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Galilean satellite ephemerides E5

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Abstract. New ephemerides of Jupiter's Galilean satellites are produced from an analysis of CCD astrometric data, Voyager-mission optical navigation images, mutual event observations, photographic plates, and eclipse timing observations. The resulting parameters, for use in the galsat computer software, are in the B1950 frame for use by the Galileo space mission. Results in the J2000 system are also available.

Key words: astrometry — celestial mechanics — ephemerides — Planets and satellites: Jupiter

1. Introduction

This paper documents the Galilean satellite ephemerides designated as E5, which were delivered in support of the Galileo space mission to Jupiter. The E5 ephemerides supersede the E4 ephemerides, which were developed (Lieske 1994a) without using CCD astrometric data in order to assess the new data type. It is believed that the E5 ephemerides are better than the E3 and E4 ephemerides and they are recommended for general usage. The parameters of E5 are given in the B1950 system so that the galsat software (Lieske 1977) can be employed directly to compute coordinates in the B1950 frame, which has been adopted for the Galileo mission.

The ephemerides E2 (Lieske 1980) were developed prior to the Voyager mission and were based solely on an analysis of earth-based observations. The E2 ephemerides utilized mutual event data from 1973 (Aksnes & Franklin 1976), photographic astrometric observations from 1967-1978 (Pascu 1977 1979), and Jovian satellite eclipse timings from 1878-1974 (Pickering 1907; Pierce 1974; Lieske 1980).

Post-Voyager mission ephemeris improvements yielded ephemerides E3, which included Voyager optical navigation astrometric data and Voyager-derived physical constants (Campbell & Synnott 1985). The E3 ephemerides employed mutual event data from 1973 and 1979 (Aksnes et al. 1984), Voyager optical navigation astrometric measurements from 1979 (Synnott et al. 1982), additional photographic observations by D. Pascu from 1973-1979, and eclipse timings from 1652 to 1983 (Lieske 1986, 1987).

The initial pre-Galileo mission ephemerides were designated E4 (Lieske 1994a) and included extended mutual event data and photographic data, but no CCD observations, since they were still in the process of being evaluated. The E4 ephemerides employed the previously mentioned Voyager data, mutual event data from 1973 and 1979 corrected for phase effects by adding δt to the observation time (Aksnes et al. 1986), photographic data and Jovian eclipse timings, as well as additional mutual event astrometric measurements from 1985 and 1991 (Aksnes et al. 1986; Franklin et al. 1991; Kaas et al. 1997; Descamps 1994; Goguen et al. 1988; Goguen 1994; Mallama 1992), and additional photographic observations from Pascu (1993) covering the interval 1980-1991. Three-years' of CCD data from Flagstaff (Monet et al. 1994; Owen 1995) were evaluated, but not employed in developing the E4 ephemerides.

The E5 ephemerides represent the most current evolution of the Galilean satellite ephemerides and incorporate all of the above data types, including an evaluation the Doppler data of Ostro et al. (1992).

The 50 parameters which define the theory of motion of the Galilean satellites (Lieske 1977) could also be transformed in a manner such that the same galsat computer program can be employed to compute rectangular coordinates with their values being in the J2000 system. Documentation and an algorithm for such transformation of all galsat-related ephemerides (e.g., Lieske 1977, 1980; Arlot 1982; Vasundhara 1994) will be issued later. In the meantime the equatorial coordinates can be transformed in the following manner.

For the Galileo mission, all input quantities are in the B1950 frame and Earth equatorial coordinates transformation from B1950 to J2000 when necessary is done by the matrix multiplication

$$r_{\mathrm{J2000}} = A r_{\mathrm{B1950}} \,, \tag{1}$$

where the matrix A could be taken from that recommended by IAU Commission 20 (West 1992),

$$A = P_{\mathrm{IAU}} R_3(-0\farcs525) \tag{2}$$

with P_{IAU} being the standard IAU precession matrix from B1950 to J2000 (Lieske 1979),

$$P_{\mathrm{IAU}} = R_3(-z_{\mathrm{A}}) R_2(\theta_{\mathrm{A}}) R_3(-\zeta_{\mathrm{A}}) \tag{3}$$

or A could be taken from the earlier discussion of Standish (1982), which was developed for transforming from DE118 to DE200,

$$A = R_3(+0\farcs00073) P_{\mathrm{IAU}} R_3(-0\farcs53160) \,. \tag{4}$$

It essentially consists of a rotation ΔE in the B1950 equatorial plane from the FK4 origin to the dynamical equinox and then precessing from B1950 to J2000 using the IAU 1976 equatorial precession parameters P_{IAU} (Lieske et al. 1977).

The matrix A could also be derived from Lieske's discussion (1994b) on the precession of orbital elements,

$$A = R_1(-\varepsilon_{\mathrm{J2000}}) R_3(L') R_1(-J_{\mathrm{A}}) R_3(-L) R_1(\varepsilon_{\mathrm{B1950}}) \,. \tag{5}$$

For the Galileo mission, the method of Standish given in Eq. (4) is employed to precess from B1950 to J2000.

The rotation matrices R_i are the standard matrices for rotations about the x, y, or z axes for $i = 1, 2, 3$:

$$\begin{aligned}
R_1 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \\
R_2 &= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \\
R_3 &= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \,.
\end{aligned} \tag{6}$$

The various matrices mentioned in Eqs. (2), (4) and (5) are presented in Table 1. The maximum difference in satellite coordinates, due to the different precessional transformations, is about 1.5 km, so any of the previously mentioned matrices could be used in a practical situation.

2. The basic parameters

In the galsat-type ephemerides, the Jovicentric Earth-equatorial coordinates of the Galilean satellites are computed as a function of 50 "galsat" parameters (Lieske 1977). The definitions of the basic parameters upon which the theory depends are given in Tables 2 and 3. It is seen that they are a combination of physical parameters and orbital elements.

In the E5 ephemerides, we employed the satellite masses ($\varepsilon_1 - \varepsilon_4$) and Jupiter pole which were determined by Campbell & Synnott (1985) from their analysis of the Voyager data. The Jupiter pole is a function of the longitude of the origin of the coordinates ψ [theory parameter β_{15}], and the inclination I_{J} of Jupiter's equator to Jupiter's orbit [theory parameter ε_{25}], with some dependence upon the Jupiter orbital inclination to the ecliptic [theory parameter ε_{26}], Jupiter's node Ω_{J} [theory parameter β_{22}], and the obliquity ε of the ecliptic [theory parameter ε_{27}]. The mass of the Jupiter system was that of JPL ephemeris DE140 (Standish & Folkner 1995) *Sun/Jupiter-system* = 1047.3486. Ephemerides E3 and E4 employed Jupiter system masses which are consistent with JPL ephemeris DE125 (Standish 1985), *Sun/Jupiter-system* = 1047.349. The Jupiter pole employed was $\alpha_{\mathrm{J}} = 268\fdg001$ and $\delta_{\mathrm{J}} = 64\fdg504$ at the theory epoch JED 2443000.5 and in the B1950 frame. The rate of ψ [theory parameter β_{15}] models the secular motion of Jupiter's pole from the theory epoch. Jupiter's oblateness parameters J_2 and J_4 were also taken from the Campbell & Synnott analysis. They correspond to theory parameters ε_{11} and ε_{12} in Table 2.

Over the years different tables of ΔT have been used for the calculation of Ephemeris Time (barycentric dynamical time TDB) minus Universal Time. The appropriate table of ΔT values depends upon what model of the Moon's tidal acceleration one adopts. The Earth's Moon was most often used to determine values of ΔT prior to 1955 because of its rapid motion. The derived values of ΔT effectively depend upon a partitioning into portions due to lunar tidal effects versus real changes in ΔT. It essentially depends upon the parameter employed to describe the lunar tidal acceleration $\dot{n}_{\mathrm{Moon}}$. The classical determination of $\dot{n}_{\mathrm{Moon}} = -22.44$ arcsec/cy^2 by Spencer Jones (1939) was employed for the E1 and E2 (Lieske 1980) ephemerides by means of the Brouwer (1952) and Martin (1969) values of ΔT, which were on the Spencer Jones system.

The Morrison and Ward (1975) value of $\dot{n}_{\mathrm{Moon}} = -26.0$ arcsec/cy^2 was used for E3, E4 and E5. Tables of ΔT given by Stephenson & Morrison (1984) can be adjusted for any $\dot{n}_{\mathrm{Moon}}$ by the technique noted in Lieske (1987) for times prior to 1955.5 by computing

$$\Delta T(\dot{n}_{\mathrm{Moon}}) = \Delta T_{\mathrm{Morrison}} - 0.911(\dot{n}_{\mathrm{Moon}} + 26) T_0^2 \ \ \mathrm{sec} \tag{7}$$

where T_0 is measured in centuries from the 1955.5 epoch of Morrison (1980). The theory parameters of E1 and E2 are consistent with the Spencer-Jones value of $\dot{n}_{\mathrm{Moon}}$, while those for E3 through E5 are consistent with that of Morrison and Ward.

Table 1. Matrices for precession from B1950 to J2000

Eq. (2): Commission 20 matrix from $P_{\mathrm{IAU}}R_3(-0''\!.525)$

$$\begin{matrix} 0.9999256794956877 & -0.0111814832204662 & -0.0004859003815359 \\ 0.0111814832391717 & 0.9999374848933135 & -0.0000271625947142 \\ 0.0048590037723143 & -0.0000271702937440 & 0.9999881946023742 \end{matrix}$$

Eq. (4): Standish matrix from $R_3(+0''\!.00073)P_{\mathrm{IAU}}R_3(\text{-}0''\!.53160)$

$$\begin{matrix} 0.9999256791774783 & -0.0111815116768724 & -0.0048590038154553 \\ 0.0111815116959975 & 0.9999374845751042 & -0.0000271625775175 \\ 0.0048590037714450 & -0.0000271704492210 & 0.9999881946023742 \end{matrix}$$

Eq. (5): Lieske matrix from $R_1(-\varepsilon_{\mathrm{J2000}})R_3(L')R_1(-J_{\mathrm{A}})R_3(-L)R_1(\varepsilon_{\mathrm{B1950}})$

$$\begin{matrix} 0.9999256795268940 & -0.0111810778339439 & -0.0004859930159015 \\ 0.0111810775053504 & 0.9999374894281627 & -0.0000272382503387 \\ 0.0048599309149990 & -0.0000271030297995 & 0.9999881900987267 \end{matrix}$$

