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Comprehensive tables for the interpretation and modeling of the light curves of eclipsing binaries⋆

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Abstract. We present parameters used in the investigation of the light curves of eclipsing binaries together with other data needed for the interpretation of their stellar and dynamical evolution. Parameters include limb-darkening coefficients and gravity darkening exponents, while data includes the apsidal motion constants, the moment of inertia, and the potential energy. The adopted stellar models are those computed by Claret (1995) for a representative chemical composition of $X = 0.70$ and $Z = 0.02$. In addition to the parameters needed for the study of the dynamical behavior and tidal evolution of binary systems, we supply the linear limb-darkening coefficients computed in 12 different photometric bands, as well as the gravity darkening exponent for each point along the evolutionary track.

We have developed a method, based on the triangles strategy by Kippenhahn et al. (1967) to compute the gravity-darkening exponent using interior models. For the first time, the gravity-darkening exponents are presented as a function of mass and age. The old values of $\beta_1 - 0.32$ and 1.0 - for convective and radiative envelopes are thus superseded by the present calculations and a smooth transition is achieved between both energy transport mechanisms.

The tables presented here assist modeling of the light curves of close binaries using limb-darkening and gravity darkening coefficients which are consistent with the observed masses, radii and effective temperatures.

In order to facilitate the use of the grid of models presented here in a variety of different research fields other than binary stars, synthetic colors ($U-B$, $B-V$, $u-b$, $b-y$) and M_v are also given.

Key words: stars: binaries: eclipsing — stars: evolution; interiors; fundamental parameters

1. Introduction

The computation of theoretical light curves for eclipsing binaries requires previous knowledge of astrophysical parameters including the limb-darkening and the gravity-darkening coefficients. It was shown in Popper (1984) that the fit of geometrical or radiative parameters to actual observations by means of synthetic light curve modeling, is not possible for second order effects like those of limb and gravity darkening. It is therefore advisable to adopt these coefficients from realistic theoretical computations instead of trying to derive them from the observations. In fact, small variations in the main geometrical and radiative parameters of the light curve may easily mask the real value of the darkening coefficients or even lead to a wrong combination of physical parameters actually reproducing the observed light variations.

Limb-darkening coefficients are computed using stellar atmosphere models while the gravity-darkening exponent requires some knowledge of the stellar interior. Several papers have been devoted in the last years to treat the effects of limb-darkening (Van Hamme 1993; Díaz-Cordovés et al. 1995; Claret et al. 1995 and references therein). But the currently adopted values of gravity-darkening are still the old ones based on the results by von Zeipel (1924) and Lucy (1967). Most of the theoretical papers dealing with gravity darkening in stars with convective envelopes are based on the work by Lucy and the results are essentially the same. Alternative formulations include Martynov (1973) and Anderson & Shu (1977). Martynov used the Planck law to derive β_1 as a function of wavelength, while Anderson & Shu argued that β_1 should be zero since the flux (almost convective) ought to be constant over equipotentials. Hereafter we denote this exponent as β_1 in order to differentiate it from the radius of gyration β. In this paper, we present new computations for β_1 using a method based on interior models which embrace convective and radiative envelopes. Such calculations are presented for the first time as a function of the mass and degree of evolution.

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⋆ Tables 1-24 are only available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/Abstract.html

For the computation of the basic stellar evolutionary parameters, we used the models previously computed by Claret 1995 for a representative chemical composition ($X = 0.70$, $Z = 0.02$). They cover a range of stellar masses (1 up to 40 $M_{\odot}$) and ages. The limb-darkening coefficients for the Strömgren, Johnson and $R\ I\ J\ H\ K$ systems were then computed, as well as the gravity-darkening exponents, for every theoretical track. For the sake of completeness, the tables produced included the most relevant parameters for the study of the dynamical behavior of binary systems, namely, the apsidal motion constants (log k_j, $j = 1, 2, 3$), the moment of inertia and the potential energy. In addition, synthetic colors and absolute V magnitude have been computed. This provides a complete and coherent table of stellar parameters allowing the modeling of light curves and the analysis of binary stars evolution with self-consistent derived values.

The paper is divided in three parts: this short Introduction; Sect. 2, where we present and discuss the stellar models with the corresponding limb-darkening coefficients and colors, and Sect. 3 that is devoted to the gravity-darkening calculations.

