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Determination of proper motions and membership of the open clusters NGC 1750 and NGC 1758

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Abstract. The kinematic state of the open clusters NGC 1750 and NGC 1758 has been studied using newly determined proper motions for 540 stars in a field of $1\overset{\circ}{.}5\times1\overset{\circ}{.}5$ in the Taurus dark cloud region. The proper motions are obtained from the reduction of PDS measurements of 20 plates that span a total time interval of 68 years, resulting in an average proper motion accuracy of 0.67 mas/yr. These proper motions are used to determine the membership probabilities of stars in the region by means of a new, improved method described in this paper. Of the 540 stars analyzed here, 332 are found to be probable members of NGC 1750, and 23 are probable members of NGC 1758. The core radii of NGC 1750 and NGC 1758 are determined to be $17\overset{\prime}{.}2$ and $2\overset{\prime}{.}25$ respectively.

Key words: open cluster: NGC 1750; NGC 1758 — astrometry

1. Introduction

There are a number of open clusters in the direction of the Taurus dark clouds. Dreyer (1888) considered that there are three overlapping open clusters: NGC 1746, NGC 1750 and NGC 1758. Besides these, the Hyades cluster is in the foreground, and the Pleiades cluster immerses itself partly into the front edge of the dark clouds. The other three clusters, as background objects, are reddened by different amounts. This group is very close to the anti-center direction of the Galaxy, with Galactic coordinates $l = 179°, b = -11°$. Their common angular diameter, given by Alter et al. (1970), is $\sim 50'$. The positions of these clusters on the celestial sphere are shown in Fig. 1.

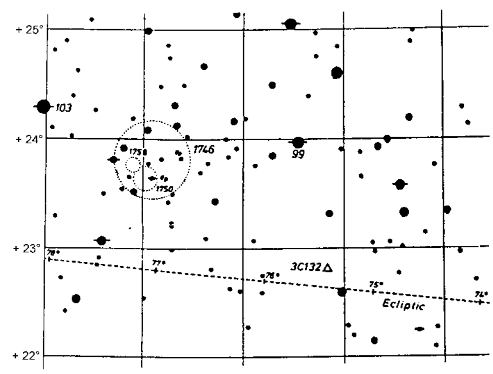

Fig. 1. Star map from the Tirion et al. (1987) atlas with the clusters NGC 1746, NGC 1750 and NGC 1758 shown

Whether or not these clusters exist is a matter of dispute. Some authors considered all three clusters as one: NGC 1746 (Alter et al. 1958, 1970; Ruprecht et al. 1981). Cuffey (1937) obtained extensive photographic photometry of stars in this area in the blue and red bands to a limiting magnitude of 13 mag. He named all of his photographed area NGC 1746. The first photoelectric photometry of stars in this area of the Taurus dark clouds was in the Vilnius photometric system (Straižys & Meištas 1980; Meištas & Straižys 1981; Černis 1987). V magnitudes, color indices, color excesses, interstellar extinction and distances were determined for 116 stars (Straižys et al. 1992), to a limit of $V \simeq 13$. They concluded that NGC 1746 was probably not a cluster, and that the distances of NGC 1750 and NGC 1758 were 510 pc and 680 pc respectively, if the two groupings were real open clusters.

Historically, proper motions have provided a reliable method for determining membership of stars in open clusters. First-epoch proper motion plates of the region of NGC 1750 and NGC 1758 were taken with the double astrograph at the Zŏ-Sè station of Shanghai Observatory in 1918–1961. We took the second-epoch plates of the same

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region with the same telescope in the 1980's. These plates are clearly very valuable for astrophysical studies in the cluster region. Proper motions of some stars in this region were published by Li (1954). However, the accuracy of his proper motions is rather low since the epoch difference of the plates he used was only 12 years and the measuring machine was not very good more than 40 years ago.

In the present paper, a new, improved approach for determining membership probabilities is presented. Relative proper motions for 540 stars within a $1^{\circ}\!.5 \times 1^{\circ}\!.5$ area centered on the Taurus dark clouds are determined using plates taken over a period of 68 years, and the two clusters are successfully separated from each other.

2. Plate measurements and reduction of proper motions

2.1. Plate material and measurements

Twenty plates of the region of NGC 1750 and NGC 1758 which were taken with the double astrograph at the Zǒ-Sè station of Shanghai Observatory are available for this study. The telescope, built by Gaultier in Paris at the beginning of this century, has an aperture of 400 mm, a focal length of 6.9 m, and a plate scale of $30'$/mm. The size of the plates is 240 by 300 mm, or $2^{\circ}\!.0 \times 2^{\circ}\!.5$. The first-epoch plates were taken from 1918 to 1961, some of which were used by Li (1954) in his study of this area. The second-epoch plates were all taken from 1983 to 1986. Table 1 lists the details for the 20 plates used in this paper. The quality in Table 1 gives the sharpness of star images on the plate. The hour angles of all second-epoch plates are within the range ± 1 hour. The hour angles are not provided in Table 1 because the starting times of the first-epoch plates were not recorded. In the column "G" and "SL" indicate respectively the image "good" and "slightly length".

All the plates were measured on a Photometric Data Systems (PDS) model 1010 automatic measuring machine at the Dominion Astrophysical Observatory in Victoria, Canada. This microdensitometer combines an accurate, high-speed photometer with a precise $x-y$ coordinate scanning system to allow the acquisition of density and position information from photographic images. Approximate positions of all the stars were obtained from measurements of one plate using a two-screw Mann measuring machine, and stored in a disk file. Then a small square area around each stellar position was scanned, using a 17 μm ($0''\!.51$) square aperture stepped by 17 μm in x and y. A 30 × 30, 40 × 40, 50 × 50, or 60 × 60 box was scanned at each stellar position depending on the brightness of the star, which determines the density and apparent diameter of its image. In order to monitor the scanning stability, a "reference loop" consisting of 15 stars spread uniformly over the plate was rescanned at the beginning, middle, and end of the measurement of each plate.