Table 2. Definition of theory parameters ε

Epsilon	Parameter	Generating value	Description
1	m_1	$449.7\ 10^{-7}(1+\epsilon_1)$	Mass of Satellite I relative to Jupiter
2	m_2	$252.9\ 10^{-7}(1+\epsilon_2)$	Mass of Satellite II relative to Jupiter
3	m_3	$798.8\ 10^{-7}(1+\epsilon_3)$	Mass of Satellite III relative to Jupiter
4	m_4	$450.4\ 10^{-7}(1+\epsilon_4)$	Mass of Satellite IV relative to Jupiter
5	S/J	$1047.355(1+\epsilon_5)$	Mass of Sun relative to Jupiter
6	n_1	$203.48895\ 4208(1+\epsilon_6)$	Mean motion of Satellite I, deg/day
7	n_2	$101.37472\ 3445(1+\epsilon_7)$	Mean motion of Satellite II, deg/day
8	n_4	$21.57107\ 1403(1+\epsilon_8)$	Mean motion of Satellite IV, deg/day
9	λ_{A}	$180^{\circ}\epsilon_9/\pi$	Amplitude of free libration, λ_{A} in deg, ϵ_9 in rad
10	n_{J}	$8.30912\ 15712\ 10^{-2}(1+\epsilon_{10})$	Mean motion of Jupiter, deg/day
11	J_2	$0.01484\ 85(1+\epsilon_{11})$	Jupiter J_2
12	J_4	$-8.107\ 10^{-4}(1+\epsilon_{12})$	Jupiter J_4
13	R_{J}	$71420(1+\epsilon_{13})$	Radius of Jupiter, km
14	P_{J}	$9.92482\ 5(1+\epsilon_{14})$	Period of Jupiter rotation, hr
15	$3(C-A)/2C$	$0.111(1+\epsilon_{15})$	Ratio of Jupiter moments of inertia
16	e_{11}	$465\ 10^{-7}(1+\epsilon_{16})$	Primary eccentricity of Satellite I, rad
17	e_{22}	$825\ 10^{-7}(1+\epsilon_{17})$	Primary eccentricity of Satellite II, rad
18	e_{33}	$15164\ 10^{-7}(1+\epsilon_{18})$	Primary eccentricity of Satellite III, rad
19	e_{44}	$73725\ 10^{-7}(1+\epsilon_{19})$	Primary eccentricity of Satellite IV, rad
20	e_{J}	$0.04846\ 02472(1+\epsilon_{20})$	Eccentricity of Jupiter
21	c_{11}	$4756\ 10^{-7}(1+\epsilon_{21})$	Primary sine inclination of Satellite I
22	c_{22}	$81490\ 10^{-7}(1+\epsilon_{22})$	Primary sine inclination of Satellite II
23	c_{33}	$31108\ 10^{-7}(1+\epsilon_{23})$	Primary sine inclination of Satellite III
24	c_{44}	$47460\ 10^{-7}(1+\epsilon_{24})$	Primary sine inclination of Satellite IV
25	I_{J}	$3.10401(1+\epsilon_{25})$	Inclination of Jupiter orbit to Jupiter equator, deg
26	J	$1.30691(1+\epsilon_{26})$	Inclination of Jupiter orbit to ecliptic, deg
27	ϵ	$23^{\circ}26'44''\!.84(1+\epsilon_{27})$	Inclination (Obliquity) of ecliptic to Earth equator deg
28	n_{S}	$3.34597\ 33896\ 10^{-2}(1+\epsilon_{28})$	Mean motion of Saturn, deg/day

Table 3. Definition of theory parameters β

Beta Parameter		Epoch value (deg)	Description
1	ℓ_1	$106\overset{\circ}{.}03042 + \beta_1$	Mean longitude of Satellite I
2	ℓ_2	$175\overset{\circ}{.}74748 + \beta_2$	Mean longitude of Satellite II
3	ℓ_3	$[120\overset{\circ}{.}60601 - \frac{1}{2}\beta_1 + \frac{3}{2}\beta_2]$	Mean longitude of Satellite III
4	ℓ_4	$84\overset{\circ}{.}51861 + \beta_4$	Mean longitude of Satellite IV
5	ϕ_λ	β_5	Free Libration $\psi_1 - 3\psi_2 + 2\psi_3 = \pi + \epsilon_9 \sin\phi_\lambda$ $= 180^\circ + \lambda_\mathrm{A} \sin\phi_\lambda$
6	π_1	$4\overset{\circ}{.}51172 + \beta_6$	Proper periapse of Satellite I
7	π_2	$74\overset{\circ}{.}53051 + \beta_7$	Proper periapse of Satellite II
8	π_3	$174\overset{\circ}{.}85831 + \beta_8$	Proper periapse of Satellite III
9	π_4	$336\overset{\circ}{.}02667 + \beta_9$	Proper periapse of Satellite IV
10	Π_J	$13\overset{\circ}{.}30364 + \beta_{10}$	Longitude of perihelion of Jupiter
11	ω_1	$242\overset{\circ}{.}73706 + \beta_{11}$	Proper node of Satellite I
12	ω_2	$95\overset{\circ}{.}28556 + \beta_{12}$	Proper node of Satellite II
13	ω_3	$125\overset{\circ}{.}14673 + \beta_{13}$	Proper node of Satellite III
14	ω_4	$317\overset{\circ}{.}89250 + \beta_{14}$	Proper node of Satellite IV
15	ψ	$316\overset{\circ}{.}73369 + \beta_{15}$	Longitude of origin of coordinates (Jupiter's pole)
16	G'	$31\overset{\circ}{.}97852\ 80244 + \beta_{16}$	Mean anomaly of Saturn
17	G	$30\overset{\circ}{.}37841\ 20168 + \beta_{17} + \delta G$	Mean anomaly of Jupiter
18	ϕ_1	$172\overset{\circ}{.}84(1 - 0.014\epsilon_{20}) + \beta_{18}$	Phase angle in solar $(A/R)^3$ with angle $2G' - G$
19	ϕ_2	$47\overset{\circ}{.}03(1 - 0.156\epsilon_{20}) + \beta_{19}$	Phase angle in solar $(A/R)^3$ with angle $5G' - 2G$
20	ϕ_3	$259\overset{\circ}{.}18 + \beta_{20}$	Phase angle in solar $(A/R)^3$ with angle $G' - G$
21	ϕ_4	$157\overset{\circ}{.}12(1 + 0.0014\epsilon_{20}) + \beta_{21}$	Phase angle in solar $(A/R)^3$ with angle $2G' - 2G$
22	Ω_J	$99\overset{\circ}{.}95326 + \beta_{22}$	Longitude ascending node of Jupiter's orbit on ecliptic

Beta Symbol		Rate (deg/day)	Description
1	$\dot{\ell}_1$	$203\overset{\circ}{.}48895\ 4208(1 + \epsilon_6)$	Mean motion of Satellite I
2	$\dot{\ell}_2$	$101\overset{\circ}{.}37472\ 3445(1 + \epsilon_7)$	Mean motion of Satellite II
3	$\dot{\ell}_3$	$[50\overset{\circ}{.}31760\ 80635\{1 - 2\epsilon_6 + 3\epsilon_7 -0.02204\ 51849\ 7(\epsilon_6 - \epsilon_7)\}]$	Mean motion of Satellite III
4	$\dot{\ell}_4$	$21\overset{\circ}{.}57107\ 1403(1 + \epsilon_8)$	Mean motion of Satellite IV
5	$\dot{\phi}_\lambda$	$\sqrt{L}\ (= 0\overset{\circ}{.}1737\ 9190 + \cdots)$	Rate of free libration (Fiche Table A.30)
6	$\dot{\pi}_1$	$(0\overset{\circ}{.}1613\ 8586 + \cdots)$	Proper periapse rate of Satellite I
7	$\dot{\pi}_2$	$(0\overset{\circ}{.}0472\ 6307 + \cdots)$	Proper periapse rate of Satellite II
8	$\dot{\pi}_3$	$(0\overset{\circ}{.}0071\ 2734 + \cdots)$	Proper periapse rate of Satellite III
9	$\dot{\pi}_4$	$(0\overset{\circ}{.}0018\ 4000 + \cdots)$	Proper periapse rate of Satellite IV
10	$\dot{\Pi}_\mathrm{J}$	0	
11	$\dot{\omega}_1$	$(-0\overset{\circ}{.}1327\ 9386 + \cdots)$	Proper node rate of Satellite I
12	$\dot{\omega}_2$	$(-0\overset{\circ}{.}0326\ 3064 + \cdots)$	Proper node rate of Satellite II
13	$\dot{\omega}_3$	$(-0\overset{\circ}{.}0071\ 7703 + \cdots)$	Proper node rate of Satellite III
14	$\dot{\omega}_4$	$(-0\overset{\circ}{.}0017\ 5934 + \cdots)$	Proper node rate of Satellite IV
15	$\dot{\psi}$	$(-0\overset{\circ}{.}0000\ 0208 + \cdots)$	Longitude of origin rate
16	$\dot{G}'$	$3\overset{\circ}{.}34597\ 33896 \cdot 10^{-2}(1 + \epsilon_{28})$	Mean motion of Saturn
17	$\dot{G}$	$8\overset{\circ}{.}30912\ 15712 \cdot 10^{-2}(1 + \epsilon_{10})$	Mean motion of Jupiter
18…22		0	

3. The observations

A variety of different observational data types were employed in developing ephemerides E5. A new and very powerful data type of CCD observations from the U.S. Naval Observatory Flagstaff Station was used for the first time, together with very accurate Voyager optical navigation data from 1979 and the mutual event observations 1973-1991, photographic observations of D. Pascu from 1967-1993 and Jovian eclipse timings from 1652-1983. Doppler observations from 1987-1991 were employed to assess the value of the Doppler data and evaluate the ephemerides.

Table 4. Observational data employed for ephemeris E5

Data span	observable type	observ.	% chg
1992-1994	CCD data, Flagstaff ra & dec	870	−52.6
1979	Voyager opnav ra & dec	366	−19.0
1973-1991	mutual events ra & dec	860	−55.5
1967-1993	photographic ra & dec	8462	−3.2
1652-1983	eclipse timings	15711	+2.7
1994	CCD data, Table Mountain	72	+68.3
1987-1991	Doppler	50	−55.6

By intercomparing various data types one learns of the strengths and weaknesses of each individual type of data and discovers inconsistencies among the data types. The data are described in Table 4, which also gives the percentage change in weighted sum-of-squares for ephemeris E5 relative to ephemeris E3. A plus sign indicates an increase and a minus sign indicates a decrease in the weighted residuals. The various data types were combined by weighting each observation by the reciprocal of its squared a priori standard deviation. A common data set (including weights) was employed to evaluate all ephemerides so that one can compare the relative merits of a given ephemeris to a common data set. Thus, although no CCD observations were employed in the development of ephemeris E3, the residuals of the CCD data employed in this paper are also given for ephemeris E3 so that the reader can make meaningful comparisons.

In order to more closely compare the various ephemerides with the different data types, we present in Table 5 the residuals of unit weight for each data type for the different ephemerides E2 through E5 by Lieske, as well as for the Bureau des Longitudes' ephemeris G5 (Arlot 1982). In comparing Table 4 with Table 5 it should be remembered that Table 4 is related to the square of the residuals while Table 5 employs the square root of the sum-of-squares. The comparison for Flagstaff CCD data, for example, for Table 5 would indicate that the Table 4 entry should be about $(29/43)^2 - 1 = -54.5\%$ for the E5 vs E3 comparison.

