ASTRONOMY & ASTROPHYSICS
SUPPLEMENT SERIES
Astron. Astrophys. Suppl. Ser. **135**, 383–390 (1999)

An improved method for the determination of the orbital parameters of a binary system that contains a pulsating component

G. Molenberghs[1], **C. Aerts**[2,⋆], **and P. De Cat**[2]

[1] Biostatistics, Limburgs Universitair Centrum, Universitaire Campus, Building D, B-3590 Diepenbeek, Belgium
[2] Instituut voor Sterrenkunde, Katholieke Universiteit Leuven, Celestijnenlaan 200 B, B-3001 Heverlee, Belgium

Received September 7; accepted November 3, 1998

Abstract. In this paper we present a method to assign statistical weights to radial-velocity measurements of a binary system of which one of the components is a variable star. The basic idea of the method is to separate the radial-velocity changes due to the intrinsic variability from those due to the orbital motion. This can be achieved if part of the data set consists of good coverages of the intrinsic variability cycle. These full coverages are used to estimate the variability for the nights on which only one or a few spectra were obtained. Our technique is applicable when the intrinsic variability has a period that is considerably shorter than the orbital period.

Once weights have been assigned, existing methods to derive the orbital parameters can be used with better accuracy compared to the case where all data points are treated as if no intrinsic variability were present (equal weights). We illustrate our method and compare the orbital solution obtained with and without assigning weights for three pulsating stars in a binary system: β Cru, ε Per, and κ Sco.

Key words: methods: statistical — stars: oscillations — binaries: spectroscopic

1. Introduction

The determination of the orbital parameters of the components of a binary has been the subject of many investigations. It is not our intention to treat these existing methods, but rather to focus on one particular case, namely the determination of the orbital parameters of a binary that consists of at least one intrinsically variable component. All existing methods developed so far do not take into account the fact that intrinsic variability, such as pulsation, can significantly influence the radial velocities. In cases where the pulsation results in a peak-to-peak amplitude which is comparable to, or a large fraction of, the orbital amplitude, accurate orbital parameters are hard to determine. In such a case, the intrinsic variability cycle needs to be fully covered with observations in order to be able to determine the average radial velocity due to the binary motion. Having a full cover of the variability cycle is, however, a condition that is often not fulfilled.

The usual criterion of having found the best orbit when the rms is minimal breaks down in the case that one of the components exhibits intrinsic variability, especially when the intrinsic and extrinsic variations have comparable amplitudes. In this paper, we develop a method to treat cases for which single radial-velocity measurements are combined with radial-velocity data that do cover a complete pulsation cycle.

Our work originates from a systematic observational study of line-profile variability in pulsating B-type stars of which more and more targets turn out to be a member of a binary system (Aerts et al. 1998b). These variable stars have pulsation periods between a few hours and a few days and are often multiperiodic. This intrinsic variability is combined with extrinsic variability due to a companion. Orbital periods range from a few days up to years. Our main aim of observing these stars is to disentangle the pulsational behaviour by studying the line-profile variations caused by the pulsation(s) in full detail. Since the study of line-profile variability demands rather large telescopes with accurate detectors, observing runs typically extend over only a few days or weeks at best. The (often unknown) binary nature of the objects can prevent a determination of the pulsational parameters. The usual strategy then is to complete the data set that was gathered to study the pulsations with single measurements that are scattered in time and that should allow a determination of the orbital parameters. Once the latter are known, the effect of the binary motion can be subtracted from the data to start a study of the pulsational behaviour.

⋆ Postdoctoral Fellow, Fund for Scientific Research, Flanders.

Well-known examples of pulsating stars that have been monitored extensively with the aim to derive the pulsational parameters and that were known or turn out to be members of a binary are σ Sco (Levee 1952; Mathias et al. 1991), ε Per (Smith et al. 1987; Gies & Kullavanijaya 1988), λ Sco (Lomb & Shobbrook 1975; De Mey et al. 1997), δ Sco (Telting & Schrijvers 1997), α Vir (Smith 1985a,b), θ^2 Tau (Kennelly et al. 1996); θ Tuc (De Mey et al. 1998). For most of these cases, the pulsational analysis was or still is limited due to the lack of accurate orbital parameters. On the other hand, more and more binaries turn out to have a variable component, while the orbital parameters were derived assuming that both components are constant stars. Examples of the latter situation are found in the case of η Ori (Waelkens & Lampens 1988; De Mey et al. 1996), and are presented for V 436 Per by Harmanec et al. (1997) and for β Sco A by Holmgren et al. (1997). The latter two stars are targets of the so-called SEFONO project introduced by Harmanec et al. (1997). This project concerns a search for forced oscillations in close binaries by means of a search for line-profile variability in one of the components. It is to be expected that more pulsating stars in well-known binaries will soon be encountered in connection with this project. Indeed, in many of the target stars line-profile variability is confirmed or suspected (Harmanec, private communication). The question then is how these variable line profiles affect the determination of the orbital parameters, since the latter have been derived before the intrinsic variability was known.