2.2. Proper motions

The reduction of the relative proper motions for 540 stars to a limiting magnitude $V \simeq 15.0$ in the region of NGC 1750 and NGC 1758 was made on the basis of the PDS measurements by means of an approach we have adopted many times before (Tian et al. 1982, 1983; Zhao et al. 1981, 1993; Su et al. 1997). There are three steps in the whole process: the first is to transform the measured results of all the plates to a common system, in order to eliminate the errors due to small differences in the orientation of different plates in scanning; the second step is to establish a reference frame, i.e. to decide upon the reference stars; the last step is to calculate proper motions of all the stars with respect to this reference frame, and their corresponding uncertainties.

Generally, any stars can be chosen as reference stars for determining relative proper motions. However, in order to obtain a good plate solution and to make the absolute proper motions of the reference frame as small as possible, our principle is to choose as many stars common to all the plate pairs as possible, except for any stars with extraordinarily large proper motions and stars located in the crowded central region. On the other hand, the distribution of star images on the plate and the magnitude distribution of the reference stars should be homogeneous. For these reasons, after two loops of the least-squares adjustment, 300 stars with residuals in both x and y coordinates less than $2\sigma_x$ and $2\sigma_y$ respectively were chosen to be reference stars from the 370 stars common to all the plate pairs, where σ_x and σ_y are the rms residuals in the x and y coordinates obtained from the least-squares adjustment.

There are two ways that can be used to determine proper motions. One is known as the plate-pairs method, and another is called the central overlap technique. Owing to the limited number of reference stars and the accuracies of the proper motions of these stars, the plate-pairs technique is used in the present study. All the linear and quadratic coordinate-dependent terms and the coma term are included in the plate solutions. The weighted mean of the proper motion of a star obtained from all of the available plate pairs should be taken as the final value of the proper motion of the star. The proper motion weight for a star in a plate pair is determined from the epoch difference of the pair. As we know, accuracies of proper motions for individual stars are different, since the time baselines, number of available pairs, weather conditions of observation, exposure times, and plate washing can be different for different plate pairs. The corresponding internal standard errors can be estimated from a comparison of the proper motions obtained from different plate pairs

Table 1. Plate material

Pairs No.	plates	Epoch (1900+)	Exp.Time min	Quality	Baselines years	Star No.
1	P60001	60.07	20	G	22.98	443
	P83001	83.05	20	G		
2	P60002	60.07	20	G	22.98	458
	P83002	83.05	20	G		
3	P530	30.13	–	SL	56.04	503
	P86009	86.17	20	G		
4	P470	18.05	–	G	67.99	532
	P86006	86.04	20	G		
5	P57005	57.07	20	G	26.03	502
	P83007	83.10	20	G		
6	P57006	57.07	20	SL	26.02	500
	P83006	83.09	20	G		
7	P526	30.06	–	G	56.07	511
	P86007	86.13	20	G		
8	P471	18.08	75	G	67.93	534
	P86001	86.01	22	G		
9	P57004	57.05	20	G	28.96	538
	P86002	86.01	20	G		
10	P527	30. 08	–	SL	55.94	506
	P86003	86.02	20	G		

for individual stars; the standard errors of the proper motions are very important for membership determination and dynamic studies of an open cluster.

Tables 2 and 3 give the accuracies of the final proper motions for stars in the region of NGC 1750 and NGC 1758 with different numbers of measured pairs (greater than 2) and different distances from the cluster center, and in different magnitude ranges, respectively. The units of the proper motions and their accuracies in this paper are mas/year. It is shown from the tables that the accuracies depend strongly on the number of plate pairs, and the greater the number of pairs, the higher the accuracies of the final proper motions of the stars. This shows that increasing the number of available plate pairs is very important for improving the accuracy of proper motions. It can also be seen from the tables that there is no obvious relation between the accuracies of the final proper motions and the distances of stars from the plate center or between the accuracies and the magnitudes of stars, which shows that the imaging quality of the telescope has been very good and that the PDS machine was quite stable. Figure 2 gives the number of stars for which different numbers of plate-pairs are available. More than 70% of proper motions are obtained from more than 6 plate pairs. The rms errors of the proper motions of all 540 stars are ϵ_x=± 0.63 mas/yr, ϵ_y =± 0.70 mas/yr, and ϵ=± 0.67 mas/yr, where $\epsilon = \sqrt{\frac{\epsilon_x^2+\epsilon_y^2}{2}}$; the rms proper-motions errors of the 70% of all stars that were observed in more than 6 plate pairs (see above) are better than ±0.50 mas/yr. This can be seen from Fig. 3, which shows the relations ϵ_x versus N, ϵ_y versus N and ϵ versus N. So what we can say is that the errors of the proper motions of stars in the region of NGC 1750 and NGC 1758 obtained by us are relatively high, because of the good stellar images taken with the 40 cm double astrograph and the excellent positioning behavior of the PDS scanning machine.

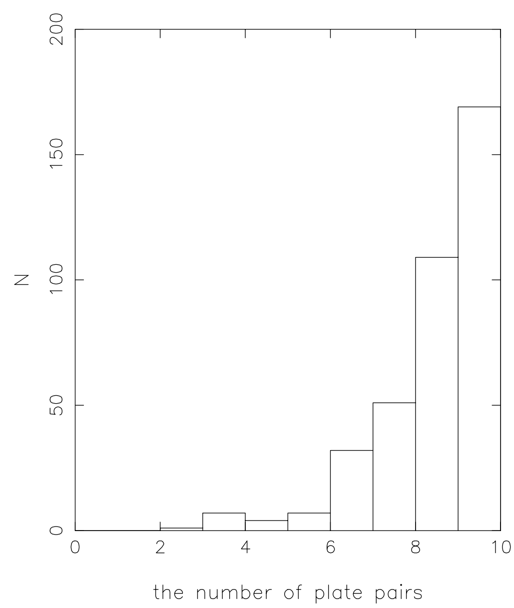

Fig. 2. The number of stars vs. the number of plate pairs available

3. Membership determination

The determination of reasonable membership criteria for open clusters is an essential prerequisite for further astrophysical research. The analysis of photometric and/or

