No Laser Guide Stars for adaptive optics in giant telescopes?

R. Ragazzoni

Astronomical Observatory of Padova, vicolo dell'Osservatorio 5, I–35122 Padova, Italy
e-mail: ragazzoni@pd.astro.it

Abstract. In order to get diffraction limited imaging on large telescopes using Laser Guide Stars one needs to fire several such artificial beacons angularly displaced with respect to the science target. Such displacement scales with the diameter of the telescope. It is pointed out that when this angle exceeds some given figure it can be more efficient to use directly some off–axis Natural Guide Stars. With realistic assumptions it turns out that next generation giant telescopes could benefit from this technique and could not require any Laser Guide Star.

Key words: telescopes — instrumentation: adaptive optics

1. Introduction

The Keck telescopes (Gillingham 1997) have demonstrated operationally the long held dream (see for instance Horn D'Arturo 1955) of providing very large mirror surfaces using a huge number of small mirrors co–aligned and co–phased actively in order to mimic a single telescope aperture. In principle there is no limit (other than the cost) to the size of these telescopes and several studies on giant telescopes ranging from 25 to 100 m (Angel 1990; Ardeberg et al. 1992; Ardeberg et al. 1998; Diericx & Ardeberg 1998; Gilmozzi et al. 1998; Owner–Petersen 1996; Sebring et al. 1998) have appeared in the literature. While photon collection gain is more than enough to make these giants an attractive idea, it is only through adaptive optics that one would obtain λ/D diffraction limited capabilities on such instruments. With the exception of some specific (but very appealing, like imaging of extra–solar planets, see for instance Angel 1994) cases, sky coverage is severely limited if one needs to use on–axis Natural Guide Stars (NGSs) as in the conventional adaptive optics schemes.

Laser Guide Stars (LGSs) are at a first glance an obvious choice to overcome such a limitation (Foy & Labeyrie 1985). Assuming that laser firing, Rayleigh pollution and absolute tip–tilt indetermination (Pilkington 1987) problems are somewhat solved through one of the several approaches that have appeared in the literature (Baruffolo et al. 1998; Belen'kii 1994, 1995, 1996; Esposito et al. 1998; Foy et al. 1992, 1995; Lukin 1996; Ragazzoni 1995, 1996ab, 1997; Ragazzoni et al. 1995; Ragazzoni & Rigaut 1998; Riccardi et al. 1997; Rigaut & Gendron 1992; Whiteley et al. 1998), one is still faced with conical anisoplanatism problem (Gardner et al. 1990; Esposito et al. 1996; Neymann 1996; Ragazzoni et al. 1998) due to the finite height of the LGS itself (Happer et al. 1994).

Fried & Belsher (1994) defined a diameter d_0 such that conical anisoplanatism can be neglected. For any reasonable ground–based site this d_0 is expected to be of the order of a few meters. Stitching or butting (Fried 1995) to solve the conical problem requires approximately $(D/d_0)^2$ LGSs. Even assuming that these LGSs could be generated by Rayleigh scattering (a somewhat less expensive and technically easier choice), at least few hundreds of LGSs should be fired and sensed making this approach a formidable one. The tomographic approach (Tallon & Foy 1990; Ragazzoni et al. 1999) to solve conical anisoplanatism is much easier. The number of LGSs is of the same order of the number of significant layers. With something like ≈ 4 LGSs fired at significantly different directions in the sky one could realize a LGSs adaptive optics system for such giant telescopes. This approach is not free from technical problems. The angles at which the LGSs are fired scales with the telescope diameter D and in order to recover these one would need to look several arcminutes away from the optical axis of the instrument. That translates into meters on the focal plane scale and would possibly require dedicated off-axis optical relays to compensate for the (presumably huge) aberrations at such large distance from the optical axis.

I pointed out (see for instance Ref. 19 in Gilmozzi et al. 1998) that the angular sky area where the LGSs are fired can be so large, when the diameter D increases, that one could have some reasonable chance of finding a

corresponding number of NGSs (see Fig. 1). This raises the hope that under certain conditions and for a telescope aperture with a diameter larger than a given size one could realize nearly all–sky coverage adaptive optics without the need of any LGS!

The absolute tip–tilt problem is automatically solved in this case. Tomographyc retrieval would not be more difficult than using LGSs (maybe even simpler because of the cylindrical shape of the NGS's beams instead of the LGS's conical ones).