In this paper, we focus on the determination of the orbital parameters in the case that one part of the radial-velocity data set consists of numerous observations that are concentrated on a few intrinsic variability cycles, while the other part of the data are single observations taken at random during subsequent variability cycles. The aim is to give each radial-velocity measurement a weight according to its ability to predict the radial velocity due to the binary motion. It is clear that the data of a fully covered pulsation cycle result in a much better predictor of the binary radial velocity than the scattered data points. We show how one can combine a single radial-velocity measurement with the fully covered cycles to give a better prediction of the radial velocity due to the binary motion. In Sect. 2, we describe the statistical methodology to derive the predictions of the binary radial-velocity and its standard error. A simulation study is performed to study the accuracy of the method. This is described in Sect. 3. The standard errors of the predictors of the binary radial velocity are then used as weights in existing methods to derive the orbital parameters. Application to the β Cep stars β Cru and κ Sco and to the star ε Per are given in Sect. 4. Finally, we end with some concluding remarks in Sect. 5.

2. Statistical methodology

Denote the radial velocity (RV) at time t, measured at wavelength λ by $v_{t\lambda}$ and the true underlying RV by v_t. Further, let β_0 be the average RV. A two-stage statistical model is then

$$
\begin{aligned}
v_{t\lambda_j} &= v_t + \varepsilon_{t\lambda_j}, & (1)\\
v_t &= \beta_0 + f(t|\gamma) + a(t), & (2)
\end{aligned}
$$

$(j = 1, \ldots, n)$, where $\varepsilon_{t\lambda_j}$ is a normally distributed error term with variance $\sigma^2_{v_t}$, due to measurement error and wavelength calibration, $f(t|\gamma)$ describes the orbital motion where γ groups the orbital parameters, and $a(t)$ is a periodic fluctuation of v_t due to the pulsation of the star. A general expression for $a(t)$ is

$$a(t) = \sum_{k=1}^{K} \alpha_k \sin(\omega_k t + \phi_k). \quad (3)$$

Here, α_k, ω_k, and ϕ_k are the amplitude, the frequency, and the phase of the radial velocity due to the kth pulsation mode. It is assumed that the frequency of $f(t|\gamma)$ is much smaller than the pulsation frequencies ω_k. This assumption ensures that it is reasonable to consider $f(t|\gamma)$ constant during a time period within which the sinusoidal terms complete their cycles. Clearly, Eqs. (1) and (2) can be combined into a single one:

$$v_{t\lambda_j} = \beta_0 + f(t|\gamma) + a(t) + \varepsilon_{t\lambda_j}. \quad (4)$$

When solely the estimation of v_t for fixed t is of interest, it is sufficient to obtain a number of replications at different wavelengths $\lambda_1, \ldots, \lambda_n$, whereafter they are averaged to yield $\overline{v}_t$. The variance is equal to

$$\sigma^2_{v_t} = \frac{1}{n-1} \sum_{j=1}^{n} (v_{t\lambda_j} - v_t)^2. \quad (5)$$

Now, $\overline{v}_t$ is an unbiased estimator for v_t since the expectation of $\varepsilon_{t\lambda}$ is assumed to be zero. Its standard deviation is estimated by replacing v_t in (5) with $\overline{v}_t$, and taking the square root. In order to obtain the standard error we divide this expression further by n.

Moreover, $\overline{v}_t$ is unbiased for the orbital motion $\beta(t) = \beta_0 + f(t|\gamma)$ since in addition $a(t)$ is zero on average. In contrast, the variance estimators are different in both situations since $\sigma^2_{v_t}$ fails to acknowledge pulsational variability. Indeed, $\sigma^2_{v_t}$ is the variance of $v_{t\lambda}$ around v_t, while we are now interested in the spread of $v_{t\lambda}$ around $\beta(t)$.