Table 2. Accuracies of proper motions for stars in different numbers of plate pairs and at different distances from the plate center in the NGC 1750/1758 region (units in mas/yr)

pairs	$r \leq 20'$				$20' < r \leq 40'$				$40' < r \leq 60'$			
	N	ϵ_x	ϵ_y	ϵ	N	ϵ_x	ϵ_x	ϵ	N	ϵ_x	ϵ_x	ϵ
3 — 5	14	1.01	0.94	0.98	21	0.95	0.95	0.95	4	0.93	0.96	0.85
6 — 7	58	0.74	0.84	0.79	88	0.72	0.78	0.75	14	0.66	0.81	0.74
8 — 10	134	0.43	0.74	0.61	170	0.45	0.66	0.56	25	0.64	0.67	0.65
3 — 10	206	0.58	0.78	0.69	279	0.60	0.72	0.67	43	0.64	0.67	0.65

Table 3. Accuracies of proper motions for stars in different magnitude ranges in the region of NGC 1750 and NGC 1758 (units in mas/yr)

V	N	ϵ_x	ϵ_y	ϵ
$V \leq 11.5$	43	0.49	0.80	0.66
$11.5 < V \leq 12.5$	86	0.64	0.84	0.75
$12.5 < V \leq 13.0$	130	0.59	0.80	0.70
$13.0 < V \leq 13.5$	157	0.56	0.72	0.66
$V > 13.5$	112	0.60	0.78	0.71
	528	0.59	0.74	0.67

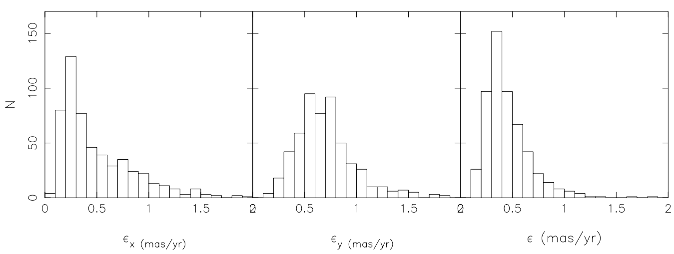

Fig. 3. Mean error of proper motions vs. proper motions

kinematic data is usually used for this purpose. Because there are a lot of binaries in open clusters, the uncertainty for photometric membership determination can be quite large (Mathieu 1984). The most popular way to distinguish cluster members from field stars is therefore based on kinematic data, especially on radial velocities and on relative proper motions obtained with a number of plates with large epoch differences. The latter technique can be more powerful than the former because it exploits the motion in two dimensions rather than in only one, and because it is less sensitive to orbital motion in unrecognized binary systems. The fundamental mathematical model set up by Vasilevskis et al. (1957) and the technique based upon the maximum likelihood principle developed by Sanders (1971) have been devised to obtain the distribution of stars in the region of a cluster and the membership probabilities of individual stars. Since then many astronomers — including those in our group — have refined this method continuously. An improved method for membership determination of stellar clusters based on proper motions with different observed accuracies was developed by Stetson (1980) and Zhao & He (1990). Then Zhao & Zhao (1994) added the correlation coefficient of the field star distribution to the set of parameters describing their distribution on the sky. The spatial distribution of cluster stars and the dependence of the distribution parameters on the magnitudes of stars were considered by Su et al. (1997). In the meantime, the fundamental principle of Sanders' method was successfully used for membership determination of clusters of galaxies. Zhao et al. (1988) and Zhao & Zhao (1994) established and developed a statistical method that can be used to determine the distribution parameters and membership of rich galaxy clusters by using radial velocities and positions of galaxies as the observational criteria. In view of possible multiple substructures in galaxy clusters, in his doctoral thesis Shao (1996) extended the above method to the situation of multiple substructures and multiple criteria. He developed a strict, rigorous, and useful mathematical model, and successfully determined the distribution parameters and membership of a galaxy cluster with a complex structure.

As we pointed out in the introduction, there may be two open clusters, NGC 1750 and NGC 1758, in the region examined in the present paper. In order to confirm this point, we will extend the maximum-likelihood method available for the multi-substructure and multi-criterion case in one-dimensional velocity space (radial velocity) to the case of two-dimensional velocity space (relative proper motions), to determine the distribution parameters and membership of the two open clusters.

3.1. Basic hypotheses of the model

Assume that the observational data consist of K kinds of components, including K_c subclusters and K_f field populations (foreground or background), where $K = K_c + K_f$. Then, the star distribution Φ in the observational data space being used as criteria, such as positions and proper motions, can be expressed as a mixture of K subdistributions Φ_c and Φ_f:

$$\Phi = \sum_{i=1}^{K} \Phi_\mathrm{k}(i) = \sum_{i=1}^{K_\mathrm{c}} \Phi_\mathrm{c}(i) + \sum_{i=1}^{K_\mathrm{f}} \Phi_\mathrm{f}(i). \quad (1)$$

Furthermore, if we use positions (two dimensions) and proper motions (two dimensions) as criteria, Φ_c and Φ_f can be expressed as follows

$$\Phi_\mathrm{c} = \sum_{i=1}^{K_\mathrm{c}} n_\mathrm{c}(i) \cdot \Phi_\mathrm{c}^\mathbf{v}(i) \cdot \Phi_\mathrm{c}^\mathbf{r}(i), \quad (2)$$

$$\Phi_\mathrm{f} = \sum_{i=1}^{K_\mathrm{f}} n_\mathrm{f}(i) \cdot \Phi_\mathrm{f}^\mathbf{v}(i) \cdot \Phi_\mathrm{f}^\mathbf{r}(i). \quad (3)$$

