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Evolution of the "efficiency" when different data sets are combined into a unified adjustment

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Abstract. Using a sample of binaries which were observed visually (VB) as well as spectroscopically for both components (SB2), we show that the efficiency almost always increases when a combined VB-SB2 solution replaces one based only on VB.

Key words: methods: numerical; statistical — binaries: visual; spectroscopic

1. Introduction

The complete solution of every parameter estimation by the adjustment of data (observations) through minimizing an objective function (e.g., the sum of the squares of the adjustment residuals) yields not only estimates for the target parameters, but also their covariance matrix. This matrix may be interpreted as the coefficient matrix of the quadratic form representing the error ellipsoid of the target-parameter estimates.

There are two quantities which measure the appropriateness of the available equations used for adjusting the data and estimating the target parameters. The first is the condition number of the system of equations from which the parameter estimates were calculated, it is essentially the ratio of the longest to the shortest principal axes of the error ellipsoid. The second is the efficiency, introduced by Eichhorn (1989), which is a measure for how well the main axes of the parameter estimates' error ellipsoid are aligned with those of a rectangular coordinate system whose axes are along the original target parameters. Efficiency and condition number are thus independent of each other: The introduction of new parameters by subjecting the original target parameters to an orthogonal transformation is equivalent to rotating the error ellipsoid with respect to the coordinate system and will always create a set of uncorrelated parameters with efficiency 1, while leaving the condition number unchanged. Independently thereof, the condition number can be rendered equal to 1 by scaling, although it is hard to appreciate any advantage one might thereby derive.

Likewise, the set of orthogonally transformed original set of target parameters, which renders the problem efficiency equal to 1, is not necessarily of interest; if, e.g., one wants to have estimates of the masses of the components of a binary with the highest possible precision, it is little comfort to know instead the sum of these masses, which can be estimated much more precisely than any of the individual masses.

Seen from a different perspective, the efficiency is a measure for the total amount of correlation between the estimates of the original target parameters as they result from the fit of a certain data set. The efficiency was first used by Eichhorn & Xu (1990) in discussing estimations of the parameters of visual binaries and is defined by

$$\epsilon = \sqrt[m]{\frac{\prod_{k=1}^{m} \lambda_k}{\prod_{k=1}^{m} q_{kk}}},$$

where λ_k are the eigenvalues of the covariance matrix $\mathbf{Q}$ of the estimated parameters, and q_{kk} are its diagonal elements. In this expression, m denotes the number of parameters in the model.

Other than by introducing more data (observations) of the same type as the original ones, the precision of the original target data (and concomitantly the efficiency) can be improved by introducing data of a different kind, whose adjustment parameters contain a nonempty subset of the original target parameters, and then fitting both sets of data simultaneously in a combined estimation. In this solution, the set of parameters will then be the union of the sets of the parameters of the original problem and those of the new one.

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Take, for example, a binary that has been observed both visually as well as for both components spectroscopically. One would then intuitively expect, that those parameters which are necessary for the fit of both data sets will be estimated with greater precision from a unified solution than from the weighted mean of the discrete estimates obtained by having fitted visual and spectroscopic observations separately. In addition, there are system parameters (e.g., the orbital inclination, the radial velocity of the system's center of gravity) that can be estimated only from analyzing one type of observations but not the other. Other parameters, such as the parallax of the system, can be estimated only from analyzing both data sets simultaneously in a unified solution. The efficiency of combined VB-SB2 solutions would therefore increase in comparison to the one achieved in any of the separate solutions.

Our aim is thus to analyze the behavior of the efficiency on a reasonably large set of double-lined spectroscopic-visual systems to check on the conjecture we have just stated, i.e. to ask: Does the efficiency increase as expected when a VB system is upgraded to a VB-SB2 one?

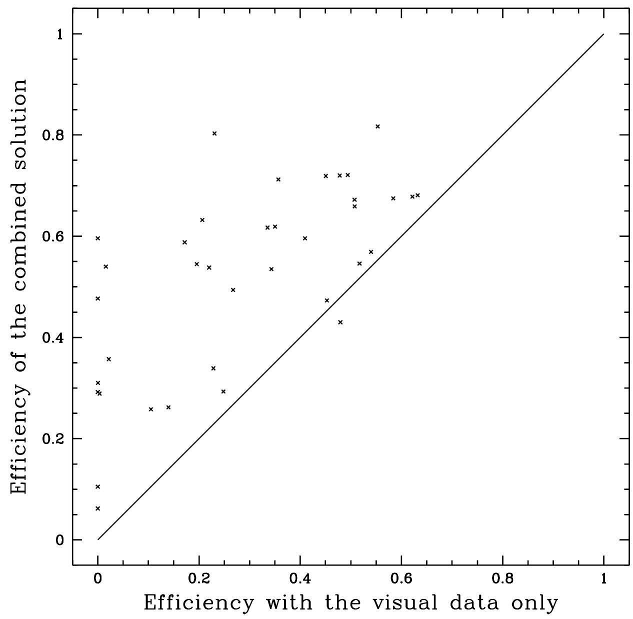

Fig. 1. Evolution of the efficiency when SB2 data are added to a set of VB observations

2. Test procedure

We therefore selected the largest set of VB-SB2 objects for which there are enough observations to undertake such an orbit determination. The resulting set contains the following 38 systems (the numbers refer to the Hipparcos catalogue Hipparcos): 677, 2941, 4463, 7580, 8903, 10064, 10644, 10952, 12390, 12623, 14328, 24608, 28360, 38382, 45170, 46404, 57565, 65378, 71683/1, 73182, 75312, 85667, 87895, 88601, 89937, 91636, 95995, 96683, 98416, 99376, 99473, 103655, 104858, 104987, 108917, 111170, 111528 and 114576. We refer to Poubaix (1998a) for the complete description of the different data sets used in this analysis.

For all systems (except HIP 111170, for which there are only two visual observations available so that a VB solution cannot be computed), there are enough observations to estimate system parameters independently, i.e., determine orbits independently from the visual and spectroscopic observations. The methods used to derive the orbital parameters were described elsewhere Pourbaix (1994, 1998a). The ten parameters used to model the VB-SB2 data supersede those of the VB observations. One can thus directly analyze the evolution of the efficiency from one data set to the other.

As shown in Fig. 1, the efficiency does grow for all binaries but one. Note particularly that even if the efficiency is almost 0 for a VB solution, the efficiency of a VB-SB2 solution for the same object can go up to an impressive 0.6. This confirms that adding radial velocities does indeed nearly always reduce the correlation between the orbital parameters. There is, however, one exception: HIP 91636. Is this just a numerical accident? Is such behavior predictable?

If we assume (for the sake of illustration) that there is strictly no correlation between the seven parameters of the visual orbit, the efficiency would be equal to 1. Radial velocities could therefore not introduce any correlations among the VB-parameters. If, however, the radial velocities cover only a small part of the curves, one could have a high correlation between V_0, the radial velocity of the system's center of gravity and κ, the fractional mass. It is then likely for the efficiency computed on the whole data set to decrease. It is indeed impossible that the visual observations exhibit such a correlation since it holds only between parameters not occurring in the visual model. This is exactly the situation with HIP 91636.

Thus we confirm that the efficiency of combined solutions is larger than that of individual ones except in freak cases, and that VB-SB2 solutions allow one the estimation of system parameters (in particular, parallaxes and masses of the individual components) which could never be obtained from adjusting only visual observations without some extraneous material.

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