This paper tries to give an answer to the following questions: i) "At which telescope diameter D_c this technique is more effective than traditional adaptive optics in terms of sky coverage?" and ii) "At which telescope diameter D_{50} (or D_{90}) are NGSs solely are able to provide some reasonable sky coverage of, say 50% (or 90%)?"

2. NGSs brightness

The limiting magnitude for NGS–based adaptive optics system can be easily computed by both numerical and analytical formulations (Rigaut et al. 1997). Matching with experimental results is relatively satisfactory (Roddier 1998). In the LGS case the extension is straightforward only accepting conical anisoplanatism degradation and omitting the tip–tilt treatment. Few experimental results are available to confirm such results. To my knowledge there is no published result on noise propagation in tomographic wavefront sensing. It should be pointed out that in the modal formulation given by Ragazzoni et al. (1999) it would be easier to approach the problem in an analytical way because error-propagation in matrices involves straightforward matrices manipulation. A careful analysis and numerical confirmation, however, still requires a thorough discussion that is beyond the scope of this work.

At the current state of knowledge it is very hard to make a firm projection of the limiting magnitude for tomographic wavefront sensing. In this section I try to outline some considerations useful for giving a rough estimate of such limiting magnitude.

I assume that the following statements are true:

1. The strongest perturbing layer is located at, or very close to, the ground level;
2. The number $\mathcal{N}$ of relevant perturbing layers is equal to the number of sensed NGSs;
3. No portion of the highest layer intersected by the scientific cylindrical beam is unsampled.

Let us suppose that, under some given conditions for the Fried parameter r_0, Greenwood frequency f_G, overall quantum efficiency of the wavefront sensing system q and its spectral bandwidth $\Delta\lambda$ expressed in nm, a certain zonal error in the wavefront correction σ_0 is obtained when a star of magnitude V_0 is used to close a *classical* adaptive optics loop. For each coherence zone (sized r_0)

of the collected wavefront and in a single coherence time (of the order of $1/f_G$), the number of collected photons N_0 will be given by (Zombeck 1990):

$$N_0 \approx 10^8 \; 10^{-0.4V_0} r_0^2 \frac{\Delta\lambda q}{f_G}. \tag{1}$$

We suppose also that the wavefront perturbation coming from the $\mathcal{N}$ layers is representative of most of the total turbulence experienced by the starlight coming into the telescope (grouping of very close layers can improve the effectiveness of the technique).

Each of these layers is numbered from 1 (the lowest) to $\mathcal{N}$ (the highest) and will be interested by an average number of NGSs given by n_i. For instance $n_1 = \mathcal{N}$ using the condition 1 and $n_{\mathcal{N}} \geq 1$ using the condition 3. For each layer one can define a r_{0i} characteristic of the wavefront passing just through that portion of turbulence.

Supposing the use of a star of magnitude V_0 to sense directly just the i–th single layer a number of photons given by:

$$N = N_0 \left(\frac{r_{0i}}{r_0} \right)^2 \tag{2}$$

for each spatial coherence zone could be used. Since the variance error is proportional to the inverse of the square root of the number of collected photons (in the Poissonian dominated error approximation that we suppose reached by some state–of–the–art wavefront sensor) it turns out that the wavefront would be sensed with a zonal error σ_i given by:

$$\sigma_i = \sigma_0 \frac{r_0}{r_{0i}}. \tag{3}$$

How do these errors couple together when a tomographic approach is used? A pessimistic approach would tell us that sensing of a layer intersected by n_i NGSs will be affected by the sum of the errors coming from these sources. On the other hand an *optimistic* approach will tell us the opposite: averaging over n_i NGSs a gain is obtained because the related errors tend to cancel out. These considerations can be written in a compact form using the following:

$$\left(\frac{\sigma}{\sigma_0} \right)^2 = \sum_{i=1}^{N} n_i^\alpha \left(\frac{r_0}{r_{0i}} \right)^2 \tag{4}$$