Since the pulsational variability and the error term $\varepsilon_{t\lambda}$ are statistically independent, Eq. (4) yields

$$
\begin{aligned}
\sigma^2_{\beta(t)} &= E[(v_{t\lambda} - \beta(t))^2] & (6)\\
&= \mathrm{var}[a(t)] + \mathrm{var}(\varepsilon_{t\lambda}) & \\
&= \mathrm{var}[a(t)] + \sigma^2_{v_t} & (7)\\
&= \mathrm{var}[a(t)] + \frac{1}{n-1} \sum_{j=1}^{n} (v_{t\lambda_j} - v_t)^2. & (8)
\end{aligned}
$$

When interest lies in the accuracy of the estimator $\widehat{\beta(t)} = \overline{v}_t$ one should compute

$$\begin{aligned}\mathrm{var}(\widehat{\beta(t)}) &= E[(\overline{v}_t - \beta(t))^2] \\ &= \mathrm{var}\left(\frac{1}{n}\sum_{j=1}^{n}[\beta(t) + a(t) + \varepsilon_{t\lambda_j}] - \beta(t)\right) \\ &= \mathrm{var}[a(t)] + \frac{1}{n}\sigma^2_{v_t}. \end{aligned} \quad (9)$$

These variances will be used as weights for the determination of the unknown orbital parameters.

Available data can consist of both full pulsation cycles as well as measurements at a single time t (but at n different wavelengths λ_j). In the second case, the second term in (9) is straightforward to evaluate but the first one is not. Indeed, even when we can assume that $\beta(t)$ can be considered constant during a pulsation cycle, $a(t)$ is generally non-zero. Unfortunately, measurements at a single t provide no information about the discrepancy between the two. In order to overcome this problem, we will propose a method to determine the first variance component from external information.

In the case of a monoperiodic pulsation, $K = 1$ in (3). When more than one sine term is present, it is often reasonable to assume that the function $a(t)$ can be approximated by a single sine function during one night, but that different approximations would be necessary for different nights. In other words, we will assume that the sinusoidal amplitude α is constant during one night, but fluctuates in a complicated fashion over longer periods of time. To model this, we assume that the amplitudes are drawn randomly from a population of amplitudes with mean μ and variance τ^2. This implies that, if several values for α have been obtained, $\alpha_1, \ \ldots, \alpha_m$ say, one can estimate the average μ, $\overline{\mu}$ say, as well as the variance

$$\tau^2 = \frac{1}{m-1}\sum_{\ell=1}^{m}(\alpha_\ell - \mu)^2.$$

These results can be used to obtain the variance of $a(t)$. Now, $a(t)$ is a sinusoidal function of t, however with variable amplitude α. To account for this extra source of variation, we use a fundamental result in mathematical statistics (Bickel & Doksum 1977, p. 36):

$$\mathrm{var}(a) = \mathrm{var}[E(a|\alpha)] + E[\mathrm{var}(a|\alpha)], \quad (10)$$

where $E(a|\alpha)$ is the expectation of $a(t)$, given a value of α and similarly $\mathrm{var}(a|\alpha)$ is the conditional variance of a random variable. The unconditional counterparts are denoted by $E(.)$ and $\mathrm{var}(.)$. Now, assuming time t is uniformly distributed within a pulsation cycle $[0, 2\pi/\omega]$, the conditional mean of $a(t)$, given an amplitude α is

$$E(a|\alpha) = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\alpha\sin[\omega(t+\phi)]\mathrm{d}t = 0$$

whence the first term in (10) cancels. Secondly, the conditional variance of $a(t)$, given an amplitude α is

$$\mathrm{Var}(a|\alpha) = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\alpha^2\sin^2[\omega(t+\phi)]\mathrm{d}t = \frac{\alpha^2}{2}$$

of which the mean is

$$\begin{aligned}E[\mathrm{var}(a|\alpha)] &= E\left(\frac{\alpha^2}{2}\right) \\ &= \frac{1}{2}E(\alpha)^2 + \frac{1}{2}\mathrm{Var}(\alpha) \\ &= \frac{1}{2}\mu^2 + \frac{1}{2}\tau^2. \end{aligned} \quad (11)$$

Substituting (11) into (10) leads to the following variance formula for the estimated β:

$$\mathrm{var}(\widehat{\beta(t)}) = \frac{1}{2}\mu^2 + \frac{1}{2}\tau^2 + \frac{1}{n}\sigma^2_{v_t}, \quad (12)$$

which is estimated by plugging in estimated values for μ, τ^2, and $\sigma^2_{v_t}$. It is instructive to contrast this quantity with the variance of a single measurement $v_{t\lambda}$ about the average RV $\beta(t)$:

$$\mathrm{var}(v_{t\lambda}) = \frac{1}{2}\mu^2 + \frac{1}{2}\tau^2 + \sigma^2_{v_t}. \quad (13)$$

When the number n of measurements $v_{t\lambda_k}$ $(k = 1, \ \ldots, n)$ increases, the determination of v_t is done with increasing precision and the third term in (12) approaches zero. In contrast, the intrinsic source of uncertainty, due to the sinusoidal fluctuation, cannot be circumvented.