Table 5. Observational rms residuals for various ephemerides

Observable type	E2	G5	E3	E4	E5
CCD Flagstaff, mas	43	40	43	32	29
Voyager opnav, mas	1309	1334	929	904	820
mutual events, mas	62	53	62	47	46
photographic, mas	107	106	106	104	104
eclipse timings, sec	55.5	74.5	53.2	53.9	53.9
CCD Table Mtn, mas	43	73	41	52	53
Doppler, Hz	15.3	18.4	13.7	11.7	11.9

3.1. CCD observations

The new CCD observations were made at the U.S. Naval Observatory Flagstaff Station (A. Monet et al. 1994) during the years 1993-1995, employing techniques developed by D. Monet and described in Monet et al. (1992) and in Monet & Monet (1992). The Flagstaff data were processed at JPL by W. Owen who produced normal-point residuals, typically from 30−50 CCD "exposures", for the author using ephemeris E3. Those residuals were then employed by the author to generate pseudo-observable "normal-point observations" by adding the residual to an artificially-constructed computed position at the mean time of the CCD exposures using the same ephemeris which was employed in computing the CCD residuals. Such a "normal point observation" could be employed with other astrometric data in an analysis of the observations, and should represent a valid description of the actual CCD observations. Additionally, the pseudo-observations will serve the purpose of archiving the CCD observations in convenient form. In processing the CCD data Owen would estimate the pointing and orientation parameters and employ a single telescope scale factor (modified for refraction and atmospheric effects) for all the Flagstaff data and he would use a single ephemeris (viz. E3) which was not adjusted in the reduction process. If that procedure is valid, then the pseudo-observables generated should behave like valid observational data, viz. the residuals should decrease if one employs a better ephemeris with the original pseudo-observables. It was for this reason that ephemeris E3 was intentionally employed – it was known to need some correction and we desired to explore the validity of the process of constructing normal point pseudo-observables. If the normal points were constructed instead on a different ephemeris, then the pseudo-observables differed by less than 15 km ($0''.005$) from those generated via ephemeris E3, even though the residuals might actually be significantly different using the two ephemerides. That 15-km reproducibility of the normal points is a good indication of the intrinsic accuracy of the CCD data.

Some less-accurate CCD data from the JPL Table Mountain Facility (Owen 1995) were also employed, although with hindsight they probably should not have been

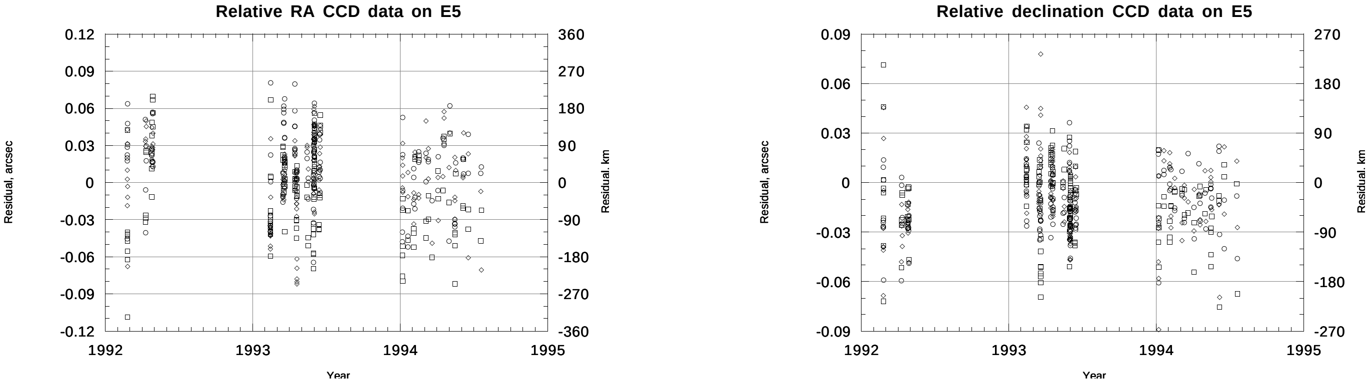

Fig. 1. Residuals in right ascension (left) and declination (right) for Flagstaff CCD observations relative to Satellite 1 using ephemeris E5. The observations of Europa relative to Io are indicated by a ○, those of Ganymede by a □, and those of Callisto by a ◇

included in developing E5. They did not exhibit the reduction of residuals with a better ephemeris, and that is believed to be due to the fact that there were too few Table Mountain data to adequately separate the orbital effects from the telescope effects.

The CCD data were processed using Lambert scattering to compute the offset between the center of light and center of figure (Lindegren 1977) and it is believed that the dominant remaining unmodeled error source in these data is due to albedo variations across the disk of the satellites. Recent estimates of the albedo variations by several scientists (Goguen 1994; Mallama 1993; Riedel 1994; Gaskell 1995) are not entirely consistent and for the Galileo-mission ephemerides it was decided to limit the processing to computation of the difference between center of light and center of figure due to Lambert scattering only, since it represents a reasonable first approximation to the scattering properties of the satellites if one excludes albedo variations (viz., effects which depend upon features on the satellites and which vary with planetocentric longitude of the central disc). The extrapolation of Voyager-derived scattering properties (which occurred at high phase angle) to the scattering properties of the satellites at low phase angle as observed from the Earth is not entirely satisfactory and the several efforts done to date are not entirely consistent with one another. It is hoped that some series of observations made from the Hubble Space Telescope will resolve the problems. Employment of Lambert scattering is a useful first-approximation. The differences between Lambert, Minnaert or Hapke scattering laws is minor compared to the albedo variations introduced by physical features on the satellites, which may introduce center-of-light relative to center-of-figure variations on the order of $75 - 100$ km.

The Flagstaff CCD data were weighted using a standard deviation of $0\farcs03$, which corresponds to about 90 km for these earth-based observations. The Table Mountain data were weighted using a standard deviation of $0\farcs05$, corresponding to about 150 km.

3.2. *Voyager optical navigation data*

During the Voyager mission in 1979, some optical navigation images of the Jovian satellites were taken from the spacecraft for use in navigating the spacecraft to the Jovian encounter. We have 183 observations of the Jovian satellites in right ascension and in declination, made during the Voyager I and Voyager II encounters (Synnott et al. 1982). The optical navigation images are analogous to earth-based astrometric observations of the satellites except that the “opnav” images are taken by an “observer” much closer to the Jovian system (typically $13 - 95$ light seconds from the satellites). At 5 10^6 km from Jupiter, one arcsec corresponds approximately to 25 km. Additionally, the spacecraft-based observations are the result of analyzing extended satellite images. By inferring the center of the satellite from observations of the limb, the Voyager data do not have the center-of-light vs center-of-figure problems which are common to disk-integrated images such as those contained in CCD observations and photographic plates and mutual events. The Voyager data were weighted using a standard deviation of $1\farcs0$ (as seen at the spacecraft’s distance from Jupiter). For spacecraft-to-satellite distances of $13 - 95$ light seconds, the $1\farcs0$ corresponds to 19 and 140 km respectively for these spacecraft-based observations. The Voyager optical navigation residuals on ephemeris E5 are depicted for right ascension and declination in Fig. 2.

3.3. *Mutual event astrometric data*

Since 1973 there have been successful campaigns to observe the mutual event seasons every six years, when the Jovian satellites eclipse and occult one another as the Sun and the Earth pass through the plane of the Jovian equator, in which the satellite orbits lie. Aksnes and colleagues

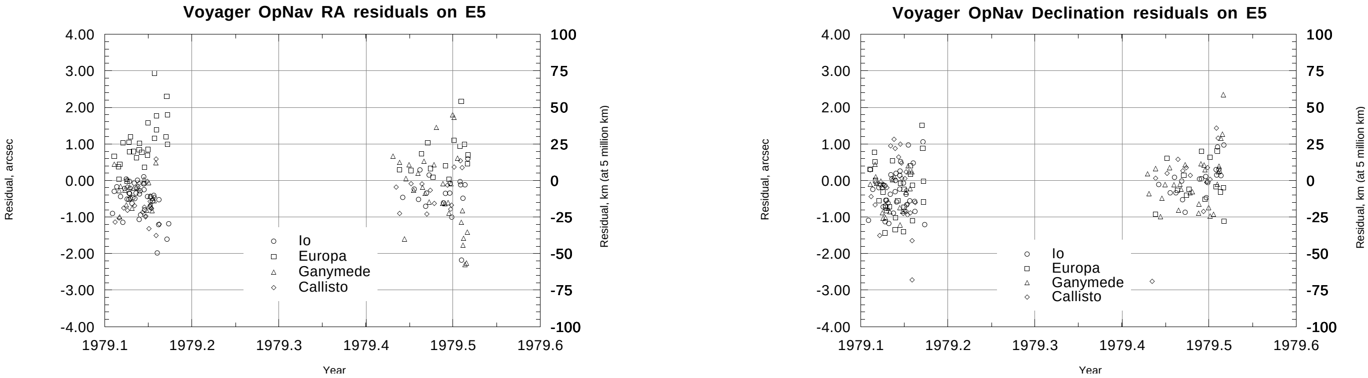

Fig. 2. Residuals in right ascension (left) and declination (right) for the Voyager optical navigation observations using ephemeris E5. The ordinate is in arcsec with an approximate corresponding linear distance scale on the right. Jupiter-relative observations of Io are indicated by ○, Europa by □, Ganymede by △, and Callisto by ◇

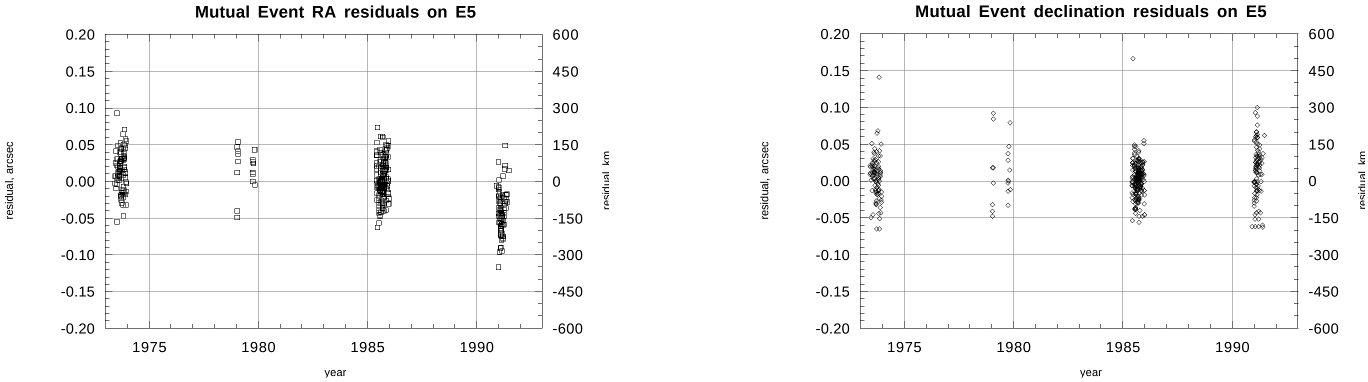

Fig. 3. Residuals in right ascension (left) and declination (right) for astrometric mutual event observations using ephemeris E5. The ordinate is in arcsec with an approximate corresponding linear distance scale on the right

(Aksnes 1974, 1984; Aksnes & Franklin 1978, 1990), along with Arlot and colleagues (Arlot 1978, 1984, 1990, 1996), have made predictions of such mutual events available to scientists throughout the world and have organized scientific programs to observe the mutual events. Aksnes' team has produced astrometric separations of the satellites, at times near the mid-event times, which are very useful for ephemeris development purposes.

