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On evolution of the pair-electromagnetic pulse of a charged black hole

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Abstract. Using hydrodynamic computer codes, we study the possible patterns of relativistic expansion of an enormous pair-electromagnetic-pulse (P.E.M. pulse); a hot, high density plasma composed of photons, electron-positron pairs and baryons deposited near a charged black hole (EMBH). On the bases of baryon-loading and energy conservation, we study the bulk Lorentz factor of expansion of the P.E.M. pulse by both numerical and analytical methods.

Key words: black hole physics — gamma-ray bursts, theory, observations

In the paper by Preparata et al. (1998), the "dyadosphere" is defined as the region outside the horizon of a EMBH where the electric field exceeds the critical value for e^+e^- pair production. In Reissner-Nordstrom EMBHs, the horizon radius is expressed as

$$r_+ = \frac{GM}{c^2}\left[1+\sqrt{1-\frac{Q^2}{GM^2}}\right]. \tag{1}$$

The outer limit of the dyadosphere is defined as the radius $r_{\rm ds}$ at which the electric field of the EMBH equals this critical field

$$r_{\rm ds} = \sqrt{\frac{\hbar e Q}{m^2 c^3}}. \tag{2}$$

The total energy of pairs, converted from the static electric energy, deposited within a dyadosphere is then

$$E^{\rm tot}_{e^+e^-} = \frac{1}{2}\frac{Q^2}{r_+}\left(1-\frac{r_+}{r_{\rm ds}}\right)\left(1-\left(\frac{r_+}{r_{\rm ds}}\right)^2\right). \tag{3}$$

In Wilson (1975, 1977) a black hole charge of the order 10% was formed. Thus, we henceforth assume a black hole charge $Q = 0.1 Q_{\rm max}, Q_{\rm max} = \sqrt{G}M$ for our detailed numerical calculations. The range of energy is of interest as a possible gamma-ray burst source.

In order to model the radially resolved evolution of the energy deposited within the e^+e^--pair and photon plasma fluid created in the dyadosphere of EMBH, we need to discuss the relativistic hydrodynamic equations describing such evolution.

The metric for a Reissner-Nordstrom black hole is

$$d^2s = -g_{tt}(r)d^2t + g_{rr}(r)d^2r + r^2d^2\theta + r^2\sin^2\theta d^2\phi\ , \tag{4}$$

where $g_{tt}(r) = -g^{-1}_{rr}(r) = -\left[1-\frac{2GM}{c^2r}+\frac{Q^2G}{c^4r^2}\right]$.
We assume the plasma fluid of e^+e^--pairs, photons and baryons to be a simple perfect fluid in the curved space-time (Eq. (4)). The stress-energy tensor describing such a fluid is given by (Misner et al. 1975)

$$T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)U^\mu U^\nu \tag{5}$$

where ρ and p are respectively the total proper energy density and pressure in the comoving frame. The U^μ is the four-velocity of the plasma fluid. The baryon-number and energy-momentum conservation laws are

$$(n_{\rm B}U^\mu)_{;\mu} = (n_{\rm B}U^t)_{,t} + \frac{1}{r^2}(r^2 n_{\rm B}U^r)_{,r} = 0, \tag{6}$$

$$(T^\sigma_\mu)_{;\sigma} = 0, \tag{7}$$

where $n_{\rm B}$ is the baryon-number density. The radial component of Eq. (7) reduces to

$$\frac{\partial p}{\partial r} + \frac{\partial}{\partial t}\left((p+\rho)U^tU_r\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2(p+\rho)U^rU_r\right) - \frac{1}{2}(p+\rho)\left[\frac{\partial g_{tt}}{\partial r}(U^t)^2 + \frac{\partial g_{rr}}{\partial r}(U^r)^2\right] = 0\ . \tag{8}$$

The component of the energy-momentum conservation Eq. (7) along a flow line is

$$U_\mu(T^{\mu\nu})_{;\nu} = (\rho U^t)_{,t} + \frac{1}{r^2}(r^2\rho U^r)_{,r} + p\left[(U^t)_{,t} + \frac{1}{r^2}(r^2U^r)_{,r}\right] = 0. \tag{9}$$

Equations (6) and (9) give rise to the relativistic hydrodynamic equations.

