α and n_β transitions. Nevertheless, for n_γ transitions, the interference term in the electron collision operator Φ_{mn} is known to be less significant, so that the CRM could provide satisfactory results for the linewidths. Hence, the SCM with the chosen lower cut-off overestimates the linewidths for $n \sim 5 - 20$, especially at low densities. Another cut-off must be determined such as to obtain agreement with the CRM. This occurs when $\rho_{\min}$ is designated to be larger. This would be in agreement with the validity of the SCM. Finally, the CRM described in this work remains an alternative tool for determining linewidths at low electron densities.

Electron and ion impact widths are compared quantitatively in Table 3 for $N_e = 10^{10}$ and $N_e = 10^{12}$ cm^{-3}.

$$\Delta\omega_i = \frac{4}{3}\pi\left(\frac{\hbar}{m_e}\right)^2 \left\langle\frac{1}{v}\right\rangle N_i$$
$$\times [\ln \rho_{\max}/\rho_{\min} + 0.5]\, F\,(n, \Delta n)\,. \tag{35}$$

For protons,

$$\rho_{\min} = \left(\frac{2}{3}\right)^{1/2} \frac{\hbar}{m_e} \left\langle\frac{1}{v}\right\rangle [F\,(n, \Delta n)]^{1/2} \tag{36}$$

$$v = [2kT\pi/\mu]^{1/2} = [2kT\pi/\,(M/2)]^{1/2}$$
$$= 2\,[kT\pi/M]^{1/2} \tag{37}$$

$$\left\langle\frac{1}{v}\right\rangle = (2\mu/\pi kT)^{1/2} = (M/\pi kT)^{1/2} \tag{38}$$

$$\Delta\omega_i = \frac{4}{3}\pi\left(\frac{\hbar}{m_e}\right)^2 \left(\frac{M}{\pi kT}\right)^{1/2} N_i\,[\ln \rho_{\max}/\rho_{\min} + 0.5]$$
$$\times F\,(n, \Delta n)$$
$$= \frac{4}{3}\sqrt{\pi}\left(\frac{\hbar}{m_e}\right)^2 \left(\frac{M}{k}\right)^{1/2} T^{-1/2} N_i\,[\ln C + 0.5]$$
$$\times F\,(n, \Delta n)\,(39)$$

$$= \frac{4}{3}\sqrt{\pi}\left(\frac{\hbar}{m_e}\right)^2 \left(\frac{M}{k}\right)^{1/2} 10^6 \equiv 3.487\ 10^{-4} \tag{40}$$

$$\Delta\omega_i = 3.487\ 10^{-4} T^{-1/2} N_i\,[\ln C + 0.5]$$
$$\times F\,(n, \Delta n) \tag{41}$$

in s^{-1}, N_i in cm^{-3} and T in K.

The Table 3 shows a domination of ion contribution to impact broadening of n_α, n_β and n_γ lines for all the atomic levels concerned with this work. It is observed that electron broadening is expected to take over for $n > 20$, as claimed by Griem (1967).

For electron densities above the impact approximation limit - for ions - and below the Inglis-Teller limit, the quasistatic approximation can be applied to ions, but the impact approximation still applies to electrons for n varying between 5 and 20.

In this case, theoreticians of lines broadening commonly define the profile of the line emitted during the transition from the level m to the level n, and broadened by electrons and ions, by:

$$I\,(\Delta\omega) = \frac{1}{\pi} Re \int_0^\infty W\,(F)\,\mathrm{d}F \sum_{\alpha\beta,\alpha'\beta',\sigma} \mathrm{d}_{\alpha\beta'}^{(\sigma)} \mathrm{d}_{\alpha'\beta}^{(\sigma)}$$
$$\times \langle\alpha\,|\,\langle\beta\,|\,\Theta^{-1}\,|\,\alpha'\rangle\,|\,\beta'\rangle \tag{42}$$

where:

$$\Theta = i\left(\omega - \omega_0 - \frac{1}{\hbar}\left(H^m\,(F) - H^n\,(F)\right)\right) + \Phi_{mn} \tag{43}$$

$W(F)$ is the ion microfield distribution and the electron Φ_{mn} the electron impact operator; $|\,\alpha\rangle$, $|\,\beta\rangle$, $|\,\alpha'\rangle$ and $|\,\beta'\rangle$ are parabolic states. In the description given by formula (42), one considers that the ions create a "static" microfield which induces a lifting of the degeneracy of the atomic energy levels - linear Stark effect - and the interaction with electrons, mainly by collisions provokes a broadening of so-formed Stark components.

