

# A zero point and accidental errors for published values of [Fe/H] for cool giants

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**Abstract.** This paper is one of a series based on published values of [Fe/H] for late-type evolved stars. Only values of [Fe/H] from high-dispersion spectroscopy or related techniques are used. The narrative in this paper begins at a point where mean values of [Fe/H] have been derived for  $\epsilon$  Vir,  $\alpha$  Boo,  $\beta$  Gem, and the Hyades giants. By using these stars as standard stars when necessary, a zero-point data base is assembled. This data base is then expanded into its final version by correcting and adding additional data in a step-by-step process. As that process proceeds, data comparisons are used to establish rms errors. Derived rms errors per datum are found to be about 0.10 – 0.12 dex, and they appear to be too large to be explained by line-to-line scatter and temperature effects.

**Key words:** methods: statistical — stars: abundances

## 1. Introduction

Published values of [Fe/H] are now available for more than 1000 evolved G and K stars. If those data are to be used successfully, two basic problems must be solved: 1) the data must be accessible, and 2) their quality must be well understood. The first of these problems has been addressed by a well-known series of catalogs by Cayrel de Strobel and her associates (see, for example, Cayrel de Strobel et al. 1997). For the second problem, however, there is no solution that is generally accepted.

One part of an acceptable solution would be a reliable zero point. As Griffin & Holweger (1989) have noted, it is not safe to assume that published values of [Fe/H] are already on a uniform zero point. Another part of an acceptable solution would be a reliable set of accidental errors. Such errors must be known for any data set if the data are to be used correctly. It is worth noting that values of [Fe/H] are almost always quoted without accidental

errors, and that there appears to be no consensus about how large those errors should be.

This paper describes procedures for a) establishing zero-point data, b) using those data to correct the zero points of other results, and c) determining accidental errors. These procedures are steps in the production of a catalog containing averaged values of [Fe/H] and rms errors. The stars listed in the catalog have spectral types of G and K and luminosity classes ranging from II through IV. An initial version of the catalog has been published by Taylor (1991, hereafter T91). That version includes zero-point and error analyses, but both must be revised because many more data are now being included<sup>1</sup>.

In Sect. 2 below, there is a review of the way the new analysis is prepared. Section 3 is concerned with the zero point and the assembly of a final data base. In Sect. 4, accidental errors are considered. In Sect. 5, a comparison is made between the new results and counterparts in T91. Section 6 concludes the paper with a brief review.

## 2. Preparation

Before applying the procedures to be described below, one must decide which published data should be adopted and what initial corrections to those data should be made. Taylor (1998a, hereafter Paper I) has considered these issues in some detail, so only a brief summary of his discussion will be given here. For each datum to be included, there is pertinent information that can be found only in the source papers. Those papers are therefore consulted; no attempt is made to rely on Cayrel de Strobel et al. (1997) alone. Almost all the initial adjustments that are then applied to the data are of the following kinds:

1. corrections to a single temperature scale,
2. non-LTE corrections, and

<sup>1</sup> For a comparable catalog for late-type dwarfs, see Taylor (1995).

### 3. corrections to a preferred system of solar equivalent widths.

These corrections are not all applied to every datum, but are instead applied as required. (The rationale for adopting this list of corrections is discussed in Sects. 5 and 6 of Taylor 1998a.)

Many published data are given relative to the Sun. However, some data are given relative to other stars instead. Zero-point stars that have often been used include  $\epsilon$  Vir,  $\alpha$  Boo,  $\beta$  Gem, and the Hyades giants. Mean values of [Fe/H] for these stars are given by Taylor (1998b, hereafter Paper II), who includes a detailed discussion of the way those mean values are derived.

### 3. Correction to a uniform zero point

#### 3.1. Data that define the zero point

The values of [Fe/H] for the zero-point stars may be thought of as an initial zero-point data base. To expand that data base, a search is made for literature results whose zero points appear to be reasonably secure. Acceptable results may be referred to the Sun directly or through the zero-point stars listed above. Data for which T91 has noted some special problem are set aside for the moment. Data are also set aside if they have been derived from the Holweger-Müller (1974) solar model and a stellar model from a published grid. This combination will be referred to here as a “non-uniform” grid of model atmospheres. Data derived with non-uniform grids require corrections ranging from  $-0.05$  dex to  $+0.25$  dex (see Sect. 6 of Paper I).