For the case of star clusters, which is different from that of galaxy clusters, only one field population should be considered, which means $K_\mathrm{f} = 1$. Therefore Eq. (3) can be simplified as

$$\Phi_\mathrm{f} = n_\mathrm{f} \cdot \Phi_\mathrm{f}^\mathbf{v} \cdot \Phi_\mathrm{f}^\mathbf{r}. \quad (4)$$

In the above equations n_c and n_f are the normalized numbers of subcluster members and field stars. They should satisfy the following condition:

$$\sum_{i=1}^{K_\mathrm{c}} n_\mathrm{c}(i) + n_\mathrm{f} = 1. \quad (5)$$

Respectively, $\Phi_\mathrm{c}^\mathbf{r}$, $\Phi_\mathrm{f}^\mathbf{r}$, $\Phi_\mathrm{c}^\mathbf{v}$, and $\Phi_\mathrm{f}^\mathbf{v}$, are the normalized distribution functions of subcluster members and field stars in the position ($\mathbf{r}$) and relative proper motion ($\mathbf{v}$) spaces. Obviously, n refers to the relative number of members of each of the different components, and Φ is refers to the shape of each distribution. Usually, the distribution of subcluster members in proper motion space can be assumed to be a (2-dimensional, isotropic) Gaussian function, and that of field stars is also Gaussian (also 2-dimensional), but with an elliptical shape. Projected onto the surface of the celestial sphere, we have no reason to reject a uniform distribution of field stars. On the other hand, the projected number-density of subcluster members should be a function of position. Some approximate formulae can be used to describe the function: for example, the King model profile or — more simply — a Gaussian (this paper) with characteristic radius r_c is often used. Thus,

$$\Phi_\mathrm{c}^\mathbf{r} = \frac{1}{2\pi r_\mathrm{c}^2} . \exp\left\{ -\frac{1}{2} \left[\left(\frac{x_i - x_\mathrm{c}}{r_\mathrm{c}} \right)^2 + \left(\frac{y_i - y_\mathrm{c}}{r_\mathrm{c}} \right)^2 \right] \right\}, \quad (6)$$

$$\Phi_\mathrm{f}^\mathbf{r} = \frac{1}{\pi r_\mathrm{max}^2} \quad (7)$$

and

$$\Phi_\mathrm{c}^\mathbf{v} = \frac{1}{2\pi(\sigma_\mathrm{c}^2 + \epsilon_{xi}^2)^{1/2}(\sigma_\mathrm{c}^2 + \epsilon_{yi}^2)^{1/2}} . \exp\left\{ -\frac{1}{2} \left[\frac{(\mu_{xi} - \mu_{x\mathrm{c}})^2}{\sigma_\mathrm{c}^2 + \epsilon_{xi}^2} + \frac{(\mu_{yi} - \mu_{y\mathrm{c}})^2}{\sigma_\mathrm{c}^2 + \epsilon_{yi}^2} \right] \right\}, \quad (8)$$

$$\Phi_\mathrm{f}^\mathbf{v} = \frac{1}{2\pi(1-\gamma^2)^{1/2}(\sigma_{x\mathrm{f}}^2 + \epsilon_{xi}^2)^{1/2}(\sigma_{y\mathrm{f}}^2 + \epsilon_{yi}^2)^{1/2}} . \exp\left\{ -\frac{1}{2(1-\gamma^2)} \left[\frac{(\mu_{xi} - \mu_{x\mathrm{f}})^2}{\sigma_{x\mathrm{f}}^2 + \epsilon_{xi}^2} - \frac{2\gamma(\mu_{xi} - \mu_{x\mathrm{f}})(\mu_{yi} - \mu_{y\mathrm{f}})}{(\sigma_{x\mathrm{f}}^2 + \epsilon_{xi}^2)^{1/2}(\sigma_{y\mathrm{f}}^2 + \epsilon_{yi}^2)^{1/2}} + \frac{(\mu_{yi} - \mu_{y\mathrm{f}})^2}{\sigma_{y\mathrm{f}}^2 + \epsilon_{yi}^2} \right] \right\}, \quad (9)$$

where ϵ_{xi} and ϵ_{yi} are the observed errors of the proper-motion components of the i-th star; and x_c, y_c (center of subcluster), r_c (characteristic radius), $\mu_{x\mathrm{c}}, \mu_{y\mathrm{c}}, \mu_{x\mathrm{f}}, \mu_{y\mathrm{f}}$ (mean values of proper motions of member and field stars), $\sigma_\mathrm{c}, \sigma_{x\mathrm{f}}, \sigma_{y\mathrm{f}}$ (intrinsic proper motion dispersions of member and field stars) and γ (correlation coefficient) are the spatial and kinematic distribution parameters (Shao & Zhao 1996).

3.2. Solution and results

There are nineteen unknown parameters $q_j (j = 1, 2, ...19)$ in Eqs. (6)–(9): $(n_\mathrm{c}(i),\ x_\mathrm{c}(i),\ y_\mathrm{c}(i),\ r_\mathrm{c}(i),\ \mu_{x\mathrm{c}}(i),\ \mu_{y\mathrm{c}}(i),\ \sigma_\mathrm{c}(i))_{i=1,2}$, $(\mu_{x\mathrm{f}},\ \mu_{y\mathrm{f}},\ \sigma_{x\mathrm{f}},\ \sigma_{y\mathrm{f}})$, and γ. The standard maximum likelihood method can be used to obtain the values of these parameters. The likelihood function of the sample can be written as:

$$L = \prod_{i=1}^{N} \Phi(i). \quad (10)$$

Now according to the maximum likelihood principle we have

$$\frac{\partial \ln L}{\partial q_i} = \frac{\partial}{\partial q_i}(\sum \ln \Phi_j) = 0 \qquad (j = 1, 2......19). \quad (11)$$

From the above equation the nineteen unknown distribution parameters can be found. Then we can determine the probability that the i-th star belongs to either of the two different open clusters by the following equations:

$$P_\mathrm{c}(i) = \frac{\Phi_\mathrm{c}(i)}{\Phi(i)} \qquad (c = 1, 2). \quad (12)$$