The early Galilean satellite ephemerides E1 and E2 (Lieske 1980) employed the Aksnes data from 1973 (Aksnes & Franklin 1976) and 1979 (Aksnes et al. 1984) and were affected by the phase offsets between eclipses and occultations which led Aksnes et al. (1986) to recommend that δt be added to the published observation times for the 1973 and 1979 data. The ephemerides E3 were generated using the recommended additions of δt to the observation times in processing the 1973 and 1979 mutual events astrometric data.

In the processing of mutual event observations by the Aksnes team in 1985 (Franklin et al. 1991) and 1991 (Kaas et al. 1997), it was intended that no value of δt would be required but that instead the authors would incorporate the phase effects into their published times and separations. However, the effects were added in the incorrect direction for the published data and hence it is recommended (Aksnes 1993; Franklin 1993; Lieske 1995) that the 1985 and 1991 Aksnes data be employed by adding *twice* the published values of the δt phase corrections to the observation times. Essentially the first addition of δt removes the erroneous application of the phase effects with the incorrect sign and the second application of δt actually corrects for the phase problem. Additionally, some infra-red astrometric mutual event separations were obtained from Goguen et al. (1988) in 1985 as well as in 1991 (Goguen 1994). Astrometric separations from the 1991 mutual event season which were employed in the development of E5 were also published by Mallama (1992a), Spencer (1993) and by Descamps (1994).

The mutual event data were weighted using standard deviations of $0\farcs020$ to $0\farcs045$, which corresponds to 60 km and 140 respectively for these earth-based observations. The typical weight corresponds to a standard deviation of $0\farcs030$ or 90 km.

The obvious offset in right ascension residuals for the 1991 mutual event season depicted in Fig. 3 is believed not to be due to ephemeris errors, but rather is due to albedo

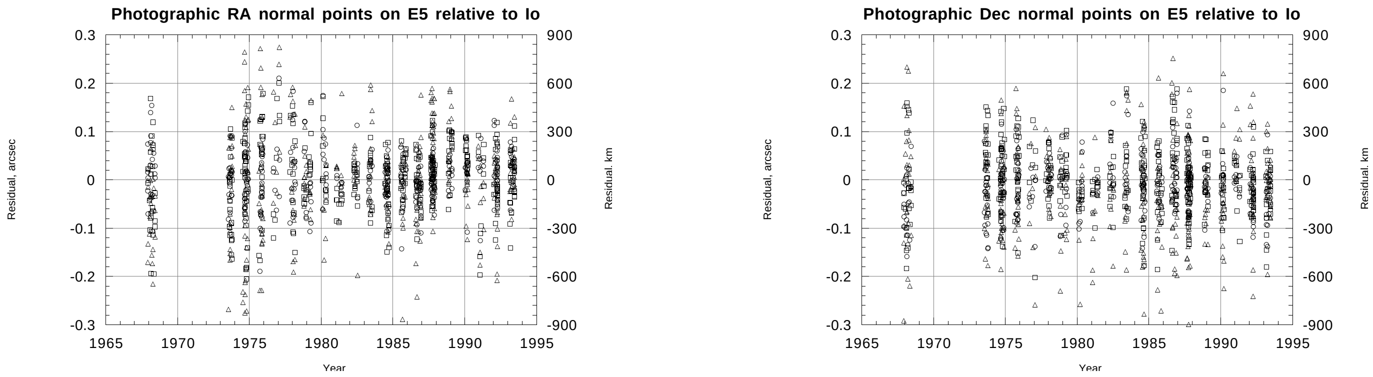

Fig. 4. Residuals in right ascension (left) and declination (right) for photographic observations relative to Io using ephemeris E5. The residuals for exposures of a given satellite on each plate have been combined to produce a normal point for each plate. Observations of Europa relative to Io are indicated by a ○, those of Ganymede by a □ and those of Callisto by a ◇

effects since almost all of the 1991 mutual event observations involved Io and were made at comparable longitudes on the satellite disk. The CCD and photographic data, for example, show no such offset and those data were sampled at various longitudes.

3.4. Photographic observations

The long and valuable series of photographic observations made by D. Pascu of the U.S. Naval Observatory have been an essential ingredient of the Galilean satellite ephemerides since the first development of the Galsat software. In an extended series of observations 1967–1993, Pascu (1977, 1979, 1993, 1994) provided astrometric observations of the satellites. He pioneered the development of neutral density filters to enable the accurate observation of the Galilean satellites on a regular basis. The Pascu data were reduced using a single scale factor (modified by adjustments for refraction for each observation) for the ensemble of observations, as determined by Pascu. Additionally, a correction to the Pascu scale was applied for a refraction-related effect, amounting to a relative change in scale of $-58''.2/206265$, which probably resulted from the manner in which the plate scale was originally determined.

The photographic data from 1967 through 1975 were weighted using a standard deviation of $0''.13$ per exposure, while those from 1976 onwards were weighted using a standard deviation of $0''.09$ per exposure, corresponding to position uncertainties of 400 km and 275 km, respectively, for each exposure. A photographic plate typically consisted of 4 exposures of each satellite.

The residuals on E5 for photographic observations are plotted in Fig. 4. In the figure, normal-point residuals are presented for each photographic plate, in order to make the comparison with the normal-point CCD observations more feasible. In the plots, the residuals for all exposures of a given satellite on a single plate are averaged into a single normal-point residual.

3.5. Jupiter eclipse timings

The Jovian eclipse timings, representing the classical observations of the Galilean satellites back to the 17th century, were discussed in Lieske (1986a,b). The early data are from the Pingré 17th century collection later published by Bigourdan (1901), and from the Delisle collection (Bigourdan 1897). The book on 17th century astronomy by Pingré published by Bigourdan was originally scheduled for publication 100 years earlier by Pingré. But Pingré's death and the French revolution intervened, and the printer's proof copies were destroyed as scrap paper. It was only 100 years later that a copy of the proofs was found and ultimately published by the Paris Academy. The manuscript collection of J.-N. Delisle contains a wealth of historically and scientifically interesting observations of Galilean satellite eclipses. These two collections effectively re-construct the "lost" Delambre collection.

We employed satellite radii of 1815, 1569, 2631 and 2400 km for Io through Callisto, respectively (Davies et al. 1985), in reducing the eclipse timings.

Additionally, the series of eclipse observations by Pickering from 1878-1903 (Pickering 1907) and those accumulated by Pierce (1974), together with those of many amateur astronomers, especially those coordinated by B. Loader and J. Westfall, were employed. Finally, a few eclipse timings by Mallama (1992b) taken in 1990–91 were analyzed.

The eclipse timing data were employed with average standard deviations between 44 s for Io and 150 s for Callisto with a mean of 63 s, which correspond to position uncertainties of 775 km for Io, 1225 km for Callisto, and 800 km on the average for all satellites. The residuals appear visually similar to those depicted in Lieske (1986a) and therefore they are not presented here again.

Table 6. Values of theory parameters ε and β for E5 in B1950 frame [see also Tables 2 and 3]

Parameter	Related to	Value	Parameter	Related to	Value
ε_1	m_1	0.046323 (±0.000813)	ε_{26}	J	−0.000137 (±0.000117)
ε_2	m_2	−0.000906 (±0.001394)	ε_{27}	ϵ	0.000000 (±0.000004)
ε_3	m_3	−0.022997 (±0.000276)	ε_{28}	n_S	0.000000 (±0.000001)
ε_4	m_4	0.258508 (±0.000537)	β_1	ℓ_1	0.°046767 (±0.00218)
ε_5	S/J	$2009.3457E-07$ ($\pm 8.12E-07$)	β_2	ℓ_2	−0.°015865 (±0.000835)
ε_6	n_1	$7.7760E-09$ ($\pm 0.549E-09$)	β_3	ℓ_3	$[= -\frac{1}{2}\beta_1 + \frac{3}{2}\beta_2]$
ε_7	n_2	$12.7230E-09$ ($\pm 1.04E-09$)	β_4	ℓ_4	−0.°074023 (±0.001950)
ε_8	n_4	$-10.4916E-09$ ($\pm 4.90E-09$)	β_5	ϕ_λ	199.°676608 (±1.57)
ε_9	λ_A	$11.2104E-04$ ($\pm 0.391E-04$) *rad*	β_6	π_1	92.°576366 (±19.9)
ε_{10}	n_J	$1.63E-05$ ($\pm 0.13E-05$)	β_7	π_2	80.°335825 (±1.35)
ε_{11}	J_2	−0.007576 (±0.000066)	β_8	π_3	13.°325727 (±0.150)
ε_{12}	J_4	−0.275934 (±0.00631)	β_9	π_4	−0.°739863 (±0.0152)
ε_{13}	R_J	−0.000308 (±0.000057)	β_{10}	Π_J	0.°166302 (±0.00344)
ε_{14}	P_J	$9.5E-06$ ($\pm 102.E-06$)	β_{11}	ω_1	69.°597506 (±0.788000)
ε_{15}	$3(C-A)/2C$	−0.170000 (±0.0676)	β_{12}	ω_2	5.°155556 (±0.0495)
ε_{16}	e_{11}	−0.995346 (±0.0291)	β_{13}	ω_3	−5.°952489 (±0.101)
ε_{17}	e_{22}	0.748031 (±0.0221)	β_{14}	ω_4	4.°726133 (±0.0772)
ε_{18}	e_{33}	−0.051182 (±0.00167)	β_{15}	ψ	−0.°215487 (±0.00545)
ε_{19}	e_{44}	−0.002434 (±0.000324)	β_{16}	G'	0.°000000 (±0.407)
ε_{20}	e_J	0.002750 (±0.000081)	β_{17}	G	−0.°140855 (±0.00279)
ε_{21}	c_{11}	0.344275 (±0.0196)	β_{18}	ϕ_1	15.°541000 (±0.411)
ε_{22}	c_{22}	−0.005970 (±0.000872)	β_{19}	ϕ_2	5.°215000 (±0.469)
ε_{23}	c_{33}	0.041611 (±0.00199)	β_{20}	ϕ_3	−1.°996000 (±0.757)
ε_{24}	c_{44}	−0.070074 (±0.000810)	β_{21}	ϕ_4	−7.°968000 (±0.293)
ε_{25}	I_J	0.005110 (±0.000079)	β_{22}	Ω_J	0.°045266 (±0.00664)

3.6. Doppler data

The Doppler observations discussed by Ostro et al. (1992) were employed to evaluate the ephemerides and explore the potential of Doppler data, but they were not included in analysis and the development of E5. The data are consistent with the observations which were analyzed, but they were not included in the analysis because of possible uncertainty in the radar scattering properties of the satellites similar to albedo effects which depend upon the planetocentric longitude. The 50 Doppler observations of the outer three Galilean satellites were made between 1987 and 1991.

The Doppler data were weighted using standard deviations of 19 Hz for Europa, 12 Hz for Ganymede and 10 Hz for Callisto for the Arecibo 13-cm S-band system data.