2. The limb-darkening coefficients and the colors

The stellar models are those of (Claret 1995). We have selected a representative chemical composition corresponding to $(X,\ Z) = (0.70,\ 0.02)$. In fact, such a chemical composition seems to fit well, as average, the properties of double-lined eclipsing binaries compiled by Andersen (1991) including the apsidal motion analysis.

The limb-darkening coefficients are computed using the stellar atmospheres models generated with the ATLAS code (Kurucz 1993). A few modifications were introduced in the method to compute the coefficients with respect to our recent publications (Díaz-Cordovés et al. 1995; Claret et al. 1995). A curve of sensitivity for a CCD detector was introduced in order to cover all bands investigated (from about 3500 Å up to 22000 Å). The limb-darkening coefficients were computed for most usual photometric systems used in investigations of eclipsing binaries covering 12 bands: $uvby$, $U\ B\ V$ and $R\ I\ J\ H\ K$. For the $J\ H\ K$ bands the sensitivity of the In-Sb detector was included.

In Fig. 1 we present an example of such a calculation showing the "evolution" of the linear limb-darkening coefficients for a 2 $M_{\odot}$ model. The main-sequence phase and the giant branches are perfectly distinguishable. Coefficients are available for each point of each track, allowing an analysis of the light curves that is both coherent and consistent with the usual final products of double-lined eclipsing binary systems: the masses, radii and effective temperatures.

In the recent years we have shown the non-linearity of the distribution of the intensities across the stellar disk (Díaz-Corbobés et al. 1995; Claret et al. 1995). For very low effective temperatures the non-linear effect is still larger (Claret 1998). Since many people still use the linear approximation, we urge such users to consider the adoption of non-linear coefficients.

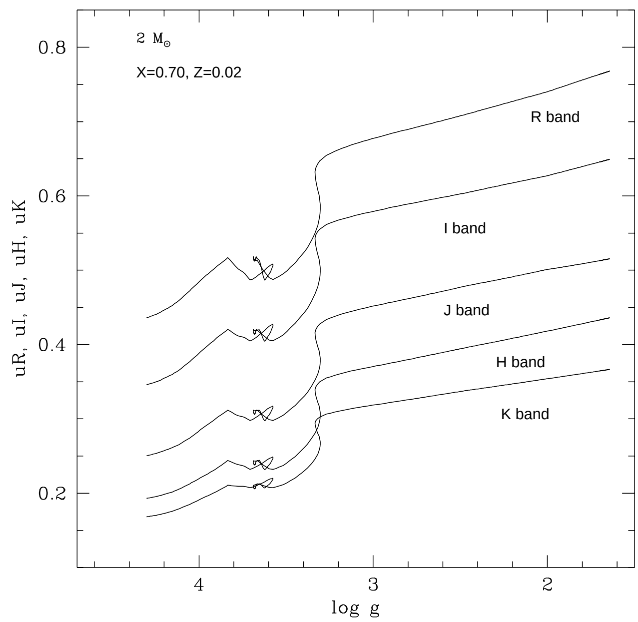

Fig. 1. Linear limb-darkening coefficients for the photometric bands $R\ I\ J\ H\ K$ for a 2 $M_{\odot}$ model as a function of the surface gravity

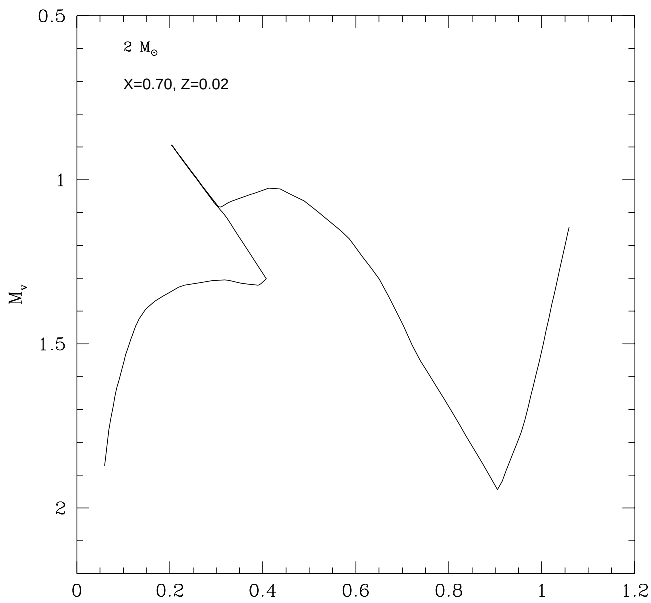

Fig. 2. M_v as a function of $B{-}V$ for a 2 $M_{\odot}$ model

When working with stellar evolution models, some basic information related directly to observations is desirable. This is the case of stellar colors. In order to carry out the computation of such parameters, we have considered