The uncertainties of the distribution parameters can be found from a square matrix $\mathbf{A}$ composed of $m \times m$ second-order derivatives $\frac{\partial^2 \ln L}{\partial q_\mathrm{l} \partial q_\mathrm{t}}$, $(l, t = 1, 2, \ldots, m)$, q referring in turn to each of the parameters and $m = 19$ being the order number of the matrix:

$$\mathbf{A} = (\frac{\partial^2 \ln L}{\partial q_\mathrm{l} \partial q_\mathrm{t}}). \quad (13)$$

Let the inverse matrix of $\mathbf{A}$ be

$$\mathbf{B} = \mathbf{A}^{-1} = (-b_\mathrm{lt}), \quad (14)$$

then the uncertainty of the parameter q_l is

$$\Delta q_\mathrm{l} = (-b_\mathrm{ll})^{1/2}. \quad (15)$$

The distribution parameters of the two open clusters and their corresponding uncertainties can be obtained and are shown in Table 4, where the units of the proper motions and proper motion intrinsic dispersions are mas/yr.

Table 4. Distribution parameters and their uncertainties for NGC 1750 and NGC 1758 (the units of μ and σ in mas/yr)

	No. stars	α_{2000} ($5^{\rm h}+$)	δ_{2000} (23^0+)	$r_{\rm c}$	μ_x	μ_y	σ	σ_x	σ_y	γ
NGC 1750	326	$3^{\rm m}44\overset{\rm s}{.}74$	$43'37\overset{''}{.}7$	$22\overset{'}{.}70$	−0.31	2.55	1.74			
		$\pm3\overset{\rm s}{.}8$	$\pm54''$	$\pm1\overset{'}{.}35$	±0.13	±0.13	±0.08			
NGC 1758	35	$4^{\rm m}39\overset{\rm s}{.}22$	$48'52\overset{''}{.}82$	$2\overset{'}{.}93$	1.16	1.11	1.18			
	±6	$\pm2\overset{\rm s}{.}4$	$\pm41''$	$\pm0\overset{'}{.}53$	±0.33	±0.28	±0.17			
field	179				1.79	−1.87		5.84	5.92	0.135

The two proper motion dispersions of the cluster members in Table 4 reflect mainly the internal velocity dispersions of the two clusters. This would also explain the two different values for the proper motion dispersion, which have also different distances from the Sun. We will present the further research about photometry, H−R diagram, distance and another astrophysical parameters of the two open clusters in the next paper. Table 5 (only available in electronic form) lists the results for all 540 stars in the region of the two open clusters: Col. 1 is the ordinal star number; Cols. 2 and 3 are $\alpha_{\rm J2000.0}$ and $\delta_{\rm J2000.0}$, based on 27 stars in the PPM Catalogue (the cross-identifications of the 27 stars are given in Table 6); Cols. 4 and 5 are the proper motions; Cols. 6 and 7 are the standard errors of the proper motions; Cols. 8, 9, and 10 are probabilities of stars belonging to NGC 1750 (P_1), NGC 1758 (P_2), and the field ($P_{\rm f}$) respectively; and Col. 11 is the number of plate pairs used in the present study. Table 7 gives the cross-identifications of 32 stars between Table 5 and Straižys(Straižys et al. 1992). Figures 4 and 5 show the proper motion vector-point diagram and the position distribution on the sky for all the measured stars respectively, where "•" denotes a member of NGC 1750 with $P_1 \geq 0.7$, "○" a member of NGC 1758 with $P_2 \geq 0.7$, and all another stars are considered field stars indicated by "×". It can be noted from the two diagrams that the centers in positional space and the centers in velocity (proper motion) space for the two open clusters are very clearly separated, which can be confirmed from the distribution parameters listed in Table 4. We can also see from the diagrams that the central concentration of NGC 1758 in positional space is more obvious than its central concentration in velocity space, which indicates that the spatial distribution of NGC 1758 plays a dominant role in its definition. The membership probability histogram (Fig. 6) shows a very clear separation between cluster members and field stars. We find that the numbers of stars with membership probabilities higher than 0.7 for NGC 1750 and NGC 1758 are 332 and 23 respectively, and their average membership probabilities are 0.93 and 0.88 respectively, i.e., contamination by field stars is expected to be only 7% and 12% for the two clusters. All of our work indicates that the determination of two open clusters is successful: there exist two real open clusters NGC 1750 and NGC 1758.

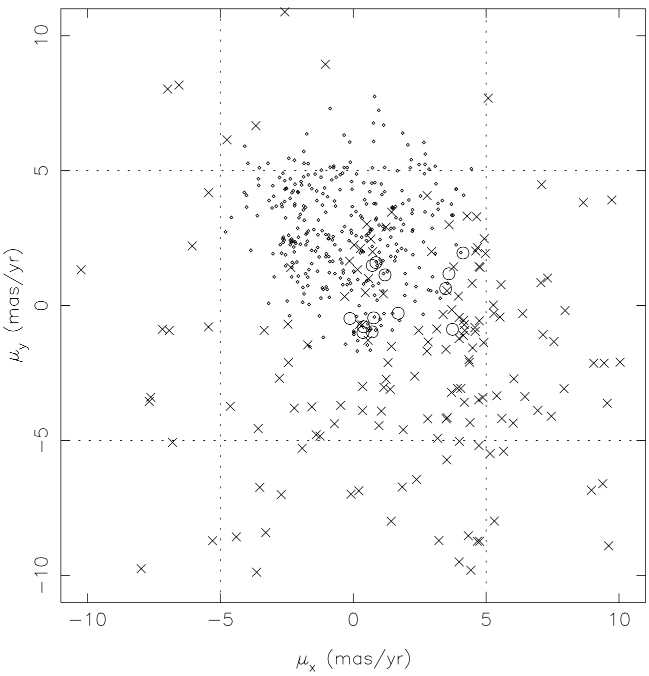

Fig. 4. The proper motion vector-point diagram of NGC 1750 and NGC 1758 ("•" denotes a member of NGC 1750 with $P_1 \geq 0.7$, "○" a member of NGC 1758 with $P_2 \geq 0.7$,"×" a field star)