There appears to be a belief among some astronomers that the history of [Fe/H] analysis has been dominated by external zeroing (see Kuroczkin & Wisniewski 1977; Twarog & Anthony-Twarog 1996)<sup>2</sup>. If this were true, presumably it would not be possible to find enough differential values of [Fe/H] to form an adequate zero-point data base. In fact, the expanded version of that data base contains results for 193 stars. Given the data for these stars, one can produce a final set of averages that does not depend on an adopted solar value of  $\log(\text{Fe}/\text{H})$ .

Alternative ways of setting the zero point include use of the Hyades as well as external zeroing. One could assume that the Hyades giants and dwarfs have the same mean value of [Fe/H] (again see Twarog & Anthony-Twarog 1996). By contrast, the approach adopted here is to test this assumption instead of adopting it at the outset. It is found that at 95% confidence, no difference as large as 0.049 dex exists between the values of [Fe/H] for the Hyades dwarfs and giants (see Sect. 7.6 of Paper II).

<sup>2</sup> In this and companion papers, the term “external zeroing” is applied if authors produce a program-star value of  $\log(\text{Fe}/\text{H})$  and then subtract from it a solar value of  $\log(\text{Fe}/\text{H})$  that they have not determined themselves.

#### 3.2. Expanding the data base

The reader is now invited to inspect Table 1. The description given to this point has reached stage 2 of the analysis (see the second line of Table 1). Details about each remaining data group considered here are given in the table. Reasons for not including the added data in the initial zero-point data base are given in footnotes to the table. The data groups are sequenced so that larger groups are added before smaller groups. This is done to make it as likely as possible that “overlap” stars can be found in both the zero-point data at a given stage and the data group to be added at the next stage. If the number of overlap stars is as large as possible, the correction to the added data will be as precise as possible. An adequate number of overlap stars is found at each step.

A search is now made for discrepant zero-point data. A few such data are found and deleted. In addition, a search is made for results that are added at stage 6 but should be included earlier. An example of such a data set is from Cottrell & Sneden (1986). Their results are initially included at stage 6 because they are based on a non-uniform grid. However, a zero-point correction derived for them turns out to be effectively zero, so they are included at stage 2 instead. Once any required editing is done, the analysis is repeated, and it is iterated until no further editing is necessary.

As each part of the analysis beyond stage 1 is performed, a number of values of [Fe/H] for the Paper II zero-point stars are encountered. Those results have not been used to calculate the mean values of [Fe/H] given in Paper II for the zero-point stars. As a result, one might conceivably find a discrepancy that could signal that the Paper II mean values should be modified. Reassuringly, no discrepancy as large as  $2\sigma$  is found.

#### 3.3. Special correction techniques

Sometimes data require more than a simple zero-point adjustment before being added to the data base. As noted in Table 1, four additional correction algorithms are required.

1) *U - B corrections.* Suppose that values of [Fe/H] from a given paper have a systematic error that varies with temperature. If the cause is continuum misplacement, the error will increase if temperature decreases or if metallicity increases. By using a parameter that does the same things, one can correct for the error.  $U - B$  fits these requirements, and since it also has a large dynamic range, it should be a good choice of correction parameter. The adopted correction equation then has the following form:

$$F = F(\text{uncorrected}) + S(U - B) + Z, \quad (1)$$

with  $F \equiv [\text{Fe}/\text{H}]$  and  $S$  and  $Z$  being constants.

This technique can be tested by applying it to the results of Gratton et al. (1982). Those authors derived

**Table 1.** Grouping the input values of [Fe/H]

Data group	Kind of correction <sup>a</sup>	Included at stage	Cumulative no. of stars
Standard-star data	None	1	7
Zero-point data for stars that are not standard stars	Standard-star (sometimes) <sup>b</sup>	2	193 <sup>c</sup>
Photometric data <sup>d</sup>	Zero-point, scale <sup>d</sup>	3	835
McWilliam 1990 <sup>e</sup>	Zero-point	3	835
Brown et al. 1989 <sup>f</sup>	$B - V$	4	1086
Luck and collaborators <sup>g</sup>	Zero-point	5	1102
Special-analysis data <sup>h</sup>	$U - B$ , $\theta$ , zero-point	6	1118

<sup>a</sup> Corrections given in papers themselves are not considered here. Only corrections derived by comparing data from different papers are listed.

<sup>b</sup> When necessary, data are referred to the Sun through the Paper II standard stars.