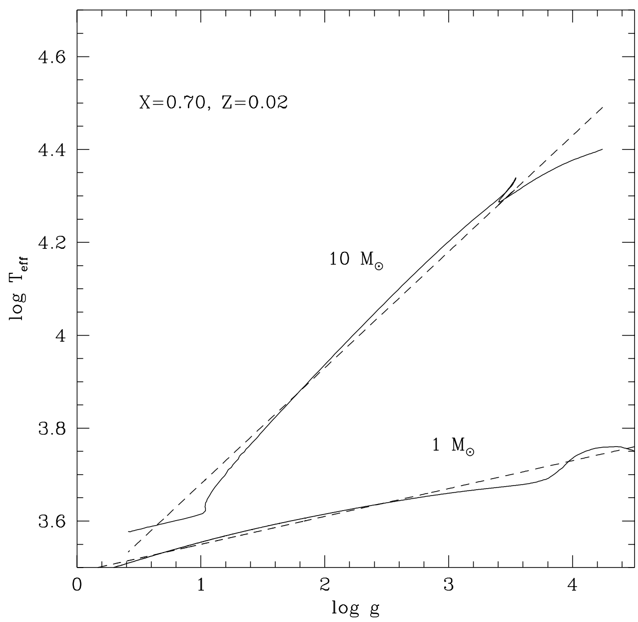

Fig. 3. Diagram log $g \times \log T_{\rm eff}$ for 1 and 10 $M_\odot$ models. The adopted chemical composition was $(X, Z) = (0.70, 0.02)$. The abcisses and coordinates are inverted with respect to the habitual HR diagram. The dashed lines represent approximations by straight lines. See text

the following semi-empirical calibrations: for luminosity class V the relationship between $T_{\rm eff}$ and $B-V$ (5000 K $\leq T_{\rm eff} \leq$ 8000 K) was that of Arribas & Martínez Roger (1988) while for hotter stars we used the Böhm-Vitense (1981) data. For luminosity classes I and III we used the data by Flower (1977). The bolometric corrections were taken from Malagnini et al. (1986). To transform colors from the Johnson to Strömgren, we used the equations by Penprase (1992). Figure 2 shows a sample of such calculations: the $B-V \times M_v$ diagram for a 2 $M_\odot$ model.

3. The interior models and the computation of the gravity-darkening exponent

The gravity-darkening exponent β_1 is an important but poorly studied parameter that is used in the analysis of the light curves of eclipsing binaries. In a stellar envelope the energy is transported by convection and/or radiation. Neither theory or intuition suggest that the values of β_1 would the same for these two possibilities. In fact, for stars in strict radiative equilibrium (pseudo-barotrope), von Zeipel (1924) has shown that the variation of brightness over the surface is proportional to the effective gravity. In mathematical form

$$F = -\frac{4acT^3}{3\kappa\rho}\frac{{\rm d}T}{{\rm d}\Phi}g^{\beta_1} \quad (1)$$

or

$${T_{\rm eff}}^4 \propto g^{\beta_1} \quad (2)$$

where $\beta_1 = 1.0$, g is the local gravity, Φ is the total potential and the remaining variables having their usual meaning in stellar structure.

One of the few papers treating convective envelopes is that by Lucy (1967). Using convective stellar envelopes he found that $\beta_1 = 0.32$ (mean value). This jump of β_1 from both radiative to convective atmospheres was awkward from the observational and theoretical points of view: the two processes of transport of energy can even exist simultaneously in a determined stellar envelope.

In order to search for this possibility of a less abrupt change, we shall first utilize very simple arguments. Consider the diagram log $g \times \log T_{\rm eff}$ for two stellar models for which convection and radiation would be predominant in their envelopes, say 1 and 10 $M_\odot$ (Fig. 3). Both models can be roughly approximated by straight lines as indicated by the dashed lines. The mean slope for these straight lines are 0.06 and 0.25 respectively for the less and more massive models. Making an analogy between these slopes and the exponent β_1, this value would be 0.24 for envelopes where convection is important while for envelopes in radiative equilibrium β_1 would be 1.0. Although the properties of the equation of transport are implicit in the problem, it is gratifying that this simple method allow us to predict, with an acceptable accuracy, both values of β_1 using only the theoretical HR diagram. However, this does not solve our problem of determining β_1 for each evolutionary state. Consequently, we have developed a more sophisticated method based on the triangles technique (see Kippenhahn et al. 1967).