4. Discussion

4.1. Effectiveness of membership determination

The clustering of celestial bodies (such as star clusters or galaxy clusters) is an important research area in astronomy and astrophysics. As membership in clusters of celestial bodies is determined, contamination by background and foreground objects through the influence of the observational projection effect can not be avoided. Ever since the concept of membership probability was established to distinguish real cluster members from field objects on the basis of observational data (proper motions, radial velocities, photometry, polarization, etc.), the method suggested by Sanders (1971) has been a successful technique. The particular method of membership determination used in the present study is an improved one. Shao & Zhao (1996) set up the concept of the effectiveness of membership determination, which can be reasonably used to judge quantitatively how effective the results of membership determination of a cluster are. They suggested a widely applicable

Table 5. Proper motions and membership probabilities of stars in the region of NGC 1750 and NGC 1758 (the units of μ and σ in mas/yr)

No.	α(j2000.0) $5^{\rm h}$	δ(j2000.0)	μ_x	μ_y	σ_x	σ_y	p_1	p_2	$P_{\rm f}$	W
1	$3^{\rm m}21\overset{\rm s}{.}73$	$+24°20'12.6''$	−0.441	1.573	0.026	0.089	0.00	0.00	1.00	10
4	3 55.34	24 19 56.5	1.004	−0.210	0.109	0.349	0.00	0.00	1.00	2
5	2 51.04	24 19 55.6	0.019	1.130	0.065	0.126	0.04	0.00	0.96	5
6	3 34.96	24 19 21.7	0.302	−1.511	0.027	0.069	0.00	0.00	1.00	10
7	2 45.09	24 19 0.3	0.097	0.298	0.024	0.110	0.97	0.00	0.03	10
8	6 17.81	24 18 36.0	−0.074	0.511	0.056	0.138	0.95	0.00	0.05	8
9	2 23.01	24 18 44.4	0.076	0.676	0.084	0.188	0.90	0.00	0.10	3
10	4 53.35	24 18 31.7	0.248	0.522	0.128	0.107	0.91	0.00	0.09	6
11	4 13.36	24 18 27.5	0.141	0.182	0.033	0.031	0.96	0.00	0.04	7
13	4 49.16	24 18 18.0	−0.050	0.429	0.026	0.089	0.97	0.00	0.03	10
14	1 58.52	24 18 23.0	0.488	−0.341	0.028	0.092	0.03	0.00	0.98	8
15	2 52.07	24 18 8.9	−0.081	0.466	0.032	0.075	0.97	0.00	0.03	9
16	5 35.98	24 17 58.6	−0.166	0.004	0.037	0.079	0.91	0.00	0.09	10
17	2 45.83	24 18 3.9	−0.303	0.444	0.028	0.083	0.94	0.00	0.06	10
18	5 27.11	24 17 24.4	−0.265	0.490	0.018	0.066	0.94	0.00	0.06	7
20	2 40.41	24 17 2.5	−0.169	0.276	0.029	0.075	0.97	0.00	0.03	10
21	5 31.18	24 16 22.4	−0.798	−0.975	0.045	0.051	0.00	0.00	1.00	9
22	5 57.73	24 15 58.6	0.367	−1.129	0.035	0.050	0.00	0.00	1.00	10
24	3 41.99	24 15 54.9	−0.240	−0.038	0.019	0.050	0.88	0.00	0.12	9
25	5 0.84	24 15 35.3	0.206	0.595	0.112	0.065	0.90	0.00	0.10	7
26	3 24.44	24 15 39.0	−0.067	0.368	0.016	0.051	0.98	0.00	0.02	10
27	1 16.52	24 15 12.2	−0.105	0.894	0.145	0.289	0.51	0.00	0.49	3
28	2 32.17	24 15 7.0	0.755	−0.133	0.139	0.097	0.01	0.00	0.99	5
29	4 25.64	24 14 58.2	−0.139	−0.479	0.015	0.036	0.05	0.00	0.95	9
30	3 58.19	24 14 43.3	0.118	0.422	0.082	0.089	0.97	0.00	0.03	9
31	2 19.50	24 14 26.9	−0.401	0.365	0.032	0.051	0.91	0.00	0.09	10
34	5 52.00	24 13 17.2	−0.257	0.431	0.045	0.078	0.94	0.00	0.06	10
36	3 15.35	24 13 23.6	0.106	−0.391	0.025	0.038	0.17	0.00	0.83	10
37	5 8.93	24 13 4.3	0.266	0.604	0.117	0.043	0.86	0.00	0.14	7
38	4 30.20	24 13 4.7	0.031	−2.188	0.027	0.038	0.00	0.00	1.00	10
39	3 16.99	24 13 8.3	−0.302	0.192	0.022	0.052	0.95	0.00	0.05	10
40	4 10.58	24 12 41.1	−0.033	0.293	0.017	0.051	0.98	0.00	0.02	10
41	3 24.04	24 12 41.2	−0.029	−0.143	0.019	0.051	0.83	0.00	0.17	10
42	4 36.50	24 12 32.9	−0.257	1.088	0.010	0.038	0.05	0.00	0.95	8
43	3 39.96	24 12 20.8	0.508	0.769	0.072	0.050	0.20	0.00	0.80	8
44	1 26.53	24 12 23.0	0.064	−0.165	0.020	0.031	0.73	0.00	0.27	8
46	5 12.97	24 11 23.9	0.002	−0.095	0.034	0.055	0.86	0.00	0.14	10