where $\alpha = 1$ represent the pessimistic case and $\alpha = -1$ the optimistic one. The summation of the errors is incoherent. The case for $\alpha = \pm 2$ represent the (unlikely, and not treated here) case of coherent superpositions of the errors. In Eq. (4) it has been implicitly assumed that the weights for each zonal sensing are of the order of the unity. Of course this is a rather unjustified assumption. At this stage one can only imagine that very different weights combine together to mimic the situation coarsely expressed by Eq. (4). On the other hand the variations due to the choice on α span less than one order of magnitude in the σ/σ_0 ratio.
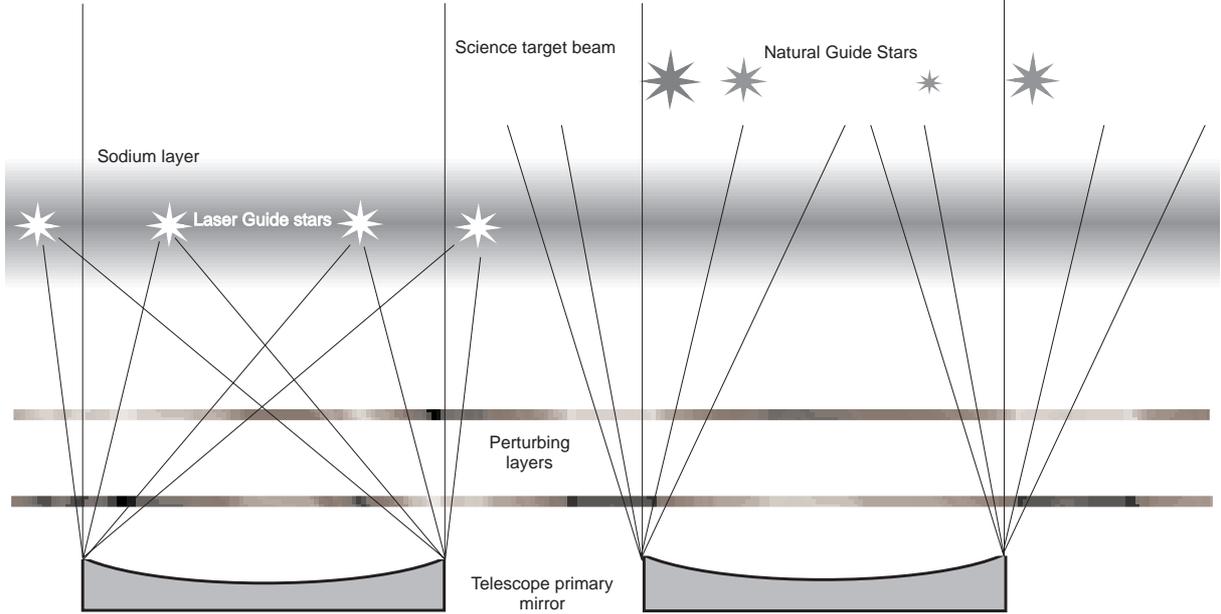
Fig. 1. In this schematic representation the LGS–based tomographic technique (left) is illustrated in a sectional view. In this case LGSs can be fired at some precise locations on the sky and some conical beams *explore* the various perturbing layers. Tilt is to be retrieved in some other way. On the right side the NGS–based case is briefly sketched: the stars have different brightness and are unevenly located on the sky. On the other hand the beams are cylindrical and tip–tilt is retained

In order to reach with the tomographic approach the same zonal error as the classical single guide star approach, a limiting magnitude for each of the $\mathcal{N}$ NGS (the condition 2 is used here) of:

$$V = V_0 + \Delta V = V_0 - 2.5 \log \left(\frac{\sigma}{\sigma_0} \right)^2 \qquad (5)$$

must be obtained (here ΔV represent the variation due to the approach used). Using $r_0 = 0.25$ m, $f_G = 200$ Hz, $q = 0.5$, $\Delta\lambda = 300$ nm and an error $\sigma_0 = 1$ radian of wavefront phase, we can determine the number of required photocounts $N_0 \approx 29$, being (Kern et al. 1989):

$$N_0 = \frac{2\pi^2}{\sigma_0^2} \qquad (6)$$

and, by inversion of Eq. (1) a limiting magnitude $V_0 = 13.0$ is obtained. In order to estimate Eq. (4) we use two of the models by Roggermann et al. (1995), namely the SLC–N and the HV–21, using $\mathcal{N} = 4$, $n_1 = n_2 = 4$, $n_3 = 2$ and $n_4 = 1$ and scaling the various r_{0i} in order to match the overall $r_0 = 0.25$ m and reported for convenience of the reader in Table 1. Equations (4) and (5) gives $\Delta V = -1.3$ (a brighter limiting magnitude) for the $\alpha = 1$ pessimistic case and $\Delta V = +1.6$ (the option to look for fainter NGSs) for the $\alpha = -1$ optimistic case in the SLC–N model. For the HV–21 model $\Delta V = -1.4$ ($\alpha = 1$) and $\Delta V = +1.5$ ($\alpha = -1$).