<sup>c</sup> This number includes only HD stars, since only data for such stars are used to rezero results in subsequent steps in the analysis.

<sup>d</sup> From Williams 1971, 1972 and Gustafsson et al. 1974. No lower limits from Williams 1972 are used. The Williams data are re-scaled and corrected to the Gustafsson et al. zero point by using Eq. (1). T91 found that an overall zero-point correction for the collected photometric data was required.

<sup>e</sup> These data are based on a non-uniform grid of model atmospheres. T91 found that a zero-point correction was required. Before averages are formed, rms errors are set to 0.07 dex if they are less than 0.07 dex.

<sup>f</sup> T91 found that a  $B - V$ -based correction was required.

<sup>g</sup> From Luck 1991, Luck & Wepfer 1995, and Luck & Challener 1995. These data are based on a non-uniform grid of model atmospheres. Results using spectroscopic values of  $\log g$  are adopted.

<sup>h</sup> Literature sources for these data will be given in a subsequent paper in this series.

high-dispersion values of [Fe/H] from the blue-violet spectral region. They give convincing evidence that their results suffer from continuum-placement error. The technique described above detects the results of this error at a confidence level exceeding 99.99%. (For more information about this test, see T91, Sect. 2.6, paragraphs 1, 2, and 4.)

2)  $B - V$  corrections. These corrections are applied to data from Brown et al. (1989), using a counterpart to Eq. (1). The Brown et al. equivalent widths were measured to the red of 6600 Å, so they are unlikely to suffer from continuum misplacement. It seems more likely that in this case, the systematic error depends on temperature alone. Ideally, one would use some version of  $R - I$  or a similar low-blanketing color index in the correction equation.  $B - V$  is chosen instead because it is readily available. The resulting correction equation appears to be quite adequate. (See T91, Sect. 3, entry for Brown et al.)

3) Corrections based on  $\theta \equiv 5040/T_{\text{eff}}$ . These corrections are applied to the data of Zacs (1994). Values of  $\theta$  derived by Zacs are the only convenient data on which to base the corrections. Again, the correction equation is a counterpart to Eq. (1).

4) Rescaling. Spectrum-synthesis results given by Williams (1971, 1972) are included in the analysis. Those

results are known to suffer from a scale-factor error. The adopted procedure for correcting that error is to transform the Williams data so that they correspond to spectrum-synthesis results from Gustafsson et al. (1974). The correction equation applied here is

$$F = 0.691F_W - 0.001, \quad (2)$$

with  $F$  now representing the data of Gustafsson et al. and  $F_W$  denoting the Williams data. No rescaling of the combined Williams and Gustafsson et al. data seems to be necessary. (For further discussion of this problem, see Sect. 2.5 and Appendix A of T91.)

It may be noted that when Eq. (1) and its counterparts are derived, one must find out whether the resulting values of  $S$  differ significantly from zero. If they do not, it is appropriate to assume that  $S = 0$  and derive a zero-point correction  $Z$  alone. To solve this problem, a least-squares routine that returns rms errors for derived coefficients is employed. The ratio of  $S$  to its derived error is then calculated. This ratio is the  $t$  statistic, and it may be used with standard tables to find out whether  $S$  differs from zero at 95% confidence or better. A similar procedure may be used if  $S = 0$  is assumed and one suspects that  $Z$  does not differ significantly from zero.

#### 3.4. Data that are not added to the data base

Some published metallicities for evolved K stars are not included in the data base. The number of such data is not disturbingly large; one finds that even without them, the final catalog includes results for 1118 stars. Data may be set aside because of continuum-placement problems or other good reasons to suspect that they suffer from systematic errors. Other reasons for setting data aside include a lack of pertinent information about the analyses used to produce them. A complete list of omitted data bases and the reasons for omitting them will be given with the catalog (see Taylor 1999).

### 4. Deriving and checking rms errors

#### 4.1. The error analysis: An overview

The error analysis may be described by referring to the stages of the zero-point analysis that are listed in Table 1. The principal steps of the error analysis are as follows.

- At stages 1–3, derive or adopt interim errors.
- Between stages 3 and 4, test those errors by comparing data bases.
- At stages 4–6, use further comparisons between data bases to derive an rms error for each added data base.

The quantity determined in this error analysis is the rms error *per datum*. The generic symbol  $\sigma_0$  will be used here to refer to that quantity. Given values of  $\sigma_0$  and numbers of contributing data, standard deviations of mean values may be calculated and included in the final catalog.