Note that, for a star which is monoperiodic, $K = 1$ and hence the same amplitude will be found during each observational period. Hence, $\tau^2 \equiv 0$. In practice, one might still find a small but non-zero value for τ^2 since the amplitudes will be determined with measurement error.

3. Simulation study

The goal of this simulation study is to assess the quality of the approximation under various orbital and pulsation models. We consider a simplified model with at most two pulsational modes:

$$v_{t\lambda} = \beta(t) + \alpha_1\sin(\omega_1 t + \phi_1) + \alpha_2\sin(\omega_2 t + \phi_2) + \varepsilon_{t\lambda}. \quad (14)$$

We performed four small simulation runs. Each time a beat period is covered with a grid of equally spaced points and for each point (14) is calculated, where the error term $\varepsilon_{t\lambda}$ is generated from a standard normal distribution. Then, we slide a window through the beat period and for the set of observations within a window, a sinusoidal approximation is calculated. From this approximation, the amplitude is retained. From these amplitudes, μ and τ^2 can be estimated and hence our proposed expression for the standard deviation (13) can be computed. The purpose of this study is to compare this with the "correct" value, which is determined as the sample standard deviation of $v_{t\lambda} - \beta(t)$ in (14). Rewrite $\omega_i = 2\pi/P_i$, where P_i is the period in days. Apart from the parameters in (14) we need to specify the number N of equally spaced times t under consideration. In addition, the number of replications n at each time needs to be specified. The settings are displayed in Table 1. All phases are chosen $\phi_1 \equiv 0$. Once we can conclude that the results are acceptable, it

Table 1. Simulation settings

Run	N	n	$\beta(t)$ (km s^{-1})	α_1 (km s^{-1})	P_1 (days)	α_2 (km s^{-1})	P_2 (days)
1	10^4	2; 5; 10; 20	0	10	0.2	0	-
2	10^5	2; 5; 10; 20	$\alpha_0 \sin(\omega_0 t)^1$	10	0.2	0	-
3	10^4	20	0	20	0.2; 0.28; 0.4; 1	20	0.2
4	10^4	20	0	20	0.28	2; 5; 15; 20	0.2

[1]: Here, $\alpha_0 = 50$ (km s^{-1}) and $d_0 = 5; 50; 100$ (days).

follows that (12) can be trusted since the only difference between (12) and (13) is that (12) further corrects for the fact that several replicates are taken at time t. In the simulation settings considered here, the discrepancy between (13) and (12) is of the order of 0.5% of (13).

In the first and the second simulation, there is only one pulsational mode and therefore the observed amplitude is constant over time, implying that $\tau^2 \equiv 0$. The second simulation allows for a non-constant $\beta(t)$ which, for simplicity, is assumed to be of sinusoidal form as well. In order to adequately cover the rapidly varying wave throughout the beat period, the number of times N was increased by a factor 10. In both simulations, the true standard deviation is about 7.14 km s^{-1}. We observe a relative error between (13) and the true value smaller than 0.23% in the first simulation and smaller than 0.15% in the second simulation. This confirms the correctness of the formula (13).

The third simulation studies the particular case of two pulsational modes with $\alpha_1 = \alpha_2 = \alpha$, implying that

$$\alpha \sin(\omega_1 t) + \alpha \sin(\omega_2 t) = \alpha(t) \sin(\omega t)$$

with ω the average of ω_1 and ω_2 and

$$\alpha(t) = 2\alpha \cos\left(\frac{\omega_1 - \omega_2}{2} t\right). \quad (15)$$

Therefore, (15) can be used to determine $\mu^2 + \tau^2$. The true standard deviations range between 20 km s^{-1} and 30 km s^{-1} and the largest discrepancy between our formula and the correct value is 0.17%.

In practice, formula (15) will not be available to determine the statistical properties of the varying amplitude. Rather, it must be estimated from a set of data. This is classically done using either Fourier transforms or minimization in the least squares sense (Bloomfield 1976) which is a special case of a statistical optimization method known as profile likelihood (Welsh 1996). We used the second method in the fourth simulation, which also enables us to cover the situation $\alpha_1 \neq \alpha_2$. Practically, we fitted a function

$$\delta + \alpha \sin(\omega t + \phi)$$

to a window of the data generated under one of the settings of the fourth study. The window consisted either of 100 or of 200 successive points (out of 1000). Windows were then shifted with increments of 10, giving us 90 or 80 different values. In some regions, and for amplitudes that are either equal or similar ($\alpha_2 = 15, 20$), the fit was difficult and yielded implausible values for α. Therefore, we applied trimming, i.e., we discarded all values α larger than 50. The results are presented in Table 2. Observe that there is some difficulty to recover the correct simulated values when $\alpha_1 = 20$ km s^{-1} and α_2 lie fairly close to each other. In those cases, some trimming was necessary. In addition, there appears to be some impact from the choice of window.