Conservatively we assume in the following $\Delta V = \pm 1.0$, that is $V = 12.0$ or $V = 14.0$ as extrema of limiting magnitudes for the NGSs to be used in the tomographic approach. While no attempt is made to use many more fainter NGSs to mimic fewer brighter NGSs (an approach

that could widen the practicability of the proposed approach) the approximate and tentative nature of these calculations must be reiterated. Also, it should be pointed out that essentially no literature is available on possible techniques for efficient wavefront sensing and correction in the tomographic mode and there is no evidence that straightforward sensing with classical wavefront sensor of each star to be coupled together in a dedicated wavefront computer represent the ultimate achievable performance. Moreover I have not speculated on the possibility of using NGSs to directly retrieve tomographic information as suggested for instance by Ribak (1995).

Table 1. The Fried parameters for the four layers model used in the text along with the right hand side of Eq. (4)

	r_{01}	r_{02}	r_{03}	r_{04}	$\left(\frac{\sigma}{\sigma_0}\right)^2_{\alpha=1}$	$\left(\frac{\sigma}{\sigma_0}\right)^2_{\alpha=-1}$
SLC–N	0.381	0.397	2.06	3.69	1.83	0.478
HV–21	0.268	1.62	1.36	5.18	1.91	0.493

3. Sky coverage

In order to fulfile condition 3 the NGSs are to be located within an angle $D/(2h_{\max})$ from the optical axes, where $h_{\max}$ is the altitude of the highest relevant perturbing layer (here and in the following we assume zenital or

nearly zenital observations). The area where the $\mathcal{N}$ NGSs stars are to be found is given, expressed in square degrees, by the following:

$$S = \pi \left(\frac{180}{\pi} \times \frac{D}{2h_{\max}} \right)^2 \approx 7.96 \, 10^{-6} \, D^2. \tag{7}$$

The numerical estimation is based upon the Roggermann et al. (1995) model, assuming $h_{\max} = 1.8 \, 10^4$ m and hence D is hereafter expressed in meters.

In contrast, recall that the usable area for *classical* NGS–based adaptive optics system is characterized by a circular zone of radius θ_0, the so called isoplanatic patch, of size $0.3r_0/h_{\mathrm{avg}}$. Assuming an average $h_{\mathrm{avg}} \approx 1.25 \, 10^3$ m for both the SLC–N and HV–21 models (the two give respectively $1.53 \, 10^3$ m and $9.73 \, 10^2$ m for h_{avg}) a numerical estimation can be made also:

$$S_{\mathrm{c}} = \pi \left(\frac{180}{\pi} \times \frac{0.3r_0}{h_{\mathrm{avg}}} \right)^2 \approx 3.71 \, 10^{-5} \tag{8}$$

where again the result is given in square degrees.

It should be pointed out in the latter that a single suitable NGS is to be found. However it is also remarkable that the points of the sky satisfying this last condition are *biased* by the presence of a relatively bright NGS within a small angle. Because of light scattering (or, at least, to the non negligible extension of the PSF) the sky background will be affected by some light negatively impacting extremely deep imaging. The classical sky coverage, or probability to find out a suitable NGS, is given by:

$$P_{\mathrm{c}} = 1 - \exp(-S_{\mathrm{c}}\rho) \tag{9}$$

regardless of the telescope diameter D. Using a limiting magnitude $V_0 = 13.0$ as reported in Sect. 2, sky coverage of $P_{\mathrm{c}} \approx 0.2\%$ for the Galactic poles ($b = 90°$) and $P_{\mathrm{c}} \approx 2\%$ for the Galactic plane ($b = 0°$) are retained.