The scatter that appears when data sets are compared is regarded here as the prime source of information about  $\sigma_0$ . No use is made of tacit errors that are too small to explain such scatter (see Sect. 7.7 of Paper II). In addition, no assumption is made that  $\sigma_0$  is related to high-dispersion analysis procedures. The reason for not making such an assumption will be given in Sect. 4.6.

#### 4.2. The error analysis: Stages 1–3

At stage 1, rms errors of means for the standard stars are carried over from Table 5 of Paper II. At stage 2, it is found that there are a number of zero-point stars with more than one datum. From the scatter in the data for those stars, a value of  $\sigma_0$  is calculated (see Appendix A). The resulting value of  $\sigma_0$  is  $0.103 \pm 0.009$  dex.

At stage 3, results added to the data base include photometric data and the McWilliam (1990) data (see Table 1). McWilliam quotes rms errors for each of his results. Those errors are adopted on an interim basis. For the photometric data, it is assumed that the observed scatter around Eq. (2) is contributed equally by the results of Gustafsson et al. (1974) and the rescaled results of Williams (1971, 1972). The resulting value of  $\sigma_0$  is  $0.097 \pm 0.009$  dex.

#### 4.3. Testing the errors: The procedure

Note that there are now three sets of interim errors to be tested. One set is the value of  $\sigma_0$  for the zero-point data. The second set of errors is for the McWilliam data, while the third set is the value of  $\sigma_0$  for the photometric data. To test those errors, a data-comparison algorithm is used. The algorithm is derived mathematically in Eqs. (B1) through (B44) of Appendix B of T91, so the description of the algorithm given here will be limited to a conceptual summary.

Let two sets of  $N$  data each (say, set A and set B) be considered. Suppose that rms errors are known for set A, but not for set B. Let an estimate be made for the set B error, and let the  $N$  differences between data sets A and B be calculated. If accidental error affects both data sets, the  $N$  differences will scatter around some mean. Suppose that one can account for that scatter by combining the known errors for data set A and the estimated error for data set B. One can then conclude that the estimate for the latter error is correct.

This procedure can also be applied to the  $N$  residuals from some general relation between data sets A and B. Since the amount of scatter depends in some degree on the adopted relation, it is good procedure to determine the relation before settling on a final value for the set B error. In practice, trial versions of Eq. (1) and its counterparts are calculated. If  $S$  differs significantly from zero (recall Sect. 3.3), the scatter around the equation is used to obtain the set B error. If  $S$  does not differ significantly from zero, the error algorithm is applied while a simple zero-point offset between data sets A and B is derived.

#### 4.4. Testing the errors: Results

Let the value of  $\sigma_0$  derived for data set B be referred to as an “external” error. This name is applied because the error is derived, in part, from data which are external to data set B. “Internal” errors have been obtained for each data set during stages 1–3. The task now at hand is to compare internal and external values of  $\sigma_0$ . A selection of results from those comparisons is given in Table 2.

The first three lines of Table 2 imply, in brief, that if one compares McWilliam (1990) and zero-point data and then compares photometric and McWilliam data, the results are satisfactory. Note that this is true, in particular, if McWilliam data with small internal errors are used. To achieve closure, it is also necessary to compare photometric and zero-point data. Given the interim error for the zero-point data, one hopes to recover the internal error for the photometric data. What actually happens, however, is that a difference of  $2.5\sigma$  appears between the internal and external photometric errors. (See the fourth line of Table 2, especially the entry set off by asterisks).

To evaluate this difference, a test is made in which the photometric data are compared to weighted means of

**Table 2.** Comparison of internal and external rms errors

Error being tested <sup>a</sup>	Error assumed to be correct <sup>a</sup>	Range of McW errors used <sup>b</sup>	No. of stars <sup>c</sup>	Internal value	External value
McW	Zpt	0.04 – 0.14	85	0.103	0.091 ± 0.016
GKA/W	McW	0.04 – 0.09	69	0.097 ± 0.009	0.128 ± 0.021
GKA/W	McW	0.04 – 0.14	181	0.097 ± 0.009	0.095 ± 0.013
GKA/W	Zpt	...	63	0.097 ± 0.009	*0.155 ± 0.020*
GKA/W	McW, Zpt	0.04 – 0.14	173 <sup>d</sup>	0.097 ± 0.009	*0.126 ± 0.012*

All numbers in Cols. 3, 5, and 6 are in dex.