Table 2. Results for simulation study 4

			var($v_{t\lambda}$)		
α_2	window	trimming	simul.	theor.	rel. err.
2	100	0%	14.26	14.36	0.70
	200	0%	14.24	14.07	−1.21
5	100	0%	14.60	16.88	15.64
	200	0%	14.60	14.26	−2.37
15	100	13%	17.70	17.89	1.05
	200	12%	17.70	17.58	−0.66
20	100	9%	20.02	20.22	0.98
	200	15%	20.02	18.71	−6.56

4. Application to three stars

We apply our method to three binaries that contain an intrinsically variable star fulfilling the condition of having an orbital period much longer than the pulsational period(s). We chose the examples in such a way as to illustrate that the inclusion of the weights can lead to negligible, noticeable, and crucial improvements of the determination of the orbital parameters. The applications show that our method leads to solutions that are at least as accurate as those found without assigning weights.

The code that we used for the determination of the best orbital parameters was first published by Bertiau & Grobben (1969). It is based on the Lehmann-Filhés method and allows that weights are given to each of the data points. The period can be fixed as well as variable.

4.1. ε Per

The star ε Per is the primary of an eccentric binary with a period of approximately two weeks. The star is also

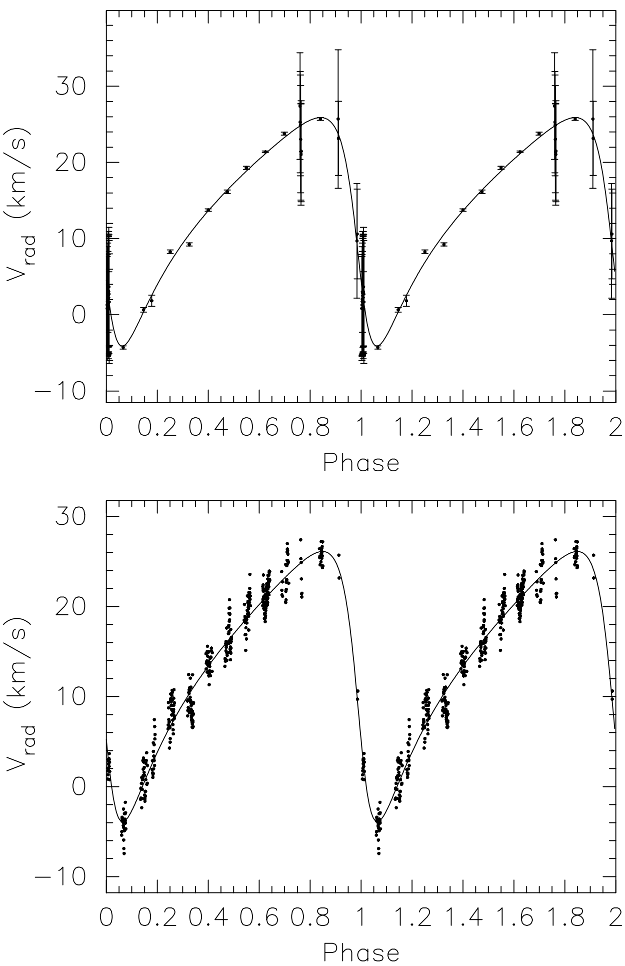

Fig. 1. Top panel: the orbital radial velocity of ε Per found by means of the data taken for De Cat et al. (in preparation). The error bars represent the standard errors. Lower panel: the orbit as determined by De Cat et al. by considering all the data of the star

known for its strong and complicated line-profile variations (Smith et al. 1987; Gies & Kullavanijaya 1988) on a time-scale of hours. It is not yet clear how many and what kind of pulsation modes are active in the star, but high-degree modes are surely involved.

De Cat et al. (in preparation) have recently obtained an extensive set of high-resolution profiles that span the complete orbital period with the aim to study the orbital motion, the pulsation of the primary, and also the coupling between the two. We refer to their study for further details but use their radial-velocity data of the $\lambda\lambda\,4553$ Å line of ε Per to determine the orbital periods with our method.

The data consist of 11 nights for which the pulsational behaviour is well-covered and 3 additional nights during which the weather conditions were poor and only a few spectra were obtained. We determined the standard error (s.e.) of these latter spectra by means of the method outlined above, i.e. by application of formula (12). To achieve this, each of the radial-velocity curves for the fully covered nights was fitted with a sine after having determined the best frequency per night for such a fit with a period-search algorithm (PDM, Stellingwerf 1978). From these fits we derived the average radial velocity of that night with its s.e. and the amplitude. The latter is used in the determination of the s.e. of the data of the uncovered nights.