We summarize the main aspects of this method. In principle, after each time step, the outer layers integration should be carried out again. However, if the external boundary conditions of the interior are unchanged at the fitting point $M_{\rm F}$, the corresponding outer layer integration would be the same as the anterior. Such calculations can be avoided saving computational time. In practice, three envelope computations are performed corresponding to three points in the HR diagram. If the next point of the evolutionary track is still within this triangle the same envelope integrations can be used without loss of accuracy (if the triangle is sufficiently small). If this condition is not fulfilled then new integrations are necessary. The conditions for which $M_{\rm f}$ can be moved are that the luminosity and chemical profile would be approximately constant in the outer layers.

To represent a distorted star with different flux distribution over the surface, we are interested in envelope models with different temperature distribution but presenting the same physical conditions at a given point where the ionization of hydrogen and helium is complete (we denote this point as $m_{\rm F}$). In order to get such models, we used the same triangle strategy but increased the number of triangles for every point of the track on the HR diagram. Now we have the three envelope integrations which fulfill the boundary conditions at $M_{\rm F}$ and additional neighboring

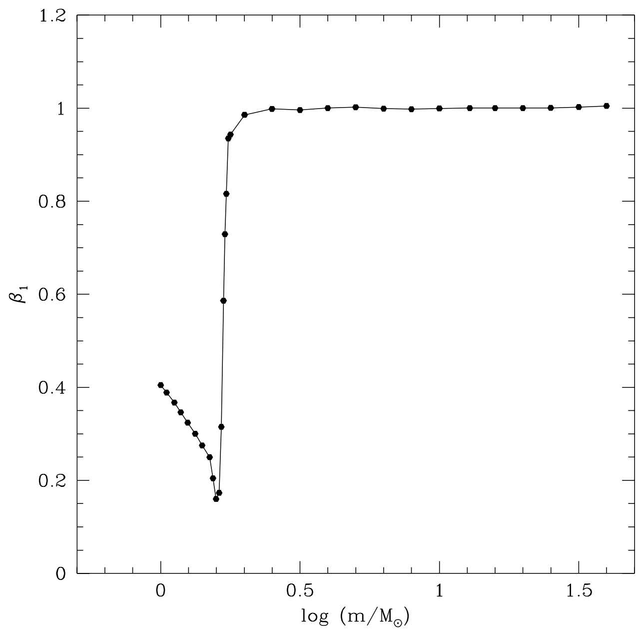

Fig. 4. Values of the exponent β_1 for homogeneous models as a function of the logarithm of mass

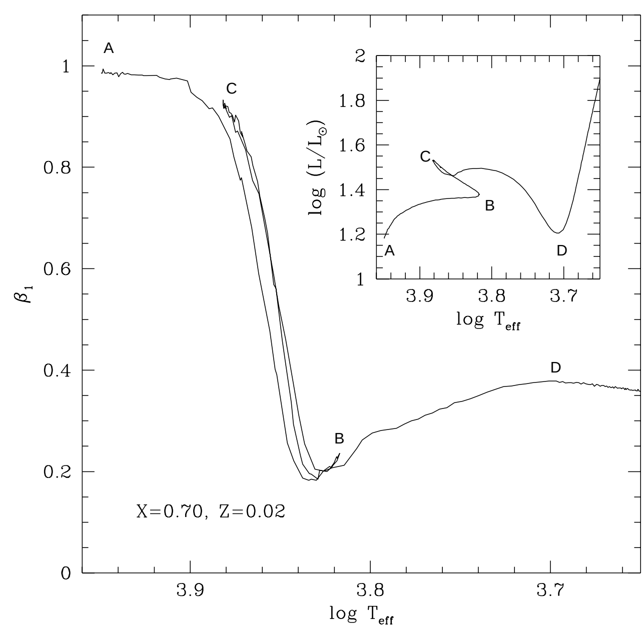

Fig. 5. The gravity-darkening exponent β_1 for a 2 $M_\odot$ model. In the upper right corner the corresponding HR diagram is shown. The letters indicate some interesting phases commented in the text

envelopes. These new envelope integrations do not necessarily fulfill the boundary conditions at $M_{\rm F}$ neither at $m_{\rm F}$. By imposing these conditions and interpolating among these extra envelopes we obtain new envelopes (with the respective of log $T_{\rm eff}$ and log g) which fulfill these physical conditions at $m_{\rm F}$ but with different temperature distribution along the radius, which is used here as independent variable. A least square method is applied to these corresponding points and the value of β_1 is then derived.

