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Gamma-ray burst beaming constraints from afterglow light curves

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Abstract. The beaming angle $\zeta_{\rm m}$ is the main uncertainty in gamma ray burst energy requirements today. We summarize predictions for the light curves of beamed bursts, and model the R band light curve of GRB 970508 to derive $\zeta_{\rm m} \gtrsim 30^\circ$. This yields an irreducible minimum energy requirement of 3.4 10^{49} ergs to power the afterglow alone.

Key words: gamma-ray: bursts

1. Beamed gamma-ray burst afterglow models

In the Rome meeting I presented a derivation of the dynamical behavior of a beamed gamma ray burst (GRB) remnant and its consequences for the afterglow light curve. (Cf. Rhoads 1999 [Paper I]). Here, I summarize these results and apply them to test the range of beaming angles permitted by the optical light curve of GRB 970508.

Suppose that ejecta from a GRB are emitted with initial Lorentz factor Γ_0 into a cone of opening half-angle $\zeta_{\rm m}$ and expand into an ambient medium of uniform mass density ρ with negligible radiative energy losses. Let the initial kinetic energy and rest mass of the ejecta be E_0 and M_0, and the swept-up mass and internal energy of the expanding blast wave be $M_{\rm acc}$ and $E_{\rm int}$. Then energy conservation implies $\Gamma E_{\rm int} \approx \Gamma^2 M_{\rm acc} c^2 \approx E_0 \approx$ constant so long as $1/\Gamma_0 \lesssim M_{\rm acc}/M_0 \lesssim \Gamma_0$.

The swept-up mass is determined by the working surface area: ${\rm d}M_{\rm acc}/{\rm d}r \approx \pi(\zeta_{\rm m} r + c_{\rm s} t_{\rm co})^2$, where $c_{\rm s}$ and $t_{\rm co}$ are the sound speed and time since the burst in the frame of the blast wave + accreted material. Once $\Gamma \lesssim 1/\zeta_{\rm m}$, $c_{\rm s} t_{\rm co} \gtrsim \zeta_{\rm m} r$ and the dynamical evolution with radius r changes from $\Gamma \propto r^{-3/2}$ to $\Gamma \propto \exp(-r/r_\Gamma)$ (Rhoads 1998, & Paper I). The relation between observer frame time $t_\oplus$ and radius r also changes, from $t_\oplus \propto r^{1/4}$ to $t_\oplus \propto \exp(r/[2r_\Gamma])$. Thus, at early times $\Gamma \propto t_\oplus^{-3/8}$, while at late times $\Gamma \propto t_\oplus^{-1/2}$. The characteristic length scale is $r_\Gamma = \left(E_0/\pi c_{\rm s}^2 \rho\right)^{1/3}$, and the characteristic observed transition time between the two regimes is $t_{\oplus,b} \approx 1.125\,(1+z)\left(E_0 c^3/[\rho c_{\rm s}^8 \zeta_{\rm m}^2]\right)^{1/3} \zeta_{\rm m}^{8/3}$, where z is the burst's redshift.

We assume that swept-up electrons are injected with a power law energy distribution $N(\mathcal{E}) \propto \mathcal{E}^{-p}$ for $\mathcal{E} = \gamma_{\rm e} m_{\rm e} c^2 > \mathcal{E}_{\rm min} \approx \xi_{\rm e} m_{\rm p} c^2 \Gamma$, with $p > 2$, and contain a fraction ξ_e of $E_{\rm int}$. This power law extends up to the cooling break, $\mathcal{E}_{\rm cool}$, at which energy the cooling time is comparable to the dynamical expansion time of the remnant. Above $\mathcal{E}_{\rm cool}$, the balance between electron injection (with $N_{\rm inj} \propto \mathcal{E}^{-p}$) and cooling gives $N(\mathcal{E}) \propto \mathcal{E}^{-(p+1)}$.

We also assume a tangled magnetic field containing a fraction ξ_B of $E_{\rm int}$. The comoving volume $V_{\rm co}$ and burster-frame volume V are related by $V_{\rm co} \approx V/\Gamma \propto M_{\rm acc}/\Gamma$, so that $B^2 = 8\pi \xi_B E_{\rm int}/V_{\rm co} \propto \Gamma^2$ and $B \propto \Gamma$.