4. Discussion

The theory parameters which result from the analysis of these data are listed in Table 6, which will produce coordinates in the B1950 frame when used with the galsat software. A future paper will document how they, and any other set of galsat parameters, can be transformed to the J2000 system in a manner such that the galsat software will directly produce J2000 coordinates. In Table 6, the uncertainties listed for the ε and β parameters are the formal errors obtained in the estimation process. By comparing the coordinates of ephemerides E3 with those of E5 and interpreting those differences to represent a 1-σ error, we obtain a scale factor which should be applied for the formal uncertainties listed in the table. That scale factor ranges between 2.5 and 3, so we recommend that the formal errors be multiplied by 3. The derived values of the angular variables for E5 are given in Table 7. The series coefficients for satellite coordinates ξ, υ and ζ are summarized in Table 8 for the E5 ephemerides.

Representing the Jupiter-equatorial projection of the orbital radius by ρ, and the true and mean longitudes by ν and ℓ, respectively, then the equatorial radial component $\xi = (\rho - a)/a$ consists of cosine terms $\xi(t) = \Sigma K_1 \cos \Theta_1(t)$, while the longitude component $\upsilon = \nu - \ell$ consists of sine terms $\upsilon(t) = \Sigma K_2 \sin \Theta_2(t)$, and the latitude component $\zeta = \bar{z}/a$ consists of sine terms $\zeta(\tau) =$

Table 7. Derived variables for ephemeris E5

Index	Variable	Value (*deg*)	Rate (*deg/day*)
1	ℓ_1	$106\overset{\circ}{.}077187$	$203\overset{\circ}{.}48895579033$
2	ℓ_2	$175\overset{\circ}{.}731615$	$101\overset{\circ}{.}37472473479$
3	ℓ_3	$120\overset{\circ}{.}558829$	$50\overset{\circ}{.}31760920702$
4	ℓ_4	$84\overset{\circ}{.}444587$	$21\overset{\circ}{.}57107117668$
5	ϕ_λ	199.676608	$0\overset{\circ}{.}17379190461$
6	π_1	$97\overset{\circ}{.}088086$	$0\overset{\circ}{.}16138586144$
7	π_2	$154\overset{\circ}{.}866335$	$0\overset{\circ}{.}04726306609$
8	π_3	$188\overset{\circ}{.}184037$	$0\overset{\circ}{.}00712733949$
9	π_4	$335\overset{\circ}{.}286807$	$0\overset{\circ}{.}00183999637$
10	$\Pi_{\rm J}$	$13\overset{\circ}{.}469942$	0.
11	ω_1	$312\overset{\circ}{.}334566$	$-0\overset{\circ}{.}13279385940$
12	ω_2	$100\overset{\circ}{.}441116$	$-0\overset{\circ}{.}03263063731$
13	ω_3	$119\overset{\circ}{.}194241$	$-0\overset{\circ}{.}00717703155$
14	ω_4	$322\overset{\circ}{.}618633$	$-0\overset{\circ}{.}00175933880$
15	ψ	$316\overset{\circ}{.}518203$	$-2\overset{\circ}{.}08362 \cdot 10^{-6}$
16	G'	$31\overset{\circ}{.}978528$	$0\overset{\circ}{.}03345973390$
17	G	$30\overset{\circ}{.}237557$	$0\overset{\circ}{.}08309257010$
18	ϕ_1	$188\overset{\circ}{.}374346$	0.
19	ϕ_2	$52\overset{\circ}{.}224824$	0.
20	ϕ_3	$257\overset{\circ}{.}184000$	0.
21	ϕ_4	$149\overset{\circ}{.}152605$	0.
22	$\Omega_{\rm J}$	$99\overset{\circ}{.}998526$	0.
	a_1		$2.819353 \cdot 10^{-3}$ a.u.
	a_2		$4.485883 \cdot 10^{-3}$ a.u.
	a_3		$7.155366 \cdot 10^{-3}$ a.u.
	a_4		$12.585464 \cdot 10^{-3}$ a.u.

$\Sigma K_3 \sin \Theta_3(\tau)$. As developed by Sampson (1921, pp. 229–230), the "time-completed" τ may be defined as

$$\tau = t + \upsilon/n, \tag{8}$$

where t is "ephemeris time" (TDB). One can employ the time-completed to compute the latitude quantity $s(t) = \bar{z}/\rho$ from the shorter series for $\zeta(t) = \bar{z}/a$ via the relationship $s(t) = \zeta(t+\upsilon/n)$. It effectively amounts to calculating the latitude perturbations as a function of true longitude rather than as a function of mean longitude.

The Jupiter equatorial coordinates $\bar{\mathbf{r}} = (\bar{x}, \bar{y}, \bar{z})^T$ are computed from the orbital components ξ, υ, ζ using the equations

$$\begin{aligned} \bar{x} &= a(1+\xi)\cos(\ell - \psi + \upsilon) \\ \bar{y} &= a(1+\xi)\sin(\ell - \psi + \upsilon) \\ \bar{z} &= a(1+\xi)s. \end{aligned} \tag{9}$$

The Earth-equatorial coordinates $\mathbf{r} = (x, y, z)^T$ are then computed from the Jupiter-equatorial coordinates via the rotation matrices

$$\mathbf{r} = R_1(-\varepsilon)R_3(-\Omega)R_1(-J)R_3(-\psi+\Omega)R_1(-I)\bar{\mathbf{r}}. \tag{10}$$

It is these Earth-equatorial coordinates $\mathbf{r}$ that are provided by the galsat software.

As described in *Theory*, the Earth-equatorial coordinates are constructed from the series for ξ, υ and ζ by the relationship

$$\begin{aligned} \xi(t) &= \Sigma K_1 \cos \Theta_1(t) \\ \upsilon(t) &= \Sigma K_2 \sin \Theta_2(t) \\ s(t) = \zeta(\tau) &= \Sigma K_3 \sin \Theta_3(\tau) \end{aligned} \tag{11}$$

where the right-hand sides are the result of computing the series given in Table 8. The third equation for $s(t)$ employs the time-completed $\tau = t + \upsilon/n$ to evaluate the series for $\zeta(\tau)$ and thus to obtain $s(t)$.

The adjustable parameters ε and β for ephemerides E5 in the B1950 frame are given in Table 6. The derived values of the angular variables for E5 are given in Table 7.

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References

Aksnes K., 1974, Icarus 21, 100
Aksnes K., Franklin F., 1976, AJ 81, 464
Aksnes K., Franklin F., 1978, Icarus 34, 188
Aksnes K., 1984, Icarus 60, 180
Aksnes K., Franklin F., Millis R., et al., 1984, AJ 89, 28; AJ 89, 1081
Aksnes K., Franklin F., Magnusson P., 1986, AJ 92, 1436
Aksnes K., Franklin F., 1990, Icarus 84, 542
Aksnes K., 1993 (personal communication)
Arlot J.-E., 1978, A&AS 34, 195
Arlot J.-E., 1982, A&A 107, 305
Arlot J.-E., 1984, A&A 138, 113
Arlot J.-E., 1990, A&A 237, 259
Arlot J.-E., 1996, A&A 314, 312
Bigourdan G., 1897, "Inventaire général et sommaire des manuscrits de la bibliothèque de l'observatoire Paris" Ann. Obs. Paris 21, F1-F60. [Delisle manuscripts are filed under the heading Manuscripts A-5-1 through A-5-8]
Bigourdan G., 1901, A.-G. Pingré: Annales Célestes du dix-septiéme siècle. Paris: Gauthier-Villars
Brouwer D., 1952, AJ 57, 126
Campbell J.K., Synnott S.P., 1985, AJ 90, 364
Davies M.E., Abalakin V.K., Bursa M., et al., 1986, Celest. Mech. 39, 103

Descamps P., 1994, A&A 291, 664
Franklin F., et al., "Galilean Satellite Observers", 1991, AJ 102, 806
Franklin F., 1993 (personal communication)
Gaskell R.W., 1995 (personal communication)
Goguen J.D., Sinton W.M., Matson D.L., et al., 1988, Icarus 76, 465
Goguen J.D., 1994 (personal communication)
Kaas A.A., Franklin F., Aksnes K., Lieske J.H., 1997, "Mutual phenomena of the Galilean satellites 1990-1991", AJ (in press)
Lieske J.H., 1977, A&A 56, 333 (referred to as Theory)
Lieske J.H., Lederle T., Fricke W., Morando B., 1977, A&A 58, 1
Lieske J.H., 1979, A&A 73, 28
Lieske J.H., 1980, A&A 82, 340 [referred to as E2 ephemerides]
Lieske J.H., 1986a, A&A 154, 61
Lieske J.H., 1986b, A&AS 63, 143
Lieske J.H., 1987, A&A 176, 146
Lieske J.H., 1994a, "Galilean Satellite Ephemerides E4" JPL Engineering Memorandum 314-545 (19 June 1994) (JPL internal publication)
Lieske J.H., 1994b, A&A 281, 281
Lieske J.H., 1995, Bull. AAS 27, 1197
Lindegren L., 1977, A&A 57, 55
Mallama A., 1992a, Icarus 95, 309
Mallama A., 1992b, Icarus 97, 298
Mallama A., 1993, J. Geophys. Res. 98, p. 18.873-18.876
Martin C.F., 1969, Ph.D. Diss., Yale Univ.
Monet A.K.B., Stone R.C., Monet D.G., et al., 1994, AJ 107, 2290
Monet D.G., Monet A.K.B., 1992, "Galilean satellite astrometry", U.S. Naval Observatory Flagstaff Station memorandum
Monet D.G., Dahn C.C., Vrba F.J., et al., 1992, AJ 103, 638
Morrison L.V., Ward C.G., 1975, MNRAS 173, 183
Morrison L.V., 1980 (personal communication)
Ostro S.J., Campbell D.B., Simpson R.A., et al., 1992, J. Geophys. Res. 97, p. 18.277
Owen W.M., 1993 (personal communication)
Owen W.M., 1995 (personal communication)
Pascu D., 1977, in: Planetary Satellites, Burns J.A. (ed.). University of Arizona Press, Tucson, p. 63
Pascu D., 1979, in: Natural and Artificial Satellite Motion Nacozy P.E., Ferraz-Mello S. (eds.). University of Texas, Austin, p. 17
Pascu D., 1993 (personal communication)
Pascu D., 1994, in: Galactic and Solar System Optical Astrometry, Morrison L.V., Gilmore G.F. (eds.). Cambridge University Press, p. 304
Pierce D.A., 1980, "Observations of Jupiter's Satellites", JPL Engineering Memorandum 900-672
Pickering E.C., 1907, Harvard Ann. 52, Part I, 1
Riedel J.E., 1994 (personal communication)
Sampson R.A., 1921, MRAS 63 (Sampson Theory)
Spencer J., 1993 (personal communication)
Spencer Jones H., 1939, MNRAS 99, 541
Standish E.M., 1982, A&A 114, 297
Standish E.M., 1985, "JPL Planetary Ephemerides DE125", JPL IOM 314.6-591
Standish E.M., Folkner W.M., 1995, "JPL Ephemerides DE400 and DE140", JPL IOM 314.10-109
Stephenson F.R., Morrison L.V., 1984, Phil. Trans. R. Soc. London, Ser. A, 313, 47
Synnott S.P., Donegan A.J., Morabito L.A., 1982, "Position Observations of the Galilean Satellites from Voyager Data", Jet Propulsion Laboratory internal document
Vasundhara R., 1994, A&A 281, 565
West R.M, 1992, in: Trans IAU XXIB, Bergeron J. (ed.). Kluwer, Dordrecht, p. 211