Since Φ_{mn} is not diagonal in the parabolic representation, the difficulty in this formulation lies in the inversion of large matrices. For memory, the size of matrices to be inverted is $(m \times n)^2 \times (m \times n)^2$. Nevertheless, we can simplify the problem by building from Φ_{mn} and operator $\widetilde{\Phi}_{mn}$ which would be diagonal in the parabolic representation (see Seaton 1990). A diagonal operator can also be obtained by simply neglecting the off-diagonal elements of Φ_{mn}. According to Griem (1960) and Sobel'man (1963,

Table 3. Electron and ion impact linewidths at $N_{\rm e} = 10^{10}$ and $N_{\rm e} = 10^{11}{\rm cm}^{-3}$ for $T = 10000$ K

n	Δn	$N_{\rm e} = 10^{10}$		$N_{\rm e} = 10^{11}$	
		$\Delta\omega_{\rm e}$	$\Delta\omega_{\rm p}$	$\Delta\omega_{\rm e}$	$\Delta\omega_{\rm p}$
	1	4.28E+05	1.34E+07	4.28E+06	1.24E+08
5	2	8.31E+05	9.29E+07	8.31E+06	8.39E+08
	3	1.55E+06	2.49E+08	1.55E+07	2.21E+09
	1	1.03E+06	1.89E+07	1.03E+07	1.74E+08
6	2	1.75E+06	1.26E+08	1.75E+07	1.13E+09
	3	2.95E+06	3.29E+08	2.95E+07	2.90E+09
	1	1.19E+07	4.98E+07	1.19E+08	4.53E+08
10	2	1.59E+07	3.02E+08	1.59E+08	2.67E+09
	3	2.15E+07	7.48E+08	2.15E+08	6.47E+09
	1	8.20E+07	1.09E+08	8.20E+08	9.78E+08
15	2	9.85E+07	6.23E+08	9.85E+08	5.42E+09
	3	1.19E+08	1.49E+09	1.19E+09	1.26E+10
	1	3.19E+08	1.90E+08	3.19E+09	1.69E+09
20	2	3.66E+08	1.05E+09	3.66E+09	8.98E+09
	3	4.17E+08	2.46E+09	4.17E+09	2.03E+10

1972), an approximate profile can be obtained by replacing Φ_{mn} by half-width $\gamma/2$ of the line i.e.:

$$I(\Delta\omega) = \int_0^\infty W(F)\,{\rm d}F \sum_{\alpha\beta} |\,{\rm d}_{\alpha\beta}\,|^2$$
$$\times \frac{1}{2\pi} \frac{\gamma}{(\Delta\omega - C_{\alpha\beta}F)^2 + \frac{\gamma^2}{4}}. \tag{44}$$

An approximate method for the evaluation of the lineshape of formula (44) consists of using the Griem's table (Griem 1960). The agreement between this method and formula (44)-based calculations has been found to be particularly good for transitions $n + \Delta n \longrightarrow n$ with $\Delta n > 1$, but not for n_α transitions. A regularly spaced extended table from the original Griem's table, permits, via a double interpolation program, a rapid determination of approximate lineshapes of $n_\beta, n_\gamma, ...$ lines.

5. Conclusion

The IR and SMM lineshapes are needed for interpretation of the observed solar lines in the infrared and submillimetric spectrum. The present work deals with the densities below the Inglis-Teller limit, above which line dissolution is important. The common semiclassical method is problematic since it overestimates the line widths at low densities and requires an improvement. The collision rate method is an alternative method for linewidth calculations in such cases, particularly for the electron case. Within the impact or quasistatic approximation for ions, the profile can be easily obtained as soon as the electronic and/or the ionic linewidth is calculated. For this purpose electron-Hydrogen atom transition rates are provided in Tables 4 to 6.

The spectral line broadening of the lower Rydberg lines is a more complex problem when the electron density becomes greater than the Inglis-Teller limit $N_{\rm IT}$. The description of a system with one or many perturbers in the Rydberg atom must be considered. Line dissolution has to be treated conveniently. In any case, even at average densities, there is always a critical electric microfield $F_{\rm c}$ - in the integration over the microfield distribution - above which, line dissolution appears (Seaton 1990). Between the impact and static approximation for ions, an intermediate region between these two approximations is expected, for which neither of the limiting approximations are valid, thus calling for an improved modelisation.