The method described above has several advantages:

1. it is a consistent method since the interior models are used to derive β_1 as well as to predicted limb-darkening coefficients, radius, effective temperature and color indices,
2. one can go as deep as necessary into the interior of the model to impose the boundary conditions,
3. changes in the chemical profile in the envelope, as for example those due to mixing, are easily taken into account,
4. it does not present possible effects of sphericity given the spherical symmetry of the structure equations,
5. it can be easily adapted to stellar evolution codes.

In Fig. 4 we present the computations for homogeneous models covering the mass range from 1.0 up to 40 $M_\odot$. The von Zeipel theorem is confirmed for the hotter models while for the remaining, a smaller value of β_1 is obtained. For more massive models the zone below the hydrogen and helium ionization is characterized by radiative equilibrium, this fact being reflected by the value of β_1. For less massive models, energy is predominantly transported by convection and the derived value of β_1 is also affected by this condition. The transition zone corresponds to effective temperatures in the interval $3.81 \lesssim \log T_{\rm eff} \lesssim 3.86$ for which radiation begins to compete with convection in the transport of energy (see also Fig. 7). It should be remarked that this range depends on the theory of convection used and in the case of mixing-length, on the value of the ratio l/λ where $\lambda = -{\rm d}\ln P/{\rm d}r$.

An interesting behavior of β_1 is expected for moderately massive models which tracks cross over the boundary between radiative and convective equilibrium. We selected a 2 $M_\odot$ model to represent this situation. In Fig. 5 one can see how β_1 depends on the effective temperature while on the upper right corner of the same figure we plot the usual HR diagram. Letters A-D indicate some points of interest. The gravity-darkening exponent for Main-Sequence models, as expected, is 1.0 and begins to decrease until the point indicated by letter B which is the boundary between both competing regimes of transport of energy. During contraction $(B - C)$ the effective temperature increases, which is reflected in an increasing β_1. After C the effective temperature decreases as well as the gravity-darkening exponent. For very deep convective envelopes, for stars at the red giant phase, β_1 seems to stabilize at 0.3.

Interesting characteristics can be also found for contracting models during the Pre Main-Sequence phase (see Fig. 6). Let us consider a 1 $M_\odot$ model. At the beginning of the Pre Main-Sequence phase the model has a very deep convective envelope which decreases in depth as star approximates to the ZAMS point (around 30 per cent of the

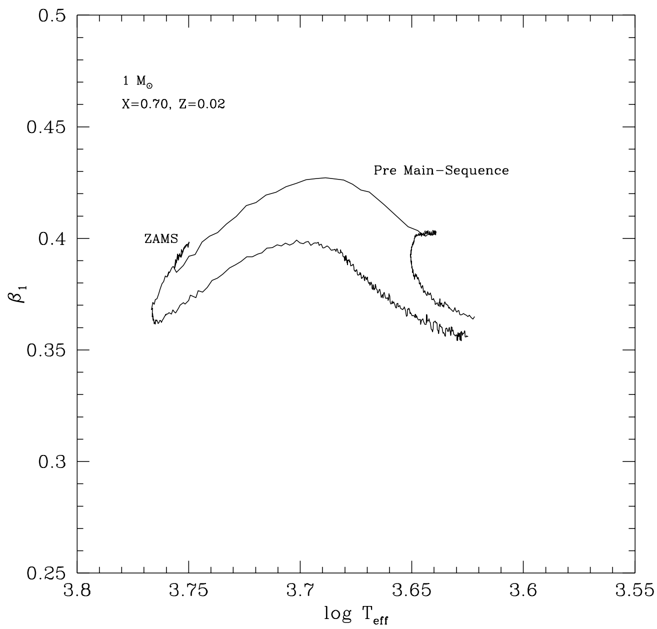

Fig. 6. The gravity-darkening exponent β_1 for a 1 $M_\odot$ model during and after the Pre Main-Sequence phase