Table 8. Series coefficients for E5

Index	E5	Argument	Ratio n/n_{sat}
XI-1: Series coefficients for $\xi_1 = (\rho_1 - a_1)/a_1$ (cosine)			
1	170	$\ell_1 - \ell_2$	.50181707
2	106	$\ell_1 - \ell_3$	.75272560
3	−2	$\ell_1 - \pi_1$	.99920691
4	−2	$\ell_1 - \pi_2$	.99976774
5	−387	$\ell_1 - \pi_3$	.99996497
6	−214	$\ell_1 - \pi_4$	.99999096
7	−66	$\ell_1 + \pi_3 - 2\Pi_J - 2G$	.99921835
8	−41339	$2\ell_1 - 2\ell_2$	1.00363413
9	3	$2\ell_1 - 2\ell_3$	1.50545120
10	−131	$4\ell_1 - 4\ell_2$	2.00726827
V-1: Series coefficients for $\upsilon_1 = \nu_1 - \ell_1$ (sine)			
1	−26	$-2\Pi_J + 2\psi - 2G$	−.00081670
2	−553	$-2\Pi_J + 2\psi$	−.00000002
3	−240	$-2\Pi_J + \omega_3 + \psi - 2G$	−.00085196
4	92	$-\omega_2 + \psi$	.00016035
5	−72	$-\omega_3 + \psi$	.00003526
6	−49	$-\omega_4 + \psi$	.00000864
7	−325	G	.00040834
8	65	$2G$	.00081668
9	−33	$5G' - 2G + \phi_2$	.00000547
10	−27	$\omega_3 - \omega_4$	−.00002662
11	145	$\omega_2 - \omega_3$	−.00012509
12	30	$\omega_2 - \omega_4$	−.00015171
13	−38	$\pi_4 - \Pi_J$	.00000904
14	−6071	$\pi_3 - \pi_4$	.00002598
15	282	$\pi_2 - \pi_3$	.00019724
16	156	$\pi_2 - \pi_4$	.00022322
17	−38	$\pi_1 - \pi_3$	.00075807
18	−25	$\pi_1 - \pi_4$	.00078405
19	−27	$\pi_1 + \pi_4 - 2\Pi_J - 2G$	−.00001454
20	−1176	$\pi_1 + \pi_3 - 2\Pi_J - 2G$	.00001144
21	1288	ϕ_λ	.00085406
22	39	$3\ell_3 - 7\ell_4 + 4\pi_4$	−.00018335
23	−32	$3\ell_3 - 7\ell_4 + \pi_3 + 3\pi_4$	−.00015737
24	−1162	$\ell_1 - 2\ell_2 + \pi_4$	.00364318
25	−1887	$\ell_1 - 2\ell_2 + \pi_3$	.00366916
26	−1244	$\ell_1 - 2\ell_2 + \pi_2$	.00386640
27	38	$\ell_1 - 2\ell_2 + \pi_1$	.00442723
28	−617	$\ell_1 - \ell_2$	.50181707
29	−270	$\ell_1 - \ell_3$	.75272560
30	−26	$\ell_1 - \ell_4$	.89399390
31	4	$\ell_1 - \pi_1$	.99920691
32	5	$\ell_1 - \pi_2$	.99976774
33	776	$\ell_1 - \pi_3$	.99996497
34	462	$\ell_1 - \pi_4$	.99999096
35	149	$\ell_1 + \pi_3 - 2\Pi_J - 2G$	.99921835
36	21	$2\ell_1 - 4\ell_2 + \omega_2 + \omega_3$	.00707264
37	−200	$2\ell_1 - 4\ell_2 + 2\omega_2$	.00694756
38	82483	$2\ell_1 - 2\ell_2$	1.00363413
39	−35	$2\ell_1 - 2\ell_3$	1.50545120
40	−3	$3\ell_1 - 4\ell_2 + \pi_3$	1.00730330
41	276	$4\ell_1 - 4\ell_2$	2.00726827

Index	E5	Argument	Ratio n/n_{sat}
22	4	$\pi_1 - \pi_3$	.00152167
23	111	$\pi_1 - \pi_4$	.00157382
24	−354	$\pi_1 + \pi_3 - 2\Pi_J - 2G$	.00002296
25	−3103	ϕ_λ	.00171435
26	55	$2\ell_3 - 2\Pi_J - 2G$	.99106590
27	−111	$3\ell_3 - 7\ell_4 + 4\pi_4$	−.00036805
28	91	$3\ell_3 - 7\ell_4 + \pi_3 + 3\pi_4$	−.00031589
29	−25	$3\ell_3 - 7\ell_4 + 2\pi_3 + 2\pi_4$	−.00026373
30	−1994	$\ell_2 - \ell_3$	.50364739
31	−137	$\ell_2 - \ell_4$	.78721450
32	1	$\ell_2 - \pi_1$	.99840803
33	2886	$\ell_2 - \pi_2$	.99953378
34	6250	$\ell_2 - \pi_3$	.99992969
35	3463	$\ell_2 - \pi_4$	.99998185
36	30	$\ell_2 - \Pi_J - G$	.99918034
37	−18	$2\ell_2 - 3\ell_3 + \pi_4$	.51096032
38	−39	$2\ell_2 - 3\ell_3 + \pi_3$	.51101248
39	98	$2\ell_2 - 2\ell_4$	1.57442901
40	−164	$2\ell_2 - 2\omega_2$	2.00064376
41	−18	$2\ell_2 - \omega_2 - \omega_3$	2.00039268
42	72	$5\ell_2 - 5\ell_3$	2.51823695
43	30	$\ell_1 - 2\ell_2 - \pi_3 + 2\Pi_J + 2G$	.00886379
44	4180	$\ell_1 - 2\ell_2 + \pi_4$	.00731293
45	7428	$\ell_1 - 2\ell_2 + \pi_3$	.00736509
46	−2329	$\ell_1 - 2\ell_2 + \pi_2$	.00776100
47	−19	$\ell_1 - 2\ell_2 + \pi_1$	.00888675
48	−185835	$\ell_1 - \ell_2$	1.00729478
49	−110	$\ell_1 - 2\ell_3 + \pi_4$	1.01460771
50	−200	$\ell_1 - 2\ell_3 + \pi_3$	1.01465987
51	39	$\ell_1 - 2\ell_3 + \pi_2$	1.01505578
52	−16	$\ell_1 - 2\ell_3 + \pi_1$	1.01618153
53	−803	$\ell_1 - \ell_3$	1.51094217
54	−19	$\ell_1 - \pi_2$	2.00682856
55	−75	$\ell_1 - \pi_3$	2.00722447
56	−31	$\ell_1 - \pi_4$	2.00727663
57	−9	$2\ell_1 - 4\ell_2 + \omega_3 + \psi$	.01451874
58	4	$2\ell_1 - 4\ell_2 + 2\omega_3$	.01444797
59	−14	$2\ell_1 - 4\ell_2 + \omega_2 + \omega_3$	.01419688
60	150	$2\ell_1 - 4\ell_2 + 2\omega_2$	.01394580
61	−11	$2\ell_1 - 4\ell_2 + 2\Pi_J + 2G$	.01622888
62	−9	$2\ell_1 - 4\ell_2 + \pi_3 + \pi_4$	.01467802
63	−8	$2\ell_1 - 4\ell_2 + 2\pi_3$	.01473017
64	915	$2\ell_1 - 2\ell_2$	2.01458956
65	96	$2\ell_1 - 2\ell_3$	3.02188434
66	−18	$4\ell_1 - 4\ell_2$	4.02917912
LAT-2: Series coefficients for $\zeta_2 = z_2/a_2$ (sine)			
1	17	$\ell_2 - 2\Pi_J + \psi - 3G$	.99754101
2	143	$\ell_2 - 2\Pi_J + \psi - 2G$	.99836066
3	−144	$\ell_2 - \omega_1$	1.00130993
4	81004	$\ell_2 - \omega_2$	1.00032188
5	4512	$\ell_2 - \omega_3$	1.00007080
6	1160	$\ell_2 - \omega_4$	1.00001735
7	−19	$\ell_2 - \psi - G$	.99918036
8	−3284	$\ell_2 - \psi$	1.00000002
9	35	$\ell_2 - \psi + G$	1.00081968
10	−28	$\ell_1 - 2\ell_3 + \omega_3$	1.01451876
11	272	$\ell_1 - 2\ell_3 + \omega_2$	1.01426768

Index	E5	Argument	Ratio n/n_{sat}
LAT-1: Series coefficients for $\zeta_1 = z_1/a_1$ (sine)			
1	46	$\ell_1 - 2\Pi_J + \psi - 2G$	.99918331
2	6393	$\ell_1 - \omega_1$	1.00065259
3	1825	$\ell_1 - \omega_2$	1.00016036
4	329	$\ell_1 - \omega_3$	1.00003527
5	93	$\ell_1 - \omega_4$	1.00000865
6	−311	$\ell_1 - \psi$	1.00000001
7	75	$3\ell_1 - 4\ell_2 + \omega_2$	1.00710791
XI-2: Series coefficients for $\xi_2 = (\rho_2 - a_2)/a_2$ (cosine)			
1	−18	$\omega_2 - \omega_3$	−.00025108
2	−27	$2\ell_3 - 2\Pi_J - 2G$	.99106590
3	553	$\ell_2 - \ell_3$	.50364739
4	45	$\ell_2 - \ell_4$	.78721450
5	−102	$\ell_2 - \pi_1$	.99840803
6	−1442	$\ell_2 - \pi_2$	.99953378
7	−3116	$\ell_2 - \pi_3$	.99992969
8	−1744	$\ell_2 - \pi_4$	.99998185
9	−15	$\ell_2 - \Pi_J - G$	.99918034
10	−64	$2\ell_2 - 2\ell_4$	1.57442901
11	164	$2\ell_2 - 2\omega_2$	2.00064376
12	18	$2\ell_2 - \omega_2 - \omega_3$	2.00039268
13	−54	$5\ell_2 - 5\ell_3$	2.51823695
14	−30	$\ell_1 - 2\ell_2 + \pi_4$	.00731293
15	−67	$\ell_1 - 2\ell_2 + \pi_3$	.00736509
16	93848	$\ell_1 - \ell_2$	1.00729478
17	48	$\ell_1 - 2\ell_3 + \pi_4$	1.01460771
18	107	$\ell_1 - 2\ell_3 + \pi_3$	1.01465987
19	−19	$\ell_1 - 2\ell_3 + \pi_2$	1.01505578
20	523	$\ell_1 - \ell_3$	1.51094217
21	30	$\ell_1 - \pi_3$	2.00722447
22	−290	$2\ell_1 - 2\ell_2$	2.01458956
23	−91	$2\ell_1 - 2\ell_3$	3.02188434
24	22	$4\ell_1 - 4\ell_2$	4.02917912
V-2: Series coefficients for $\upsilon_2 = \nu_2 - \ell_2$ (sine)			
1	98	$-2\Pi_J + 2\psi - 2G$	−.00163936
2	−1353	$-2\Pi_J + 2\psi$	−.00000004
3	551	$-2\Pi_J + \omega_3 + \psi - 2G$	−.00171013
4	26	$-2\Pi_J + \omega_2 + \psi - 2G$	−.00196122
5	31	$-\omega_2 + \psi$	.00032186
6	255	$-\omega_3 + \psi$	.00007078
7	218	$-\omega_4 + \psi$	.00001733
8	−1845	G	.00081966
9	−253	$2G$	.00163932
10	18	$2G' - 2G + \phi_4$	−.00097920
11	19	$2G' - G + \phi_1$	−.00015954
12	−15	$5G' - 3G + \phi_1$	−.00080867
13	−150	$5G' - 2G + \phi_2$	.00001098
14	102	$\omega_3 - \omega_4$	−.00005344
15	56	$\omega_2 - \omega_3$	−.00025108
16	72	$\pi_4 - \Pi_J$	.00001815
17	2259	$\pi_3 - \pi_4$	.00005216
18	−24	$\pi_3 - \pi_4 + \omega_3 - \omega_4$	−.00000129
19	−23	$\pi_2 - \pi_3$	.00039591
20	−36	$\pi_2 - \pi_4$	.00044807
21	−31	$\pi_1 - \pi_2$	.00112575