Table 3. Orbital parameters for ε Per found by the traditional approach and based on our method that includes the assignment of weights. The data are taken from De Cat et al. (in preparation)

Element	Without weights	With weights
P (days)	13.43 ± 0.07	13.4 ± 0.1
v_γ (km s^{-1})	14.0 ± 0.2	14.0 ± 0.1
K (km s^{-1})	15.0 ± 0.2	15.0 ± 0.2
E_0 (HJD)	2450384.52 ± 0.02	2450384.55 ± 0.04
e	0.494 ± 0.008	0.49 ± 0.01
ω (degrees)	113 ± 1	115 ± 1
$a_1 \sin i$ (a.u.)	0.01615	0.01610
$f(M)(M_\odot)$	0.00312	0.00312

The orbital solution that we find is in very good agreement with the solution found by De Cat et al. (see Fig. 1 and Table 3). In fact, all orbital parameters agree within their s.e. We refer to De Cat et al. for a full description of the analysis of the complete data set.

This example shows that adding points of poorly covered nights to well-determined nightly averages over a pulsation cycle leads to the same accuracy of the orbital parameters if one assigns a proper weight to the scattered data points. In the example of ε Per there is no problem to determine the solution without assigning weights because the data nicely cover the complete orbital cycle and a couple of additional nights provide little extra information. When the latter is not the case our method becomes particularly helpful (see the last example).

4.2. κ Sco

The β Cep star κ Sco was monitored as part of a systematic spectroscopic study of pulsating stars in multiple systems (see De Mey 1997; Aerts et al. 1998b). It consists of a β Cep-type primary and a yet unknown secondary. This object has a pulsational behaviour that is similar to the one of the multiple β Cep star λ Sco and was observed during the same observing runs that were devoted to both stars (see De Mey et al. 1997, for the results on λ Sco). The data set consists of 9 fully covered nights and 78 scattered data points. We refer to De Mey (1997) for a full description of the data set.

Two pulsational frequencies are known in the literature for κ Sco and we fitted the fully covered nights with a sine fit for the main frequency 5.004 c/d (Lomb & Shobbrook 1975). We do not find different results than