<sup>a</sup> “McW” is McWilliam 1990, “Zpt” refers to the zero-point data, and “GKA/W” refers to Gustafsson et al. 1974 and Williams 1971, 1972.

<sup>b</sup> “McW” is McWilliam 1990.

<sup>c</sup> This is the number of stars used in the comparison.

<sup>d</sup> No stars used to derive Eq. (2) are used in this test. As a result, the quoted internal and external results are from completely independent determinations.

the McWilliam and zero-point data. The weights are values of  $\sigma_0^{-2}$  for the contributing data, and the means are formed after the two sets of data are reduced to a common zero point. This time, the difference between the external and internal errors is less than  $2\sigma$  (see the fifth line of Table 2). The result in the fourth line of Table 2 is therefore regarded as a statistical fluctuation. In subsequent stages of the analysis, the internal errors for stages 1 – 3 are adopted without change.

#### 4.5. The error analysis: Stages 4–6

At stage 4 of the analysis, the data-comparison algorithm is applied to derive a value of  $\sigma_0$  for the results of Brown et al. (1989). Using again the terminology of Sect. 4.3, let the Brown et al. data make up set B. For set A, an interim set of weighted averages is used. Contributing data for the averages are from stages 1–3 of the analysis. As before, the weights are values of  $\sigma_0^{-2}$ . The weights are used to assign rms errors to the set A averages. (The equations applied here are Eqs. (C3) and (C5) of Appendix C of T91.)

At stage 5, the above procedure is repeated for the “Luck et al.” data (from Luck 1991; Luck & Challener 1995; and Luck & Wepfer 1995). Interim averages from stages 1–4 are employed, and the Luck et al. data are first corrected for the way in which a model-atmosphere grid was used to derive them (see Appendix B). At stage 6, interim averages from stages 1–5 are used. The data-comparison algorithm is now applied to derive values of  $\sigma_0$  for data sets requiring special error analyses (note again the last line of Table 1). In a special preliminary reduction, the data of Helfer & Wallerstein (1968) are tested at this stage (see below).

For the Brown et al. and Luck et al. results, the derived values of  $\sigma_0$  are  $0.11 \pm 0.01$  dex and  $0.12 \pm 0.02$  dex, respectively. Note that those errors and the others given so far are quite similar. The quoted error for the Luck

et al. data is for values of [Fe/H] which those authors obtain while calculating spectroscopic values of  $\log g$ . Luck et al. also give values of [Fe/H] derived by using so-called “physical” gravities. For those latter data,  $\sigma_0 = 0.07 \pm 0.02$  dex, so their precision may be somewhat better than that of their “spectroscopic” counterparts. Nevertheless, the “spectroscopic” data are adopted when available. (This procedure is a response to the problem posed by the metallicity of  $\mu$  Leo and will be discussed in a subsequent paper.)

At stage 6, unusually large values of  $\sigma_0$  are obtained for some data sets. Even if those data sets require no systematic corrections, they are excluded from the zero-point data set. Identifying noisy data is another reason for iterating the analysis (recall Sect. 3.2).

#### 4.6. Accidental errors and analysis procedures

As noted above, no use is made here of errors inferred from high-dispersion analysis procedures. In particular, such errors are not substituted for errors derived from data scatter. There is admittedly a certain surface plausibility in assuming that there is a link between small errors and reliable analysis procedures. However, it should be noted that such a link may be urged on the basis of supposition or even outright bias. To forestall such problems, an appeal to numerical demonstration is required.

To gain some insight into this issue, the data of Helfer & Wallerstein (1968) have been tested. Those data are from differential curve-of-growth (DCOG) analyses. The shortcomings of principle of such analyses seem to be beyond reasonable doubt, judging from comments made by Griffin (1969, 1975). It is of obvious interest to find out whether those shortcomings lead to an inflated accidental error.

In a test reduction, the Helfer-Wallerstein data were held out of the analysis until stage 6. An error was

**Table 3.** T91 and revised corrections

Data base	Quantity <sup>a</sup>	T91	Rederived
Photometric data	$Z$	$-0.07 \pm 0.02$	$-0.06 \pm 0.02$
McWilliam 1990 <sup>b</sup>	$Z$	$0.04 \pm 0.01$	$0.07 \pm 0.01$
Brown et al. 1989	$S$	$-0.59 \pm 0.09$	$-0.60 \pm 0.05$
Brown et al. 1989	$Z$	$0.57 \pm 0.08$	$0.60 \pm 0.06$
Luck et al. <sup>c</sup>	$Z$	...	$-0.05 \pm 0.01$
Luck et al. <sup>d</sup>	$Z$	...	$-0.13 \pm 0.01$

All numbers in Cols. 3–4 are in dex.