Index	E5	Argument	Ratio n/n_{sat}
XI-3: Series coefficients for $\xi_3 = (\rho_3 - a_3)/a_3$ (cosine)			
1	24	$-\omega_3 + \psi$	.00014259
2	−9	$\omega_3 - \omega_4$	−.00010767
3	10	$\pi_3 - \pi_4$	.00010508
4	294	$\ell_3 - \ell_4$	.57130175
5	18	$\ell_3 - \pi_2$	.99906071
6	−14388	$\ell_3 - \pi_3$	.99985835
7	−7919	$\ell_3 - \pi_4$	.99996343
8	−23	$\ell_3 - \Pi_J - G$	.99834864
9	−20	$\ell_3 + \pi_4 - 2\Pi_J - 2G$	.99673384
10	−51	$\ell_3 + \pi_3 - 2\Pi_J - 2G$	.99683892
11	39	$2\ell_3 - 3\ell_4 + \pi_4$	.71394181
12	−1761	$2\ell_3 - 2\ell_4$	1.14260349
13	−11	$2\ell_3 - 2\pi_3$	1.99971671
14	−10	$2\ell_3 - \pi_3 - \pi_4$	1.99982179
15	−27	$2\ell_3 - 2\Pi_J - 2G$	1.99669728
16	24	$2\ell_3 - 2\omega_3$	2.00028527
17	9	$2\ell_3 - \omega_3 - \omega_4$	2.00017760
18	−24	$2\ell_3 - \omega_3 - \psi$	2.00014268
19	−16	$3\ell_3 - 4\ell_4 + \pi_4$	1.28524355
20	−156	$3\ell_3 - 3\ell_4$	1.71390524
21	−42	$4\ell_3 - 4\ell_4$	2.28520699
22	−11	$5\ell_3 - 5\ell_4$	2.85650873
23	6342	$\ell_2 - \ell_3$	1.01469677
24	9	$\ell_2 - \pi_3$	2.01455512
25	39	$2\ell_2 - 3\ell_3 + \pi_4$	1.02943011
26	70	$2\ell_2 - 3\ell_3 + \pi_3$	1.02953519
27	10	$\ell_1 - 2\ell_2 + \pi_4$	.01473334
28	20	$\ell_1 - 2\ell_2 + \pi_3$	.01483842
29	−153	$\ell_1 - \ell_2$	2.02939354
30	156	$\ell_1 - \ell_3$	3.04409031
31	11	$2\ell_1 - 2\ell_2$	4.05878708
V-3: Series coefficients for $\upsilon_3 = \nu_3 - \ell_3$ (sine)			
1	10	$-\pi_3 + \pi_4 - \omega_3 + \psi$	.00003751
2	28	$-2\Pi_J + 2\psi - 2G$	−.00330281
3	−1770	$-2\Pi_J + 2\psi$	−.00000008
4	−48	$-2\Pi_J + \omega_3 + \psi - 2G$	−.00344540
5	14	$-\omega_2 + \psi$	.00064845
6	411	$-\omega_3 + \psi$	.00014259
7	345	$-\omega_4 + \psi$	.00003492
8	−2338	G	.00165136
9	−66	$2G$	.00330272
10	10	$G' - G + \phi_3$	−.00098639
11	22	$2G' - 2G + \phi_4$	−.00197278
12	26	$2G' - G + \phi_1$	−.00032142
13	11	$3G' - 2G + \phi_2 + \phi_3$	−.00130781
14	9	$3G' - G + \phi_1 - \phi_2$	.00034355
15	−19	$5G' - 3G + \phi_1$	−.00162923
16	−208	$5G' - 2G + \phi_2$	.00002213
17	159	$\omega_3 - \omega_4$	−.00010767
18	21	$\omega_2 - \omega_3$	−.00050586
19	121	$\pi_4 - \Pi_J$	.00003657
20	6604	$\pi_3 - \pi_4$	.00010508
21	−65	$\pi_3 - \pi_4 + \omega_3 - \omega_4$	−.00000259
22	−88	$\pi_2 - \pi_3$	.00079765
23	−72	$\pi_2 - \pi_4$	.00090273
24	−26	$\pi_1 - \pi_3$	.00306570

Table 8. continued

Index	E5	Argument	Ratio n/n_{sat}
25	-9	$\pi_1 - \pi_4$	.00317078
26	16	$\pi_1 + \pi_4 - 2\Pi_J - 2G$	-.00005881
27	125	$\pi_1 + \pi_3 - 2\Pi_J - 2G$	.00004627
28	307	ϕ_λ	.00345390
29	-10	$\ell_4 - \pi_4$	.42866169
30	-100	$\ell_3 - 2\ell_4 + \pi_4$	.14264006
31	83	$\ell_3 - 2\ell_4 + \pi_3$	.14274514
32	-944	$\ell_3 - \ell_4$	.57130175
33	-37	$\ell_3 - \pi_2$	.99906071
34	28780	$\ell_3 - \pi_3$	.99985835
35	15849	$\ell_3 - \pi_4$	.99996343
36	7	$\ell_3 - \pi_4 + \omega_3 - \omega_4$	.99985576
37	46	$\ell_3 - \Pi_J - G$	.99834864
38	51	$\ell_3 + \pi_4 - 2\Pi_J - 2G$	.99673384
39	11	$\ell_3 + \pi_3 - 2\Pi_J - 3G$	.99518756
40	97	$\ell_3 + \pi_3 - 2\Pi_J - 2G$	.99683892
41	1	$\ell_3 + \pi_1 - 2\Pi_J - 2G$	.99990462
42	-101	$2\ell_3 - 3\ell_4 + \pi_4$	.71394181
43	13	$2\ell_3 - 3\ell_4 + \pi_3$	.71404689
44	3222	$2\ell_3 - 2\ell_4$	1.14260349
45	29	$2\ell_3 - 2\pi_3$	1.99971671
46	25	$2\ell_3 - \pi_3 - \pi_4$	1.99982179
47	37	$2\ell_3 - 2\Pi_J - 2G$	1.99669728
48	-24	$2\ell_3 - 2\omega_3$	2.00028527
49	-9	$2\ell_3 - \omega_3 - \omega_4$	2.00017760
50	24	$2\ell_3 - \omega_3 - \psi$	2.00014268
51	-174	$3\ell_3 - 7\ell_4 + 4\pi_4$	-.00074150
52	140	$3\ell_3 - 7\ell_4 + \pi_3 + 3\pi_4$	-.00063642
53	-55	$3\ell_3 - 7\ell_4 + 2\pi_3 + 2\pi_4$	-.00053134
54	27	$3\ell_3 - 4\ell_4 + \pi_4$	1.28524355
55	227	$3\ell_3 - 3\ell_4$	1.71390524
56	53	$4\ell_3 - 4\ell_4$	2.28520699
57	13	$5\ell_3 - 5\ell_4$	2.85650873
58	42	$\ell_2 - 3\ell_3 + 2\ell_4$	-.12790672
59	-12055	$\ell_2 - \ell_3$	1.01469677
60	-24	$\ell_2 - \pi_3$	2.01455512
61	-10	$\ell_2 - \pi_4$	2.01466020
62	-79	$2\ell_2 - 3\ell_3 + \pi_4$	1.02943011
63	-131	$2\ell_2 - 3\ell_3 + \pi_3$	1.02953519
64	-665	$\ell_1 - 2\ell_2 + \pi_4$	.01473334
65	-1228	$\ell_1 - 2\ell_2 + \pi_3$	.01483842
66	1082	$\ell_1 - 2\ell_2 + \pi_2$	.01563606
67	90	$\ell_1 - 2\ell_2 + \pi_1$	.01790411
68	190	$\ell_1 - \ell_2$	2.02939354
69	218	$\ell_1 - \ell_3$	3.04409031
70	2	$2\ell_1 - 4\ell_2 + \omega_3 + \psi$	.02925086
71	-4	$2\ell_1 - 4\ell_2 + 2\omega_3$	.02910827
72	3	$2\ell_1 - 4\ell_2 + 2\omega_2$	.02809655
73	2	$2\ell_1 - 4\ell_2 + \pi_3 + \pi_4$	.02957175
74	2	$2\ell_1 - 4\ell_2 + 2\pi_3$	.02967683
75	-13	$2\ell_1 - 2\ell_2$	4.05878708
LAT-3: Series coefficients for $\zeta_3 = z_3/a_3$ (sine)			
1	37	$\ell_3 - 2\Pi_J + \psi - 3G$	.99504587
2	321	$\ell_3 - 2\Pi_J + \psi - 2G$	.99669724
3	-15	$\ell_3 - 2\Pi_J + \psi - G$	.99834860
4	-45	$\ell_3 - 2\Pi_J + \psi$	.99999996
5	-2797	$\ell_3 - \omega_2$	1.00064849