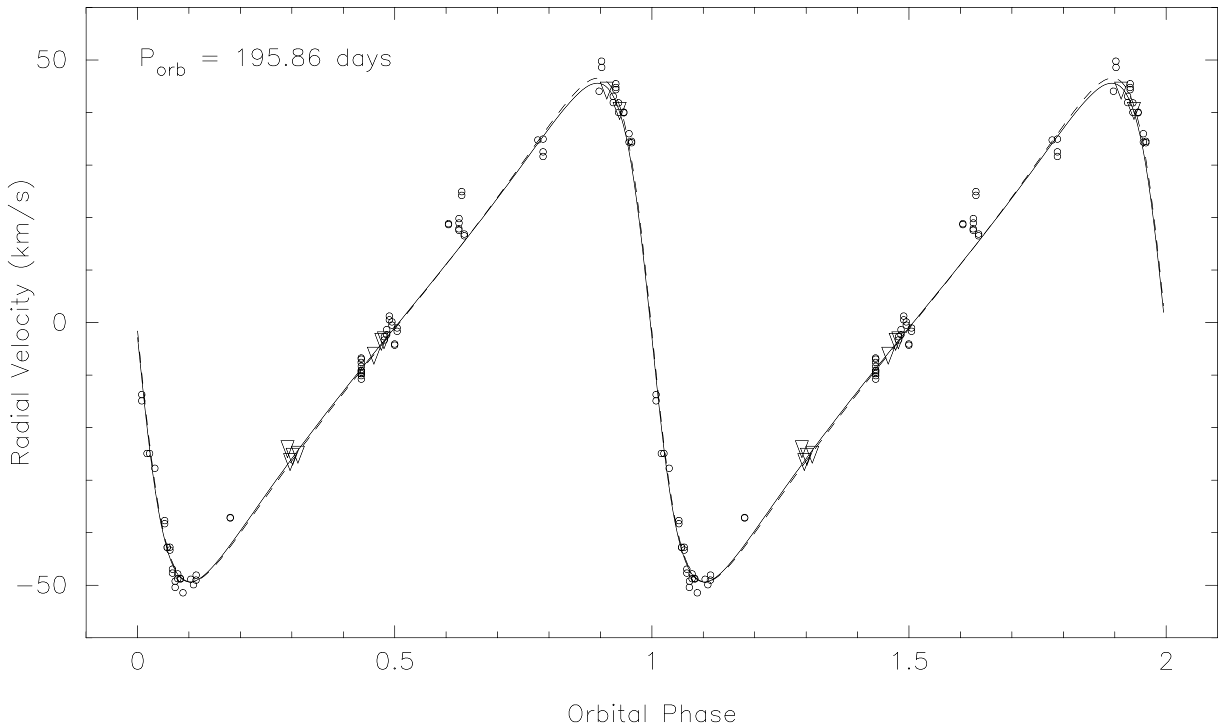

Fig. 2. The orbital radial velocity of the β Cep star κ Sco. The data are taken from De Mey (1997). The symbol $\circ$ denotes the scattered data points while $\bigtriangledown$ stands for the average radial velocity in the case of a fully covered night. The full line denotes the orbital solution determined by considering all the radial-velocity data without taking into account weights while the dotted line is the orbital solution found by our proposed method

Table 4. Orbital parameters for κ Sco found by the traditional approach and based on our method that includes the assignment of weights

Element	Without weights	With weights
P (days)	195.866 $\pm$ 0.010	195.858 $\pm$ 0.007
v_γ (km s^{-1})	−1.30 $\pm$ 0.17	−1.45 $\pm$ 0.15
K (km s^{-1})	47.96 $\pm$ 0.35	47.48 $\pm$ 0.28
E_0 (HJD)	2442147.79 $\pm$ 0.55	2440777.41 $\pm$ 0.43
e	0.477 $\pm$ 0.005	0.478 $\pm$ 0.005
ω (degrees)	90.25 $\pm$ 0.79	91.14 $\pm$ 0.69
$a_1 \sin i$ (a.u.)	0.759	0.751
$f(M)(M_\odot)$	1.520	1.472

the ones listed below by considering the second frequency given by Lomb & Shobbrook, nor when we first search for the best frequency per night and make a fit with this frequency.

De Mey (1997) determined the orbital period from the complete radial-velocity data set with the PDM and CLEAN methods and found 195.77 days. She used this fixed period to search for the other orbital parameters. We refer to her work for the results.

At first, we used the same data set as De Mey (1997) to search for the orbital parameters in the traditional way without fixing the orbital period. This leads to a period of 195.87 $\pm$ 0.01 days and slightly different orbital parameters as found by De Mey (1997). We refer to the second column of Table 4 for the values of the other orbital parameters. The rms of this solution is 2.38 km s^{-1} while the orbit given by De Mey (1997) corresponds to an rms of 2.44 km s^{-1}. We therefore take our solution given in Table 4 and compare it with the results obtained with the approach outlined in Sect. 2. The results of this comparison are given in Table 4 and are shown in Fig. 2. It can be noted from the figure that the solution found by means of our new approach gives a slightly better fit to the data and results in orbital parameters with substantially smaller standard errors.

For this example, the introduction of weights is less crucial for the point estimates than for the standard errors. The reason is that we were able to extend our initial data set with many scattered follow-up data that were obtained to determine the orbit. If the number of scattered points is sufficiently high and well-spread, they will not so much change the average, but may affect precision estimation. Moreover, this star has an orbital velocity amplitude much larger than the amplitude of the pulsation velocity. The last example shows that, when these two conditions do not hold, it can be essential to include weights in a proper way.

Table 5. Orbital parameters for β Cru based on our method that includes the assignment of weights

Element	Without weights	With weights
P (days)	–	1828.0 ± 2.5
v_γ (km s^{-1})	–	10.3 ± 0.2
K (km s^{-1})	–	5.9 ± 0.8
E_0 (HJD)	–	2449879 ± 38
e	–	0.38 ± 0.09
ω (degrees)	–	293 ± 9
$a_1 \sin i$ (a.u.)	–	0.91
$f(M)(M_\odot)$	–	0.030

4.3. β Cru

Finally, the β Cep star β Cru is considered. The line-profile variations of this star were recently studied very thoroughly by Aerts et al. (1998a), who were for the first time able to determine the orbital parameters of this binary. Aerts et al. have gathered over 1000 spectra for β Cru, but their time-spread is very poor from a point of view of determining the orbital parameters (only a few nights with each some $50 - 250$ spectra were obtained).

The application of our method to the data presented by Aerts et al. (1998a) illustrates that assigning weights as we propose here can make the difference between succeeding and failing to determine the orbital parameters in the case that the time spread of the data is very limited with respect to the orbital period. Indeed, Aerts et al. (1998a) failed to find a suitable orbital solution for β Cru before the application of our method. The reason is that the few follow-up measurements that were gathered with the specific aim to study the orbital motion were almost neglected in the calculation of the orbit compared to the more than 1000 spectra obtained for the study of the pulsational behaviour. Moreover, β Cru is an example in which the amplitude of the orbital motion is comparable to the one of the pulsational velocity. In such a situation, it is crucial to substitute fully covered nights by a single data point and add this measurement to the follow-up data, each of them with a proper weight. In fact, the development of our method originates from the purpose to determine an orbital solution for the complicated case of β Cru from the data presented by Aerts et al. (1998a). We list the solution for the orbit of β Cru in Table 5 and refer to Aerts et al. (1998a) for further details of this application.