We now turn to the analysis of e^+e^- pairs initially created in the Dyadosphere. Let $n_{e^\pm}$ be the proper densities of electrons and positrons ($e^\pm$). The rate equation for $e^\pm$ is

$$(n_{e^\pm} U^\mu)_{;\mu} = \overline{\sigma v} \left[n_{e^-}(T) n_{e^+}(T) - n_{e^-} n_{e^+} \right], \tag{10}$$

where σ is the mean pair annihilation-creation cross-section, v is the thermal velocity of $e^\pm$, and $n_{e^\pm}(T)$ are the proper number-densities of $e^\pm$, given by appropriate Fermi integrals with zero chemical potential. The equilibrium temperature T is determined by the thermalization processes occurring in the expanding plasma fluid with a total proper energy-density ρ, governed by the hydrodynamical Eqs. (6,9). We have

$$\rho = \rho_\gamma + \rho_{e^+} + \rho_{e^-} + \rho^{b}_{e^-} + \rho_B, \tag{11}$$

where ρ_γ is the photon energy-density and $\rho_{e^\pm}$ is the proper energy-density of $e^\pm$. In Eq. (11), $\rho^{b}_{e^-} + \rho_B$ are baryon-matter contributions. We can also, analogously, evaluate the total pressure p. We define the total proper internal energy density ϵ and the baryon mass density ρ_B in the comoving frame, and have the equation of state (Γ is thermal index)

$$\epsilon \equiv \rho - \rho_B, \quad \rho_B \equiv n_B m c^2, \quad \Gamma = 1 + \frac{p}{\epsilon}\cdot \tag{12}$$

The calculation is initiated by depositing the total energy (3) between the Reissner-Nordstrom radius r_+ and the dyadosphere radius r_{ds}. The calculation is continued as the plasma fluid expands, cools and the e^+e^- pairs recombine, until it becomes optically thin:

$$\int_R (n_{pairs} + n^{b}_{e}) \sigma_T dr = \frac{2}{3} \tag{13}$$

where σ_T is the Thomson cross-section, n^b_e is the number-density of ionized electrons and integration is over the radial size of the expanding plasma fluid in the comoving frame. Here, we only present $n^b_e = 0, \rho_B = 0$ case.

We use a computer code (Wilson et al. 1997, 1998) to evolve the spherically symmetric hydrodynamic equations for the baryons, e^+e^--pairs and photons deposited in the Dyadosphere. In addition, we use an analytical model to integrate the spherically symmetric hydrodynamic equations with the following geometries of plasma fluid expansion: (i) spherical model: the radial component of four-velocity $U(r) = U\frac{r}{\mathcal{R}}$, where U is four-velocity at the surface ($\mathcal{R}$) of the plasma, (ii) slab 1: $U(r) = U_r =$ const., the constant width of expanding slab $\mathcal{D} = R_\circ$ in the coordinate frame of the plasma; (iii) slab 2: the constant width of expanding slab $R_2 - R_1 = R_\circ$ in the comoving frame of the plasma.

We compute the relativistic Lorentz factor γ of the expanding e^+e^- pair and photon plasma. We compare these hydrodynamic calculations with simple models of the expansion. In Fig. 1 we see a comparison of the Lorentz factor of the expanding fluid as a function of radius for all of the models. We can see that the one-dimensional code (only a few significant points are pesented) matches the expansion pattern of a shell of constant coordinate thickness (slab 1).

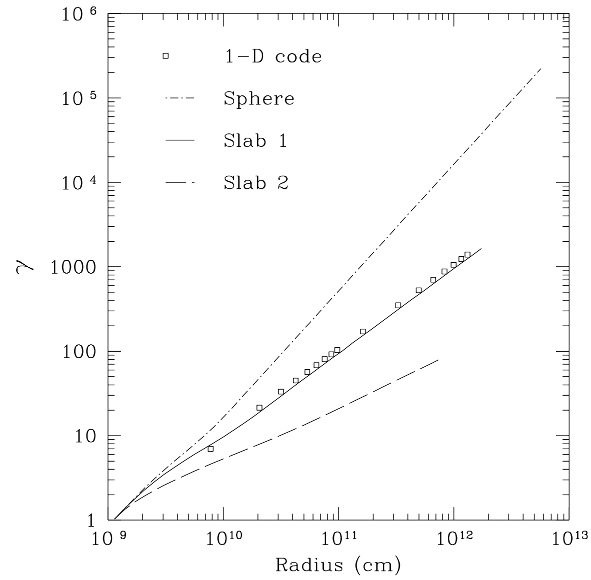

Fig. 1. Lorentz γ as a function of radius. Three models for the expansion pattern of the e^+e^- pair plasma are compared with the results of the one dimensional hydrodynamic code for a 1000 $M_\odot$ black hole with charge $Q = 0.1 Q_{max}$. The 1-D code has an expansion pattern that strongly resembles that of a shell with constant coordinate thickness

We have shown that a relativistically expanding P.E.M. pulse can originate from the Dyadosphere of a EMBH. The P.E.M. pulse can produce gamma-ray bursts having the general characteristics of observed bursts. For example, the burst energy for a 1000 $M_\odot$ BH is 3 10^{54} ergs with a spectral peak at 500 keV and a pulse duration of 40 seconds (Ruffini et al. 1999). This oversimplified model is encouraging enough to demand further study of the Dyadosphere created by EMBHs.

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