<sup>a</sup> Symbols are defined in Eq. (1). If a value of  $S$  does not differ significantly from 0, it is assumed to be  $\equiv 0$  and is not quoted.

<sup>b</sup> The “T91” entry was derived by Taylor 1996.

<sup>c</sup> This entry is for data based on “spectroscopic” gravities.

<sup>d</sup> This entry is for data based on “physical” gravities.

then derived for those data by using the data-comparison algorithm. The derived error for the Helfer-Wallerstein data was found to be  $0.126 \pm 0.027$  dex. Recall that for the stage 2 zero-point data as a whole,  $\sigma_0 = 0.103 \pm 0.009$  dex. Plainly the two values of  $\sigma_0$  are effectively the same.

Model-atmosphere procedures are likely to resemble each other more closely than any of them resemble DCOG analysis. For this reason, the result just given suggests that there should be no general correlation between error size and type of model-atmosphere analysis. A correlation of this sort may appear in specific instances; note that such a correlation may have been found for the two kinds of Luck et al. analysis (recall Sect. 4.5). However, it seems clear that one should insist on numerical demonstration in all cases before deciding that such a correlation exists.

## 5. Updating T91: The results of the new analysis

### 5.1. Zero points and scale factors

It is of interest to see how different the results of the new analysis are from their T91 counterparts. Zero points and scale factors will be considered first. A selection of values of  $S$  and  $Z$  (recall Eq. (1)) is given in Table 3, with T91 results in the third column and new results in the fourth column. Table 3 also contains the first values of  $Z$  derived for the Luck et al. data.

From the Table 3 data that have T91 counterparts, one gets the overall impression that little has changed. Note especially the entries in the table for the photometric and McWilliam data. Since those data are added at stage 3, the tabular entries show how much the overall zero point established at stage 2 has changed. Despite a marked increase in the size of the data base, that change appears to be no more than  $0.01 - 0.02$  dex. Apparently there is good reason to hope that future changes will be no larger than this.

### 5.2. The overall zero point: An estimate of accidental error

To estimate the rms error of the overall zero point, one may use the stage 2 results and either the photometric or the McWilliam data.  $Z$  is known with better precision for the McWilliam data, so those data will be used here (note the first and second lines of Table 3). To three decimal places, the rms error of  $Z$  for the McWilliam data is 0.014 dex. If the McWilliam data and the stage 2 data contribute about equally to this error, then  $\sigma \sim 0.010$  dex for the overall zero point. This result is essentially unchanged from T91.

### 5.3. Values of $\sigma_0$ : Sizes and possible sources

For values of  $\sigma_0$ , there is a greater change than the one found for systematic effects. In T91, the “default error” per datum is found to be  $0.16 \pm 0.02$  dex. Typical counterpart values obtained here are in the range 0.10–0.12 dex. These smaller errors are obtained from a noticeably larger data base than was available for the T91 analysis. There is therefore good reason to hope that the revised errors are more reliable than the T91 errors. Testing of the revised errors is planned as further data become available.

One would like to know whether the derived errors can be explained by an appeal to obvious sources. Line-to-line scatter is clearly one such source. Estimates of error from this source can be derived from data in Table 7 of Gratton & Sneden (1990). For photographic spectra, the estimated error contribution is 0.06 dex. For Reticon and CCD spectra, the corresponding number is 0.02 dex.

Errors in assigned temperatures also contribute to  $\sigma_0$ . To understand this error source, one should recall that  $\sigma_0$  has been derived by comparing data from various stars. When such comparisons are made, systematic errors from a temperature calibration should largely cancel out because they should be about the same for all stars. On the other hand, if photometric indices are used to assign temperatures (as they often are here), the errors introduced by such indices will not cancel out because they vary randomly from star to star. An estimated size for this second kind of error is therefore required.