5. Concluding remarks

We have presented a new approach to determine the orbital parameters of a binary in the case that one of the components is an intrinsically variable star. The method is applicable when the orbital period is considerably longer than the period(s) of the intrinsic variability. Our approach turns out to be very useful when a limited amount of data is available, i.e. when only a few nights of extensive data, that cover the intrinsic variability cycle, and a number of data points scattered throughout the orbital phase have been obtained. Such a situation typically occurs when the intrinsic variable target star of an observing run that covers the pulsational period turns out to be the component of an unknown binary. It is then often difficult to extend the data set in such a way that the orbital solution can be found without any problem. The method is also helpful when the observing run extends over the complete orbital period but when some nights are badly covered because of instrumental problems or poor atmospheric conditions.

Imbert (1987, 1994) previously suggested a method that takes into account the intrinsic variability of Cepheids in binaries when determining the orbital parameters. He achieves this by describing the pulsational radial-velocity variations by an *artificial* Keplerian orbit. This method only works well when the following conditions hold: the pulsation is monoperiodic, the pulsation period is well-known, and the pulsation cycles are well covered with data. It is therefore very restricted compared to our method and not applicable to most non-radial pulsators. Moreover, it assumes an artificial motion while we make use of a physically justified model. Finally, and most importantly, all observations are treated with equal importance in his method, contrary to our algorithm. The strength of our method is that it allows to use measurements of both badly and well covered cycles in a statistically justified way.

It is to be expected that the algorithm proposed here will be used more often in the near future now that it turns out that many pulsating stars belong to a multiple system (see e.g. Aerts et al. 1998b) and that known binaries turn out to have at least one variable component (Harmanec et al. 1997), the latter's intrinsic variations being neglected in the determination of the orbital solution so far.

Acknowledgements. P. De Cat acknowledges the financial support of the Belgian Fund for Joint Basic Research under Research Project No. G.0398.98.

References

Aerts C., De Cat P., Cuypers J., et al., 1998a, A&A 329, 137
Aerts C., De Mey K., De Cat P., Waelkens C., 1998b, in: A Half Century of Stellar Pulsation Interpretations: A tribute to Art N. Cox, Bradley P. & Guzik J. (eds.), PASP Conf. Ser. 380
Bertiau F.C., Grobben J., 1969, Ricerche Astr., Vatican Obs. 8, 1
Bickel P.J., Doksum K.A., 1977, Mathematical Statistics. Prentice Hall
Bloomfield P., 1976, Fourier Analysis of time series: an introduction. John Wiley & Sons, New York
De Mey K., 1997, PhD Thesis. Katholieke Universiteit Leuven, Belgium

De Mey K., Aerts C., Cranmer S.R., et al., 1997, A&A 324, 1096
De Mey K., Aerts C., Waelkens C., Van Winckel H., 1996, A&A 310, 164
De Mey K., Daems K., Sterken C., 1998, A&A 336, 527
Gies D.R., Kullavanijaya A., 1988, ApJ 326, 813
Harmanec P., Hadrava P., Yang S., et al., 1997, A&A 319, 867
Holmgren D., Hadrava P., Harmanec P., et al., 1997, A&A 322, 565
Imbert M., 1987, A&A 175, 30
Imbert M., 1994, A&AS 105, 1
Kennelly E.J., Walker G.A.H., Catala C., et al., 1996, A&A 313, 571
Levee R.D., 1952, ApJ 115, 402
Lomb N.R., Shobbrook R.R., 1975, MNRAS 173, 709
Mathias P., Gillet D., Crowe R., 1991, A&A 252, 245
Smith M.A., 1985a, ApJ 297, 206
Smith M.A., 1985b, ApJ 297, 224
Smith M.A., Fullerton A.W., Percy J.R., 1987, ApJ 320, 768
Stellingwerf R.F., 1978, ApJ 224, 953
Telting J.H., Schrijvers C., 1997, in: A Half Century of Stellar Pulsation Interpretations: A tribute to Art N. Cox, Bradley P. & Guzik J. (eds.), PASP Conf. Ser. 149
Waelkens C., Lampens P., 1988, A&A 194, 143
Welsh A.H., 1996, Aspects of Statistical Inference. John Wiley & Sons