The resulting spectrum has peak flux density $F_{\nu,\oplus,m} \propto \Gamma B M_{\rm acc}/\max(\zeta_{\rm m}^2, \Gamma^{-2})$ at an observed frequency $\nu_{\oplus,\rm m} \propto \Gamma B \mathcal{E}_{\rm min}^2/(1+z) \propto \Gamma^4/(1+z)$. Additional spectral features occur at the frequencies of optically thick synchrotron self absorption (which we shall neglect) and the cooling frequency $\nu_{\oplus,\rm cool}$ (which is important for optical observations of GRB 970508). The cooling break frequency follows from the relations $\gamma_{\rm cool} \approx (6\pi m_{\rm e} c)/(\sigma_{\rm T} \Gamma B^2 t_\oplus)$ (Sari et al. 1998; Wijers & Galama 1998) and $\nu_{\oplus,\rm cool} \propto \Gamma B \gamma_{\rm cool}^2 \propto (\Gamma^4 t_\oplus^2)^{-1}$. In the power law regime, $F_{\nu,\oplus,m} \propto t_\oplus^0$, $\nu_{\oplus,\rm m} \propto t_\oplus^{-3/2}$, and $\nu_{\oplus,\rm cool} \propto t_\oplus^{-1/2}$; while in the exponential regime, $F_{\nu,\oplus,m} \propto t_\oplus^{-1}$, $\nu_{\oplus,\rm m} \propto t_\oplus^{-2}$, and $\nu_{\oplus,\rm cool} \propto t_\oplus^0$. The spectrum is approximated by a broken power law, $F_\nu \propto \nu^{-\beta}$, with $\beta \approx -1/3$ for $\nu < \nu_{\oplus,\rm m}$, $\beta \approx (p-1)/2$ for $\nu_{\oplus,\rm m} < \nu < \nu_{\oplus,\rm cool}$, and $\beta \approx p/2$ for $\nu > \nu_{\oplus,\rm cool}$.

The afterglow light curve follows from the spectral shape and the time behavior of the break frequencies. Asymptotic slopes are given in Table 1. For the $\Gamma \sim 1/\zeta_{\rm m}$ regime, we study the evolution of break frequencies numerically. The results for $\nu_{\oplus,\rm m}$ and $F_{\nu,\oplus,m}$ are given in Paper I. For $\nu_{\oplus,\rm cool}$, a good approximation is

$$\nu_{\oplus,\rm cool} = \left[5.89\ 10^{13}\,(t_\oplus/t_{\oplus,b})^{-1/2} + 1.34\ 10^{14}\right]{\rm Hz}$$
$$\times \left(\frac{1}{1+z}\right)\left(\frac{c_{\rm s}}{c/\sqrt{3}}\right)^{17/6}\left(\frac{\xi_B}{0.1}\right)^{-3/2}$$

Table 1. Light curve exponents α as a function of frequency and time. Here $F_{\nu,\oplus} \propto t_\oplus^\alpha$

	$\nu_{\rm abs} < \nu < \nu_{\rm m}$	$\nu_{\rm m} < \nu < \nu_{\rm cool}$	$\nu_{\rm cool} < \nu$
$t_\oplus \ll t_{\oplus,b}$	1/2	3/4 − 3p/4	1/2 − 3p/4
$t_\oplus \gg t_{\oplus,b}$	−1/3	$-p$	$-p$

Table 2. Fitted break times $t_{\oplus,b}$ and magnitudes $R_{\rm c}(t_0)$ (at fiducial observed time $t_0 = 3.231$ day) for beamed GRB afterglow models for three pairs of acceptable host galaxy magnitude $R_{\rm H}$ and electron power law index p. The fit included all 43 data points with $1.3\,{\rm day} \le t_\oplus \le 95\,{\rm day}$ in the compilation by Garcia et al. (1998)

Model	$R_{\rm H}$	p	$\log(t_{\oplus,b}/\,{\rm day})$	$R_{\rm c}(t_0)$	χ^2/d.o.f.
1	25.74	2.36	9	20.32	4.34
2	25.55	2.20	5	20.33	3.80
3	25.36	2.04	3.75	20.32	3.55

$$\times \left(\frac{\rho \cdot {\rm cm}^3}{10^{-24}\,{\rm g}}\right)^{-5/6} \left(\frac{E_0/10^{53}\,{\rm erg}}{\zeta_{\rm m}^2/4}\right)^{-2/3} \left(\frac{\zeta_{\rm m}}{0.1}\right)^{-4/3} .$$

2. Application to GRB 970508

In the best-sampled GRB afterglow light curve yet available (the GRB 970508 R band data), the optical spectrum changed slope at $t_\oplus \sim 1.4$ day, suggesting the passage of the cooling break through the optical band (Galama et al. 1998). We explore the range of acceptable beaming angles for this burst by fitting the afterglow light curve for $1.3\,{\rm day} \le t_\oplus \le 95\,{\rm day}$ assuming that $\nu_{\oplus,{\rm cool}} < c/0.7\,\mu$m.