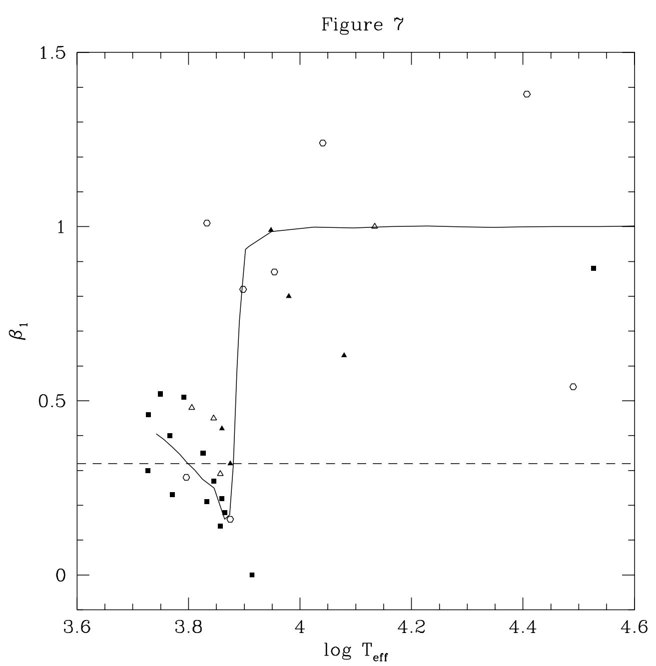

Fig. 7. Observational values of the gravity-darkening exponent (Rafert & Twigg 1980) as a function of the effective temperatures. Open circles represent detached systems, full triangles semi-detached systems, full squares denotes contact systems while open triangles denotes other systems. The full line represent the theoretical β_1 for homogeneous models

normalized radius). At the interval which corresponds to $3.75 \lesssim \log T_{\rm eff} \lesssim 3.77$ the depth of the convective layer is almost constant. Outside this interval this value increases again. Note that during the Pre Main-Sequence β_1 is systematically larger than the computed after ZAMS point for a given effective temperature. This is a consequence of that, although the Pre and Post-ZAMS models present similar effective temperatures, the physical conditions in the envelopes are different in both cases. When the depth of the convective envelopes are of the same order, β_1 converges to similar values (note the behavior of β_1 in the beginning of the Pre Main-Sequence and the red giant phase or in the neighborhood of ZAMS).

Given the actual level of knowledge supplied by light curve synthesis is difficult task to compare the model calculations with observational data. Indeed, as in the case of the limb-darkening coefficients, gravity darkening is a second order parameter which can not be determined directly from observations. However, a significant effort was made by Rafert & Twigg (1980) to derive such parameters from light curves analysis. Although these values include many of the serious problems mentioned above, they serve as indicators . The comparison shown in Fig. 7 (the error bars were omitted for sake of clarity) can be considered within the inherent modeling difficulties. Evolutive effects may also change the comparison.

Other attempts to determine empirically the gravity-darkening are described by Kitamura et al. (1996). They report values of β_1 sensibly larger than unity for Roche-lobe filling secondaries of semi-detached systems. However, the complexity of the physics involved in these kind of systems may include a variety of complications which make comparison of observation and theory difficult.

We do not consider the comparison shown in Fig. 7 as definitive. This is due in part to the observational problems involved in obtaining β_1 as well as the theoretical uncertainties. Indeed, in the method we have developed (and in another ones) to derive β_1, cause-effect is not explicitly considered since the extra envelopes do not necessarily represent rotating or tidal distorted ones and this may be of critical importance. Another physical phenomena, as for example circulation should be also included (Smith & Worley 1974).

Finally, a few words on the organization of the tables. Each table contains seven lines for each point of the evolutionary track. We tabulate the identification number, age (in years), $\log L/L_\odot$, $\log g$ (in cgs units), $\log T_{\rm eff}$, mass (in solar units), the apsidal motion constants $\log k_n$ ($n = 2$, 3, 4), $\alpha = -\,\Omega{\rm R}/({\rm G}M^2)$ where Ω is the potential energy, the radius of gyration β and the gravity-darkening exponent β_1 in the first line. In the second line, we tabulate the linear limb-darkening coefficients for u v b y, U B V and R I J H K bands. In the lines 3-6 we tabulate the non-linear limb-darkening coefficients (quadratic and root-square approximations) for the same bands. In the seventh line, $M_{\rm bol}$, M_v, $U-B$, $B-V$, $u-b$ and $b-y$ are tabulated.

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