Table 6. The cross-identification of stars between the PPM catalogue and Table 5

Table 5	PPM	Table 5	PPM	Table 5	PPM
465	93970	7	93997	343	94015
470	93971	430	93998	253	94016
246	93977	475	94002	82	94018
601	93978	354	94005	346	94025
101	93989	340	94006	423	94026
200	93992	211	94007	49	94033
180	93993	337	94012	344	94034
172	93994	329	94013	153	94039
69	93996	349	94014	229	94048

Table 7. The cross-identification of 32 stars between Table 5 and Straižys (Straižys et al. 1992)

Table5	Straižys	Table5	Straižys	Table5	Straižys	Table5	Straižys
153	87	221	81	268	91	344	68
177	70	222	63	271	67	346	56
197	65	228	64	274	84	349	46
201	82	229	93	280	71	354	41
207	79	232	50	315	57	380	59
211	43	253	52	329	45	405	90
214	83	262	67	337	44	409	58
220	92	265	88	340	42	423	60

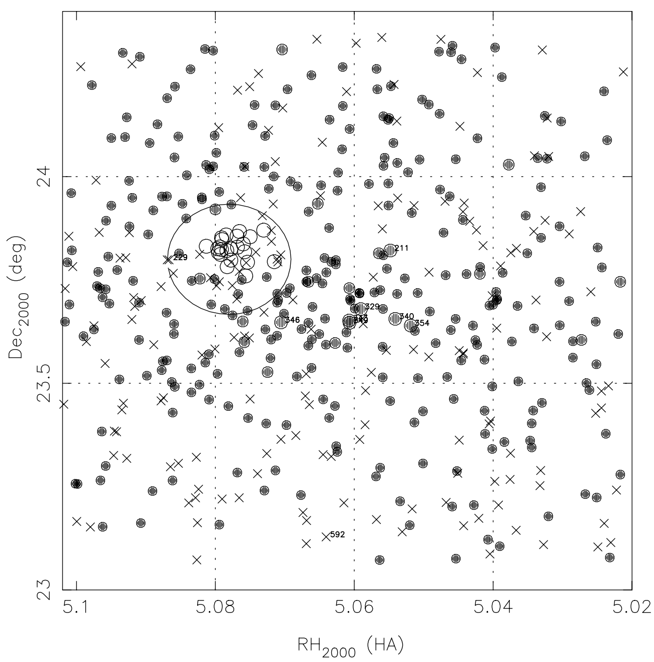

Fig. 5. The position distribution of stars in NGC 1750 and NGC 1758 ("•" denotes a member of NGC 1750 with $P_1 \geq 0.7$, "∘" a member of NGC 1758 with $P_2 \geq 0.7$, "×" a field star)

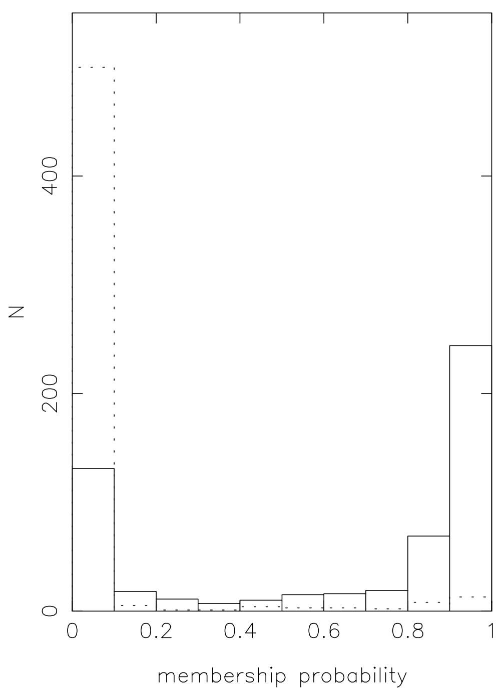

Fig. 6. The histogram of membership probability of NGC 1750 and NGC 1758 (solid line is the stars of NGC 1750, dotted line NGC 1758)

index E which can be used to measure the effectiveness of membership determination:

$$E = 1 - N\sum_{i=1}^{N}\{P(i)\,[1-P(i)]\} \Big/ \left\{\sum_{i=1}^{N}P(i)\sum_{i=1}^{N}[1-P(i)]\right\}. \quad (16)$$

The bigger E is, the more effective the membership determination is. If $\overline{P}$ is the average membership probability of all the bodies in a sample, i.e., $\overline{P} = \sum_{i=1}^{N} P(i)/N$, then Eq. (16) can be written as follows:

$$E = \sum_{i=1}^{N}\left[P^2(i) - \overline{P}^2\right] \Big/ \left(N\overline{P} - N\overline{P}^2\right). \quad (17)$$

From Eq. (17) we can determine that the effectiveness of membership determination is 0.66 and 0.76 for NGC 1750 and NGC 1758 respectively, under the assumption of only one cluster, the effectiveness of membership determination is 0.60. It indicates that existence of two cluster is more reasonable than one cluster. It is shown in the Fig. 3 of Shao's paper (Shao & Zhao 1996) that the effectiveness of membership determination of 43 open clusters are from 0.20 to 0.90 and the peak value is 0.55. Compared with the their work, we can see that the effectiveness of membership determination for two open clusters present in this paper is now significantly higher in both cases.

4.2. Surface density distribution

The surface density distribution for the cluster members can be defined by the following equations:

$$\rho_c = \frac{\sum P_c(i)}{\Delta S} \pm \frac{\sqrt{\sum P_c(i)}}{\Delta S}. \quad (18)$$

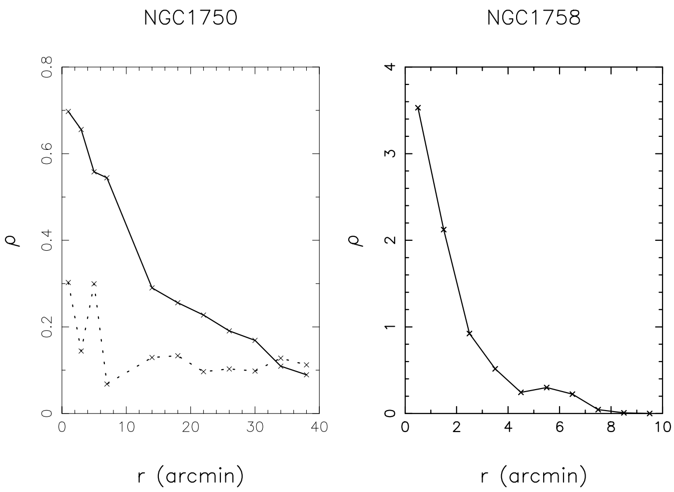

Fig. 7. The surface density distribution (dotted line is the field stars)