The error size may be obtained from the following relation:

$$\sigma_{0,T} = \sigma_\theta |d[\text{Fe}/\text{H}]/d\theta|. \quad (3)$$

To obtain a specific result from this equation, a star in the level 2 data base with  $T_{\text{eff}} = 5000$  K will be considered. For such a star,  $|d[\text{Fe}/\text{H}]/d\theta|$  is typically 3.3 (see Eq. (4) of Taylor 1998a). A value of  $\sigma_\theta$  may be obtained from errors derived for photometric indices (see Table III of Taylor et al. 1987 and Appendix B of Taylor 1996). For level 2 data, the mean value of  $\sigma_\theta$  is about 0.009. The resulting value of  $\sigma_{0,T}$  is then 0.03 dex.

If this error is added in quadrature to the line-scatter errors, the results are about 0.07 dex for data from photographic plates and about 0.04 dex for data from Reticons and CCDs. Recalling that the derived value of  $\sigma_0$  is about 0.10 dex for level 2 data, one can see at once that some important error source is being missed. Identifying that source turns out to be something of a problem. In principle, the missing error could be produced by inconsistencies in the way that spectra for diverse program stars are reduced. This explanation is not very attractive because it is hard to name a specific inconsistency that might plausibly be causing the problem. Nevertheless, it might be worthwhile to look into this possibility by exposing and reducing a series of spectrograms for a given star.

## 6. Summary

The analysis described above is centered on data which define a zero point. Enough of those data are found to allow the analysis to proceed. Data that are not zero-point data are corrected and added to the overall data base in a step-by-step process. Only an acceptably small number of data cannot be included in this process. As it proceeds, values of  $\sigma_0$  are determined by data comparisons. For the most part, the resulting values of  $\sigma_0$  lie between 0.10 and 0.12 dex. These errors appear to be too large to be explained by line-to-line scatter and temperature effects.

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## Appendix A: The rms error for the zero-point data

Suppose that for star  $i$ , there are  $n_i > 1$  zero-point data. Here one uses a standard expression to calculate

$$v_i = (n_i - 1)^{-1} \sum_{j=1}^{n_i} (f_j - \langle f_j \rangle)^2. \quad (\text{A1})$$

$f_j$  is the  $j$ th value of [Fe/H] for star  $i$ , and the variance  $v_i$  of the values of [Fe/H] is the square of the standard deviation per datum.

Since  $v_i$  is calculated from a finite data set,  $v_i$  itself has a finite variance. If the  $f_j$  are normally distributed, the variance of  $v_i$  is inversely proportional to  $\nu_i \equiv n_i - 1$  (see Keeping 1962, Eq. [5.11.14], p. 110). Inverse-variance weighting will then yield a weight which is proportional to  $\nu_i$ , so the expression for the mean variance becomes

$$v = \frac{\sum_{i=1}^N \nu_i v_i}{\sum_{i=1}^N \nu_i}, \quad (\text{A2})$$

with  $N$  being the total number of contributing stars. The associated number of degrees of freedom is given by

$$\nu_0 = \sum_{i=1}^{N_i} n_i - N. \quad (\text{A3})$$

In part, this analysis is described here because it does not appear to be common knowledge that  $n_i - 1$  weighting should be used in this context.

## Appendix B: Interpolation corrections for the Luck et al. data

If authors do not interpolate their model-atmosphere grids to force equality between model and derived metallicities, small corrections to the latter may be required. Because Castro et al. (1996) discuss such corrections for the Luck et al. results, those corrections receive special attention in this analysis. Castro et al. point out that for the solar-abundance model used by Luck et al.,  $\log \epsilon(\text{Fe}) = 7.50$  (see Bell et al. 1976). Since the solar abundance used by Luck et al. is 7.67, the effective value of [Fe/H] for their nominal solar model is  $-0.17$  dex. For the purpose at hand, this number is adopted as the zero point of the Bell et al. grid.

Castro et al. also calculate a scale factor for the interpolation correction. That scale factor is set aside here, and a counterpart given by Luck et al. themselves is used (see Table 7 of Luck & Challener 1995). The complete correction may be expressed as follows: if  $N$  is an integer and  $(0.5N - 0.25) \text{ dex} \leq [\text{Fe}/\text{H}]_{\text{LC95}} < (0.5N + 0.25) \text{ dex}$ ,

$$[\text{Fe}/\text{H}](\text{corr}) = 1.12[\text{Fe}/\text{H}]_{\text{LC95}} + 0.02 + 0.06N. \quad (\text{B1})$$

This correction is applied before the zero-point adjustments given in Table 3 are calculated. It may be noted that to first order, the effect of the correction is a simple change in zero-point adjustment.

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