Appendix A: Calculation of $H(n, \Delta n)$

$$H(n, \Delta n) = \sum_{l,m} |\,\langle n + \Delta n, l + 1, m \mid Z \mid n, l, m \rangle\,|^2$$
$$\times h(n, l, \Delta n) \tag{A1}$$

Table 4. Total electron transition rates of level n at various temperatures from 1000 to 6000 K

n	$T = 1000$ K	$T = 2000$ K	$T = 4000$ K	$T = 5000$ K	$T = 6000$ K
5	1.5882E-06	3.0592E-06	5.6691E-06	6.8276E-06	7.9094E-06
6	4.4936E-06	8.6374E-06	1.5707E-05	1.8763E-05	2.1575E-05
7	1.1077E-05	2.1008E-05	3.7359E-05	4.4251E-05	5.0529E-05
8	2.3681E-05	4.4458E-05	7.7622E-05	9.1303E-05	1.0367E-04
9	4.6104E-05	8.5588E-05	1.4686E-04	1.7170E-04	1.9395E-04
10	8.3518E-05	1.5319E-04	2.5858E-04	3.0064E-04	3.3792E-04
11	1.4384E-04	2.6029E-04	4.3222E-04	4.9980E-04	5.5890E-04
12	2.3402E-04	4.1853E-04	6.8511E-04	7.8830E-04	8.7730E-04
13	3.6537E-04	6.4604E-04	1.0435E-03	1.1949E-03	1.3238E-03
14	5.5324E-04	9.6681E-04	1.5414E-03	1.7564E-03	1.9374E-03
15	8.1002E-04	1.4005E-03	2.2066E-03	2.5029E-03	2.7503E-03
16	1.1551E-03	1.9769E-03	3.0802E-03	3.4788E-03	3.8096E-03
17	1.6104E-03	2.7292E-03	4.2073E-03	4.7325E-03	5.1667E-03
18	2.2055E-03	3.7012E-03	5.6460E-03	6.3263E-03	6.8874E-03
19	2.9602E-03	4.9231E-03	7.4370E-03	8.3046E-03	9.0192E-03
20	3.9093E-03	6.4459E-03	9.6469E-03	1.0739E-02	1.1638E-02
21	5.0969E-03	8.3328E-03	1.2357E-02	1.3716E-02	1.4832E-02
22	6.5496E-03	1.0624E-02	1.5620E-02	1.7293E-02	1.8665E-02
23	8.3165E-03	1.3389E-02	1.9524E-02	2.1564E-02	2.3234E-02
30	3.4241E-02	5.2684E-02	7.3223E-02	7.9824E-02	8.5037E-02
31	4.0720E-02	6.2297E-02	8.6094E-02	9.3702E-02	9.9673E-02
32	4.8114E-02	7.3216E-02	0.1006	0.1094	0.1162
33	5.6541E-02	8.5593E-02	0.1171	0.1270	0.1347

Table 5. Total electron transition rates of level n at various temperatures from 7000 to 12000 K

n	$T = 7000$ K	$T = 8000$ K	$T = 9000$ K	$T = 10000$ K	$T = 12000$ K
5	8.9258E-06	9.8884E-06	1.0806E-05	1.1685E-05	1.3332E-05
6	2.4192E-05	2.6652E-05	2.8972E-05	3.1162E-05	3.5165E-05
7	5.6326E-05	6.1713E-05	6.6719E-05	7.1366E-05	7.9691E-05
8	1.1497E-04	1.2532E-04	1.3480E-04	1.4347E-04	1.5883E-04
9	2.1403E-04	2.3213E-04	2.4850E-04	2.6336E-04	2.8953E-04
10	3.7106E-04	4.0057E-04	4.2702E-04	4.5096E-04	4.9303E-04
11	6.1070E-04	6.5642E-04	6.9722E-04	7.3410E-04	7.9870E-04
12	9.5448E-04	1.0222E-03	1.0826E-03	1.1371E-03	1.2321E-03
13	1.4347E-03	1.4318E-03	1.6183E-03	1.6961E-03	1.8308E-03
14	2.0924E-03	2.2280E-03	2.3484E-03	2.4565E-03	2.4517E-03
15	2.9616E-03	3.1462E-03	3.3097E-03	3.4557E-03	3.7033E-03
16	4.0917E-03	4.3378E-03	4.5548E-03	4.7473E-03	5.0706E-03
17	5.5367E-03	5.8584E-03	6.1407E-03	6.3893E-03	6.8030E-03
18	7.3649E-03	7.7783E-03	8.1388E-03	8.4540E-03	8.9741E-03
19	9.6250E-03	1.0149E-02	1.0601E-02	1.0995E-02	1.1639E-02
20	1.2398E-02	1.3049E-02	1.3610E-02	1.4093E-02	1.4881E-02
21	1.5773E-02	1.6574E-02	1.7258E-02	1.7846E-02	1.8797E-02
22	1.9816E-02	2.0788E-02	2.1614E-02	2.2321E-02	2.3460E-02
23	2.4626E-02	2.5795E-02	2.6782E-02	2.7623E-02	2.8974E-02
30	8.9203E-02	9.2583E-02	9.5374E-02	9.7712E-02	0.1014
31	0.1044	0.1083	0.1114	0.1141	0.1182
32	0.1216	0.1259	0.1295	0.1325	0.1371
33	0.1408	0.1457	0.1497	0.1531	0.1583