The second term of the right side of the above equation is the uncertainty, σ_i, which follows the Poisson distribution; at the same time the surface density distribution of the field stars is:

$$\rho_{\rm f} = \frac{\sum P_{\rm f}(i)}{\Delta S}. \qquad (19)$$

In Eqs. (18) and (19) the sums are performed for the stars in the area ΔS using the membership probabilities for each of the two clusters ($P_{\rm c}(i)$, $i = 1, 2$) and the field ($P_{\rm f}$) in turn. The surface densities $\rho_{\rm c}$ and $\rho_{\rm f}$ are calculated for each different ΔS, which is defined as an annulus with varying radial distance from the cluster center, and $\rho_{\rm c}$ is calculated separately for each of the two clusters. Table 8 gives the surface density distributions $\rho_{\rm c}$ of the member stars and the corresponding uncertainty σ in the two distributions.

Figure 7 shows the surface density distributions of members of the two open clusters and of the common field stars respectively. It is seen that the surface densities of member stars in the two clusters decrease rapidly with distance from the cluster center, and the radial variation is more obvious for NGC 1758 than for NGC 1750. We can see from these figures that both NGC 1750 and NGC 1758 have good central concentration, while on the other hand the surface density of field stars is quite uniform in the whole region. At the same time, these figures indicate that the two star clusters defined in the present study actually exist independently, though they overlap each other on the sky.

4.3. The radii of NGC 1750 and NGC 1758

In order to study the fundamental dynamics, we can use the surface density distribution to fit the radius of a cluster on the basis of King's model. King (1962) gave an empirical formula for the surface density of a stellar system

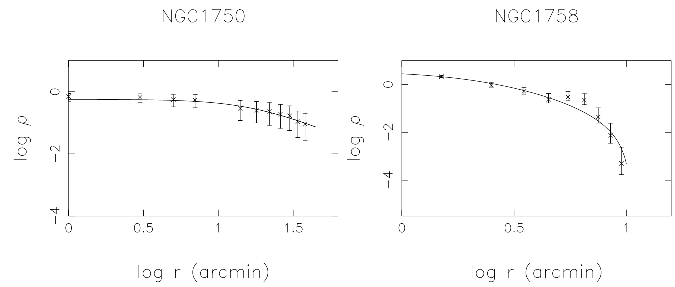

Fig. 8. The fitting results obtained from King's empirical density law

$$\rho = \rho_0 \left[\frac{1}{(1 + r^2/r_{\rm c}^2)^{1/2}} - \frac{1}{(1 + r_{\rm t}^2/r_{\rm c}^2)^{1/2}} \right]^2, \qquad (20)$$

where ρ is the density, and ρ_0, $r_{\rm c}$ and $r_{\rm t}$ are the fitting parameters, which have clear physical meanings: $r_{\rm c}$ and $r_{\rm t}$ are the core radius and the tidal radius of a cluster, and ρ_0 is the central surface density; $c = r_{\rm t}/r_{\rm c}$ can be used to describe the central concentration of the cluster. The fitting parameters can be obtained from a χ^2 test:

$$\chi^2 = \sum_i \frac{1}{\sigma_i^2} \left[\rho_{\rm ob}(i) - \rho_{\rm exp}(i)\right]^2 \qquad (21)$$

where $\rho_{\rm ob}$ is the observed value of the surface density in an annulus and σ_i is its uncertainty, which are defined in Eq. (18) and are listed in Table 7. $\rho_{\rm exp}$ is the theoretical value of the surface density from derived from Eq. (20). The fitting results are: $\rho_0 = 0.57/{\rm arcmin}^2$, $r_{\rm c} = 17'.2$ with a significance level of 89% for NGC 1750; $\rho_0 = 5.26/{\rm arcmin}^2$, $r_{\rm c} = 2'.3$, $r_{\rm t} = 10'.4$ with a significance level of 91% for NGC 1758. In the previous section we obtained Gaussian characteristic radii for the two open clusters of $22'.70 \pm 1'.35$ for NGC 1750 and $2'.93 \pm 0'.53$ for NGC 1758 from the maximum likelihood solution. We can say that the results of two different methods are basically consistent. The solution for the dynamic radius $r_{\rm t}$ of NGC 1750 does not converge, and we believe that the main reason for this is that King's model is applicable to a star system with full relaxation, such as globular clusters or old open clusters with strong concentration, whereas the concentration of NGC 1750 is not very obvious. From the fitting results we see that the central concentration of NGC 1758 is 4.58, which means NGC 1758 is of higher concentration. This behavior can also be seen from the fitting curves shown in Fig. 8.

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Table 8. The surface densities of the member stars and the corresponding uncertainties in two open clusters

NGC 1750											
r(arcmin)	1	3	5	7	14	18	22	26	30	34	38
ρ(1/arcmin)	0.70	0.66	0.58	0.54	0.29	0.25	0.23	0.19	0.17	0.11	0.09
σ	0.42	0.27	0.17	0.14	0.05	0.04	0.04	0.03	0.03	0.02	0.02
NGC 1758											
r(arcmim)	0.5	1.5	2.5	3.5	4.5	5.6	6.5	7.5	8.5	9.5	
ρ(1/arcmin)	3.53	2.12	0.92	0.52	0.24	0.30	0.22	0.04	0.01	0.00	
σ	1.87	0.84	0.43	0.27	0.16	0.16	0.13	0.05	0.02	0.01	

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