The range of acceptable energy distribution slopes p for swept-up electrons is taken from the optical colors. Precise measurements for $2\,{\rm day} \lesssim t_\oplus \lesssim 5\,{\rm day}$ give $F_\nu \propto \nu^{-\beta}$ with $\beta = 1.10 \pm 0.08$ (Zharikov et al. 1998), so that $p = 2.20 \pm 0.16$. We take this value to hold throughout the range $1.3\,{\rm day} \le t_\oplus \le 95\,{\rm day}$, thus assuming that p does not change as the afterglow evolves. We subtract the host galaxy flux ($R_{\rm H} = 25.55 \pm 0.19$; Zharikov et al. 1998) from all data points before fitting.

We fixed values of $R_{\rm H}$ and p, and then executed a grid search on the break time $t_{\oplus,b}$ and normalization of the model light curve. Results are summarized in Table 2 and Fig. 1. The final χ^2 per degree of freedom is ~ 4.

These large χ^2 values make meaningful error estimates on parameters difficult. Let us suppose χ^2 is large because details omitted from the models (clumps in the ambient medium or blast wave instabilities) affect the light curve, and so attach an uncertainty of 0.1 mag to each predicted flux. Adding this in quadrature to observational uncertainties when computing χ^2, we obtain $\chi^2/{\rm d.o.f.} \sim 1$. Error estimates based on changes in χ^2 then rule out $\log(t_{\oplus,b}/\,{\rm day}) < 3.5$ at about the 90% confidence level even for our "maximum beaming" case ($p = 2.04$, $R_{\rm H} = 25.36$).

To convert a supposed break time $t_{\oplus,b}$ into a beaming angle $\zeta_{\rm m}$, we need estimates of the burst energy per steradian and the ambient density. Wijers & Galama (1998) infer $E_0/\Omega = 3.7\ 10^{52}\,{\rm erg}/(4\pi\,{\rm Sr})$ and $\rho = 5.8\ 10^{-26}\,{\rm g/cm}^3$. Combining these values with $t_{\oplus,b} \gtrsim 10^{3.5}$ day gives $\zeta_{\rm m} \gtrsim 0.5\,{\rm rad} \approx 30\,{\rm deg}$. E_0/Ω and ρ are substantially uncertain, but because $\zeta_{\rm m} \propto (\rho/E_0)^{1/8}$, the error budget for $\zeta_{\rm m}$ is dominated by uncertainties in p rather than in E_0 or ρ.

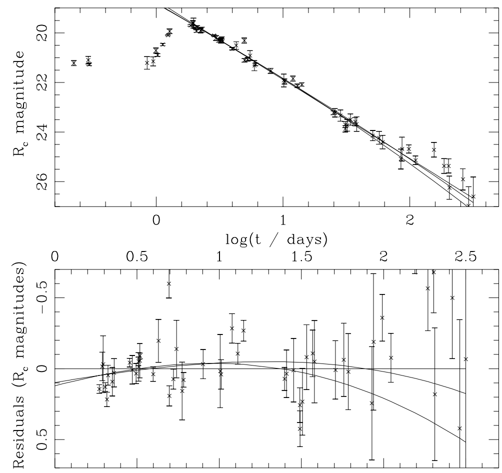

Fig. 1. Upper panel: The Cousins R band light curve for GRB 970508 with the three fits shown in Table 1. Lower panel: Residuals for the data and for models 2 and 3 (in order of increasing curvature) relative to model 1. A host galaxy flux corresponding to $R_{\rm H} = 25.55$ has been subtracted from all data points

This beaming limit implies $\Omega \ge 0.75\,{\rm Sr}$, which is 6% of the sky. GRB 970508 was at $z \ge 0.835$ (Metzger et al. 1997). We then find gamma ray energy $E_\gamma = 2.8\ 10^{50}\,{\rm erg} \times (\Omega/0.75\,{\rm Sr})(d_{\rm L}/4.82\,{\rm Gpc})^2\,(1.835/[1+z])$. If the afterglow is primarily powered by different ejecta from the initial GRB, as when a "slow" wind ($\Gamma_0 \sim 10$) dominates the ejecta energy, then our beaming limit applies only to the afterglow emission. The optical fluence implies $E_{\rm opt} = 3.4\ 10^{49}\ {\rm erg} \times (\Omega/0.75\,{\rm Sr})(d_{\rm L}/4.82\,{\rm Gpc})^2\,(1.835/[1+z])$. The irreducible minimum energy is thus $3.4\ 10^{49}$ erg, using the smallest possible redshift and beaming angle. We have reduced the beaming uncertainty, from the factor $\sim \Gamma_0^2 \sim 300^2 \sim 10^5$ allowed by γ-ray observations alone to a factor $(4\pi\,{\rm Sr})/(0.75\,{\rm Sr}) \sim 20$, and thus obtain the most rigorous lower limit on GRB energy requirements yet.

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