Table 6. Total electron transition rates of level n at various temperatures from 13000 to 25000 K

n	$T = 13000$ K	$T = 14000$ K	$T = 15000$ K	$T = 20000$ K	$T = 25000$ K
5	1.4101E-05	1.4835E-05	1.5534E-05	1.8592E-05	2.1104E-05
6	3.6991E-05	3.8712E-05	4.0338E-05	4.7349E-05	5.3013E-05
7	8.3434E-05	8.6943E-05	9.0248E-05	1.0439E-05	1.1554E-05
8	1.6569E-04	1.7211E-04	1.7815E-04	2.0370E-04	2.2319E-04
9	3.0120E-04	3.1210E-04	3.2231E-04	3.6478E-04	3.9610E-04
10	5.1172E-04	5.2911E-04	5.4530E-04	6.1136E-04	6.5858E-04
11	8.2724E-04	8.5361E-04	8.7800E-04	9.7554E-04	1.0435E-03
12	1.2736E-03	1.3118E-03	1.3469E-03	1.4846E-03	1.5786E-03
13	1.8892E-03	1.9424E-03	1.9909E-03	2.1790E-03	2.3051E-03
14	2.7211E-03	2.7930E-03	2.8582E-03	3.1080E-03	3.2729E-03
15	3.8084E-03	3.9030E-03	3.9885E-03	4.3126E-03	4.5237E-03
16	5.2066E-03	5.3285E-03	5.4381E-03	5.8507E-03	6.1156E-03
17	6.9757E-03	7.1298E-03	7.2679E-03	7.7844E-03	8.1112E-03
18	9.1897E-03	9.3816E-03	9.5532E-03	1.0190E-02	1.0587E-02
19	1.1904E-02	1.2140E-02	1.2350E-02	1.3126E-02	1.3600E-02
20	1.5204E-02	1.5490E-02	1.5745E-02	1.6677E-02	1.7237E-02
21	1.9187E-02	1.9530E-02	1.9836E-02	2.0944E-02	2.1596E-02
22	2.3924E-02	2.4333E-02	2.4695E-02	2.5999E-02	2.6751E-02
23	2.9522E-02	3.0005E-02	3.0431E-02	3.1951E-02	3.2808E-02
30	0.1028	0.1041	0.1052	0.1087	0.1103
31	0.1199	0.1212	0.1225	0.1263	0.1280
32	0.1389	0.1405	0.1418	0.1461	0.1478
33	0.1603	0.1620	0.1635	0.1681	0.1699

where $h(n, l, \Delta n)$ is given by formula (17). As $h(n, l, \Delta n)$ is independent of m, one can write:

$$H(n, \Delta n) = \sum_l h(n, l, \Delta n)$$
$$\times \sum_m |\langle n + \Delta n, l + 1, m \mid Z \mid n, l, m \rangle|^2. \qquad (A2)$$

Further,

$$\sum_m |\langle n + \Delta n, l + 1, m \mid Z \mid n, l, m \rangle|^2$$
$$= \frac{l+1}{3} \left(R_{nl}^{n+1, l+1} \right)^2. \qquad (A3)$$

So

$$H(n, \Delta n) = \sum_{l=0}^{n-1} \frac{l+1}{3} \left(R_{nl}^{n+1, l+1} \right)^2 h(n, l, \Delta n). \qquad (A4)$$

The calculation of $F(n, \Delta n)$, is carried out in a similar way, after replacing $h(n, l, \Delta n)$ by $f(n, l, \Delta n)$.

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