In the tomographic case $\mathcal{N}$ stars are to be found and the probabilities composed in a multiplicative manner:

$$P = [1 - \exp(-S\rho)]^{\mathcal{N}}. \tag{10}$$

Using the numerical estimation given in Eq. (7):

$$P \approx \left[1 - \exp\left(-7.96 \, 10^{-6} D^2 \rho \right) \right]^{\mathcal{N}} \tag{11}$$

where one can note the dependence both from ρ and D. Inversion of Eq. (11) for D gives the following:

$$D_{\mathrm{P}} \approx \sqrt{\frac{-\ln\left(1 - P^{1/\mathcal{N}}\right)}{7.96 \, 10^{-6}\rho}}. \tag{12}$$

Imposing $P = 0.50$ or $P = 0.90$ one can find the diameter where 50% and 90% of sky coverage is reached:

$$D_{50} \approx \frac{482}{\sqrt{\rho}}; \quad D_{90} \approx \frac{680}{\sqrt{\rho}}. \tag{13}$$

Solving Eq. (12) for the classical NGS–based probabilities ($P = 0.02$ for $b = 0°$ and $P = 0.002$ for $b = 90°$) one can find out also the critical diameter D_{c} as defined in the first section. All these results are summarized in Table 2.

Equation (10) and followings do not impose any particular geometry for the $\mathcal{N}$ stars. Hence there is some chance that these stars are placed on the sky in a way that avoids properly sensing some portion of the highest layers. This problem is only mentioned here.
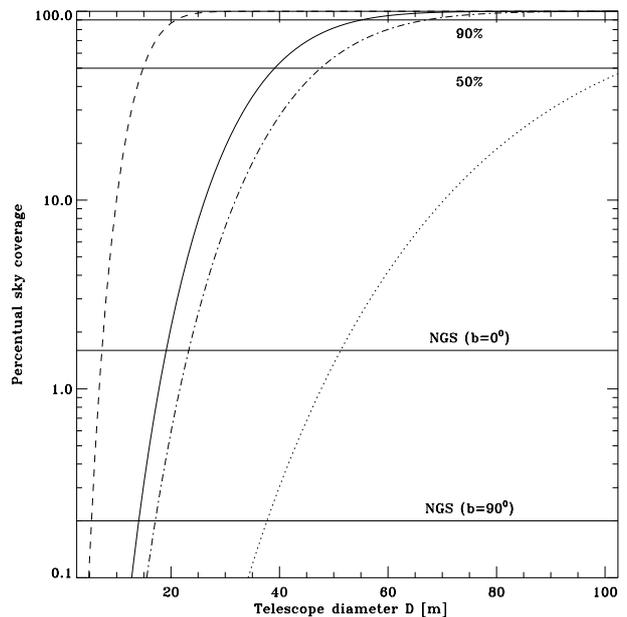

Fig. 2. Sky coverage for various conditions vs. telescope diameter. Continuous line is for $V = 12$, $b = 0°$; dotted line is for $V = 12$, $b = 90°$; dashed line is for $V = 14$, $b = 0°$; dotted and dashed line is for $V = 14$, $b = 90°$. Note for comparison the sky coverage obtained by classical NGS–based adaptive optics. The lines indicating 50% and 90% of sky coverage are also reported

Table 2. The diameter of the telescopes, expressed in meters, where NGS–based tomographic technique provides sky coverages close to the NGS–based classic adaptive optics (D_{c}), where the sky coverage reaches 50% (D_{50}) and 90% (D_{90}) for different limiting magnitude V and galactic latitude b. In the second column the average number of stars per square degree, brighter than V is given

	ρ	D_{c}	D_{50}	D_{90}
$V = 12$, $b = 0°$	151	19.2	39.2	55.3
$V = 12$, $b = 90°$	21	37.7	105	148
$V = 14$, $b = 0°$	1047	7.29	14.9	21.0
$V = 14$, $b = 90°$	102	17.1	47.7	67.3

4. Conclusions

The results shown here, in spite of their preliminarity and somewhat roughness of their derivation, are showing a scenario where LGSs are no longer necessary for giant telescopes. The borderline lies somewhere between few tens of m to one hundred meters, depending upon several assumptions. The exact threshold diameter depends upon a number of seeing and performance requirement parameters, and it is hard to outline further conclusions without assuming some specific case study. It is also noticeable that multiple wavefront sensing is still in its infancy and it cannot be excluded that some substantial improvement can be obtained by some novel technique. What are

fundamental limits of this technique and is it possible that the threshold diameter could make this technique an attractive alternative even for current 8 m class telescopes is beyond the scope of this paper but it is, of course, a very interesting and somewhat fascinating issue to be attacked urgently.

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