Index	E5	Argument	Ratio n/n_{sat}
V-4: Series coefficients for $\upsilon_4 = \nu_4 - \ell_4$ (sine)			
1	8	$-\pi_3 - \pi_4 + 2\psi$	-.00041590
2	-9	$-\pi_3 - \pi_4 + \omega_4 + \psi$	-.00049737
3	27	$-\pi_3 + \pi_4 - \omega_4 + \psi$	-.00016365
4	-409	$-2\pi_4 + 2\psi$	-.00017079
5	310	$-2\pi_4 + \omega_4 + \psi$	-.00025226
6	-19	$-2\pi_4 + \omega_3 + \psi$	-.00050341
7	8	$-\pi_4 - \Pi_J + 2\psi$	-.00008549
8	-5	$-\pi_4 - \Pi_J + \omega_4 + \psi$	-.00016696
9	63	$-\pi_4 + \Pi_J - \omega_4 + \psi$	-.00000384
10	8	$-2\Pi_J + 2\psi - 3G$	-.01155630
11	73	$-2\Pi_J + 2\psi - 2G$	-.00770427
12	-5768	$-2\Pi_J + 2\psi$	-.00000019
13	16	$-2\Pi_J + \omega_4 + \psi - 2G$	-.00778573
14	-97	$-\omega_3 + \psi$	.00033262
15	152	$-2\omega_4 + 2\psi$	.00016293
16	2070	$-\omega_4 + \psi$	.00008146
17	-5604	G	.00385204
18	-204	$2G$	.00770407
19	-10	$3G$	.01155611
20	24	$G' - G + \phi_3$	-.00230090
21	11	$G' + \phi_1 - 2\phi_2$	.00155114
22	52	$2G' - 2G + \phi_4$	-.00460180
23	61	$2G' - G + \phi_1$	-.00074976
24	25	$3G' - 2G + \phi_2 + \phi_3$	-.00305066
25	21	$3G' - G + \phi_1 - \phi_2$	.00080138
26	-45	$5G' - 3G + \phi_1$	-.00380042
27	-495	$5G' - 2G + \phi_2$	.00005162
28	-44	$\omega_3 - \omega_4$	-.00025116
29	5	$\pi_4 - \Pi_J - G$	-.00376674
30	234	$\pi_4 - \Pi_J$	.00008530
31	11	$2\pi_4 - 2\Pi_J - 2G$	-.00753348
32	-10	$2\pi_4 - \omega_3 - \omega_4$	.00058487
33	68	$2\pi_4 - 2\omega_4$	.00033372
34	-13	$\pi_3 - \pi_4 - \omega_4 + \psi$	.00032658
35	-5988	$\pi_3 - \pi_4$	.00024511
36	-47	$\pi_3 - \pi_4 + \omega_3 - \omega_4$	-.00000604
37	-3249	$\ell_4 - \pi_3$	.99966959
38	48	$\ell_4 - \pi_4 - 2\Pi_J + 2\psi$	.99991451
39	10	$\ell_4 - \pi_4 - \omega_4 + \psi$	.99999616
40	33	$\ell_4 - \pi_4 - G$	.99606266
41	147108	$\ell_4 - \pi_4$	.99991470
42	-31	$\ell_4 - \pi_4 + G$	1.00376674
43	-6	$\ell_4 - \pi_4 + \omega_4 - \psi$	.99983324
44	-61	$\ell_4 - \pi_4 + 2\Pi_J - 2\psi$	.99991489
45	10	$\ell_4 - \Pi_J - 2G$	.99229593
46	178	$\ell_4 - \Pi_J - G$	.99614796
47	-363	$\ell_4 - \Pi_J$	1.00000000
48	5	$\ell_4 + \pi_4 - 2\Pi_J - 5G' + 2G - \phi_1$	1.00003368
49	12	$\ell_4 + \pi_4 - 2\Pi_J - 4G$	.98467715
50	124	$\ell_4 + \pi_4 - 2\Pi_J - 3G$	.98852919
51	1088	$\ell_4 + \pi_4 - 2\Pi_J - 2G$	.99238122
52	-55	$\ell_4 + \pi_4 - 2\Pi_J - G$	.99623326
53	-12	$\ell_4 + \pi_4 - 2\Pi_J$	1.00008530
54	-13	$\ell_4 + \pi_4 - \omega_4 - \psi$	1.00016696
55	6	$\ell_4 + \pi_4 - 2\psi$	1.00008549
56	17	$\ell_4 + \pi_3 - 2\pi_4$	1.00015981
57	-28	$\ell_4 + \pi_3 - 2\Pi_J - 2G$	.99262634

Index	E5	Argument	Ratio n/n_{sat}
6	32402	$\ell_3 - \omega_3$	1.00014263
7	6847	$\ell_3 - \omega_4$	1.00003496
8	-45	$\ell_3 - \psi - G$	.99834868
9	-16911	$\ell_3 - \psi$	1.00000004
10	51	$\ell_3 - \psi + G$	1.00165140
11	10	$2\ell_2 - 3\ell_3 + \psi$	1.02939350
12	-21	$2\ell_2 - 3\ell_3 + \omega_3$	1.02925091
13	30	$2\ell_2 - 3\ell_3 + \omega_2$	1.02874505
XI-4: Series coefficients for $\xi_4 = (\rho_4 - a_4)/a_4$ (cosine)			
1	-19	$-\omega_3 + \psi$	.00033262
2	167	$-\omega_4 + \psi$	.00008146
3	11	G	.00385204
4	12	$\omega_3 - \omega_4$	-.00025116
5	-13	$\pi_3 - \pi_4$	.00024511
6	1621	$\ell_4 - \pi_3$	.99966959
7	-24	$\ell_4 - \pi_4 - 2\Pi_J + 2\psi$	.99991451
8	-17	$\ell_4 - \pi_4 - G$	.99606266
9	-73546	$\ell_4 - \pi_4$	.99991470
10	15	$\ell_4 - \pi_4 + G$	1.00376674
11	30	$\ell_4 - \pi_4 + 2\Pi_J - 2\psi$	.99991489
12	-5	$\ell_4 - \Pi_J - 2G$	.99229593
13	-89	$\ell_4 - \Pi_J - G$	.99614796
14	182	$\ell_4 - \Pi_J$	1.00000000
15	-6	$\ell_4 + \pi_4 - 2\Pi_J - 4G$	.98467715
16	-62	$\ell_4 + \pi_4 - 2\Pi_J - 3G$	.98852919
17	-543	$\ell_4 + \pi_4 - 2\Pi_J - 2G$	.99238122
18	27	$\ell_4 + \pi_4 - 2\Pi_J - G$	.99623326
19	6	$\ell_4 + \pi_4 - 2\Pi_J$	1.00008530
20	6	$\ell_4 + \pi_4 - \omega_4 - \psi$	1.00016696
21	-9	$\ell_4 + \pi_3 - 2\pi_4$	1.00015981
22	14	$\ell_4 + \pi_3 - 2\Pi_J - 2G$	.99262634
23	13	$2\ell_4 - \pi_3 - \pi_4$	1.99958429
24	-271	$2\ell_4 - 2\pi_4$	1.99982940
25	-25	$2\ell_4 - 2\Pi_J - 3G$	1.98844389
26	-155	$2\ell_4 - 2\Pi_J - 2G$	1.99229593
27	-12	$2\ell_4 - \omega_3 - \omega_4$	2.00041428
28	19	$2\ell_4 - \omega_3 - \psi$	2.00033281
29	48	$2\ell_4 - 2\omega_4$	2.00016312
30	-167	$2\ell_4 - \omega_4 - \psi$	2.00008166
31	142	$2\ell_4 - 2\psi$	2.00000019
32	-22	$\ell_3 - 2\ell_4 + \pi_4$	.33272835
33	20	$\ell_3 - 2\ell_4 + \pi_3$	.33297346
34	974	$\ell_3 - \ell_4$	1.33264305
35	24	$2\ell_3 - 3\ell_4 + \pi_4$	1.66537139
36	177	$2\ell_3 - 2\ell_4$	2.66528609
37	4	$3\ell_3 - 4\ell_4 + \pi_4$	2.99801444
38	42	$3\ell_3 - 3\ell_4$	3.99792914
39	14	$4\ell_3 - 4\ell_4$	5.33057219
40	5	$5\ell_3 - 5\ell_4$	6.66321524
41	-8	$\ell_2 - 3\ell_3 + 2\ell_4$	-.29836073
42	92	$\ell_2 - \ell_4$	3.69956841
43	105	$\ell_1 - \ell_4$	8.43341914

Index	E5	Argument	Ratio n/n_{sat}
58	-33	$2\ell_4 - \pi_3 - \pi_4$	1.99958429
59	676	$2\ell_4 - 2\pi_4$	1.99982940
60	36	$2\ell_4 - 2\Pi_J - 3G$	1.98844389
61	218	$2\ell_4 - 2\Pi_J - 2G$	1.99229593
62	-5	$2\ell_4 - 2\Pi_J - G$	1.99614796
63	12	$2\ell_4 - \omega_3 - \omega_4$	2.00041428
64	-19	$2\ell_4 - \omega_3 - \psi$	2.00033281
65	-48	$2\ell_4 - 2\omega_4$	2.00016312
66	167	$2\ell_4 - \omega_4 - \psi$	2.00008166
67	-142	$2\ell_4 - 2\psi$	2.00000019
68	148	$\ell_3 - 2\ell_4 + \pi_4$	.33272835
69	-94	$\ell_3 - 2\ell_4 + \pi_3$	.33297346
70	-390	$\ell_3 - \ell_4$	1.33264305
71	9	$2\ell_3 - 4\ell_4 + 2\pi_4$	.66545669
72	-37	$2\ell_3 - 3\ell_4 + \pi_4$	1.66537139
73	6	$2\ell_3 - 3\ell_4 + \pi_3$	1.66561651
74	-195	$2\ell_3 - 2\ell_4$	2.66528609
75	6	$3\ell_3 - 7\ell_4 + 2\pi_4 + \omega_4 + \psi$	-.00198192
76	187	$3\ell_3 - 7\ell_4 + 4\pi_4$	-.00172966
77	-149	$3\ell_3 - 7\ell_4 + \pi_3 + 3\pi_4$	-.00148455
78	51	$3\ell_3 - 7\ell_4 + 2\pi_3 + 2\pi_4$	-.00123943
79	-10	$3\ell_3 - 7\ell_4 + 3\pi_3 + \pi_4$	-.00099432
80	6	$3\ell_3 - 6\ell_4 + 3\pi_4$	.99818504
81	-8	$3\ell_3 - 4\ell_4 + \pi_4$	2.99801444
82	-41	$3\ell_3 - 3\ell_4$	3.99792914
83	-13	$4\ell_3 - 4\ell_4$	5.33057219
84	-44	$\ell_2 - 3\ell_3 + 2\ell_4$	-.29836073
85	89	$\ell_2 - \ell_4$	3.69956841
86	106	$\ell_1 - \ell_4$	8.43341914
LAT-4: Series coefficients for $\zeta_4 = z_4/a_4$ (sine)			
1	8	$\ell_4 - 2\Pi_J - \omega_4 + 2\psi$	1.00008137
2	8	$\ell_4 - 2\Pi_J + \psi - 4G$	.98459175
3	88	$\ell_4 - 2\Pi_J + \psi - 3G$	.98844379
4	773	$\ell_4 - 2\Pi_J + \psi - 2G$	.99229583
5	-38	$\ell_4 - 2\Pi_J + \psi - G$	.99614787
6	5	$\ell_4 - 2\Pi_J + \psi$	.99999990
7	9	$\ell_4 - \omega_1$	1.00615611
8	-17	$\ell_4 - \omega_2$	1.00151270
9	-5112	$\ell_4 - \omega_3$	1.00033272
10	-7	$\ell_4 - \omega_4 - G$	.99622952
11	44134	$\ell_4 - \omega_4$	1.00008156
12	7	$\ell_4 - \omega_4 + G$	1.00393360
13	-102	$\ell_4 - \psi - G$	.99614806
14	-76579	$\ell_4 - \psi$	1.00000010
15	104	$\ell_4 - \psi + G$	1.00385213
16	-10	$\ell_4 - \psi + 5G' - 2G + \phi_2$	1.00005172
17	-11	$\ell_3 - 2\ell_4 + \psi$	.33264295
18	7	$\ell_3 - 2\ell_4 + \omega_4$	.33256149