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Membership determination of stars using proper motions in the region of the open cluster M 11*

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Abstract. Relative proper motions of 872 stars in the open cluster M 11 region are reduced using 10 plate pairs taken over time baselines of 16 ~ 70 years with the double astrograph telescope of Shanghai Observatory. The scale is $30''$/mm. The plates were measured with the PDS machines in the Purple Mountain Observatory in Nanjing and the Institute of Technology and Communication in Luoyang, China. The average proper motion accuracy is about 1.1 mas/yr with 85% of the data better than 1 mas/yr.

Membership probabilities of 785 stars within $25'$ centred on M 11 are determined based on their proper motions. The method used is suggested by Su et al. (1995) with some improvements of Zhao & He (1990), in which the space distribution and magnitude dependencies for cluster stars are taken into account. The results are significantly good. The total integrated membership probabilities for all these stars is 547 and the number of stars with probabilities higher than 0.7 is 541. It can be found after the membership determination that there exists mass segregation in M 11. Some comparisons and discussion are also given.

Key words: star cluster: open — individual: M 11 — proper motions: membership determination — dynamics

1. Introduction

Open clusters play an important role in understanding stars, stellar systems and our Galaxy. Some cluster regions have become focal points for studies of star formation, for instances, the Trapezium (Prosser et al. 1994), IC 348 (Lada & Lada 1995), NGC 2362 (Wilner & Lada 1991) and ρ Oh (Wilking et al. 1989; Rieke et al. 1989). Secondly, although their HR diagrams are basic for distance determination of celestial bodies, there still are some unsolved questions, such as the mechanisms of blue stragglers, gaps in main sequences, double main sequences, and so on (Milone & Latham 1994; Raboud & Mermilliod 1994). Furthermore, being typical stellar systems, open clusters leave many open questions, although great efforts have been devoted to dynamics, internal motions, etc. (Carraro & Chiosi 1994; Su 1994; Drukier 1995). Finally, open clusters are often considered as special tracers to investigate the structure and evolution of our Galaxy, the relationship between the disk and halo, and whether or not there existed some star formation bursts during the lifetime of the Galactic disk (Janes & Phelps 1994; Phelps & Janes 1994).

As we know, the concentration to the Galactic plane for open clusters is very strong. Their scale heights perpendicular to the Galactic plane are 55 pc and 375 pc for "young" and "old" open clusters, respectively (Janes & Phelps 1994). That is to say, there are many field stars mixed in the observational data for clusters. If membership is not reasonably determined for an open cluster, great uncertainties will be brought out in features, such as age, distance, structure, dynamics, stellar mass function, etc. Thus, it is the firststep that membership determination must be done before doing deeper researches.

M 11(NGC 6705, $\alpha_{1950} = 18^{\rm h}48^{\rm m}4^{\rm s}$, $\delta_{1950} = -6°20'$) is a famous middle-age open cluster with distance of 1.6 kpc, mass of $4.0 \sim 5.5\ 10^3\ M_\odot$ and age of $2.0\ 10^8$ yrs (Mathieu 1984; Su 1994; Su et al. 1997). Its Trumple classification is II 2r and/or 2b-a. Because of its large number of members and very good central concentration, M 11 is one of the best-choice candidates to study open clusters.

McNamara et al. (1977) used 15 plate pairs to derive proper motions of 1890 stars in the central region within 0.25 square degree of M 11. Most of these stars have V magnitudes. The completeness of their sample is down to $V \sim 16.5$. Using these proper motion data, they also determined M 11 membership. They took 874 stars with

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* Table 4 is only available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsweb.u-strasbg.fr/Abstract.html

membership probabilities higher than 0.5 as M 11 members. This is the best sample of M 11 at that time. McNamara & Sanders (1977) also used this sample to analyze the internal motions of M 11. They found that there exists obvious space-mass segregation, but the velocity-mass segregation is unclear.

There are some other researches for M 11, including photometric (Johnson et al. 1957; Patenaude 1978; Mathieu 1984), analyses of internal motion (Su 1994; Su et al. 1997; Zhao & He 1987), as well as structure and dynamics (Su 1994; Mathieu 1984).

In the present paper, we use 10 plate pairs taken with the double astrograph at the Zo-Se station of Shanghai Observatory to derive proper motion data of 872 stars. The time baselines of the plate pairs are 16∼70 years. According to the method suggested by Stetson (1979), we can reasonably estimate B magnitudes for individual stars; V magnitude values of stars can be obtained from McNamara et al. (1977) through the cross-identification for common stars. The completeness magnitude for the data is about $15\stackrel{m}{.}5$ in the B band. Stars within 25 arcmin centred on the open cluster M 11 region are used to do membership determination, from which some useful information can be found. Because all of our plates were taken at one telescope and the time baselines are also rather long, the accuracy of proper motions will be expected to be good and it is an independent and useful sample of M 11 for further researches.

The descriptions of the plates and the reduction of the proper motion data are in Sect. 2. Membership determination is in Sect. 3. Some comparisons and discussion are given in Sect. 4. The last section is the summary.

2. Plates and reduction of the data

2.1. Plate material and PDS measurement

There are 20 plates of the open cluster M 11 region used in the present study. All the plates were taken with the double astrograph at the Zo-Se station of Shanghai Observatory. This old telescope and its site have a detailed description (Chevalier 1905). Zo-Se is a small hill, which is one of the highest ones in Shanghai. The distance from Zo-Se to the center of Shanghai is nearly 30 km. Now it is the modern observation base of Shanghai Observatory.

The telescope, built by Gaultier in Paris, has an aperture of 40 cm and a focal length of 6.9 m, with a plate scale of $30''$/mm. Each plate is 24 by 30 cm. The plates can be divided roughly into three epochs. The first one is between 1916 and 1923, the second is in 1964, and the third is between 1980 and 1986. Details of the plate pairs are listed in Table 1.

These plates were measured with PDS machines in the Purple Mountain Observatory in Nanjing and the Institute of Technology and Communication in Luoyang, China. A pixel size of 20 by 20 microns, a step length of 20 micron, a speed of 25 μ/s and a scan type of R were used throughout. All the scanned programs are provided by Wang et al. (1990) and Wang & Chen (1992), Wang (1993, 1994) which are based on Lee & Van Altena (1983).

2.2. Proper motions

According to the method adopted by Zhao et al. (1993, 1980); Tian et al. (1982, 1983) and the references therein, we can derive the relative proper motions of stars based on the results of the PDS measurements. The whole process can be divided into three steps: the first one is to determine the reference stars, i.e., to establish a reference frame; the second to calculate proper motions; the last to estimate uncertainties of the data. Many authors, including ourselves, have done efforts in this field, so we give only a brief description here.

Theoretically, one can choose any stars freely to be reference stars to reduce relative proper motions. Reference stars are, in fact, normally chosen such that their proper motions are small. If this is not the case, program star proper motions might be distorted if they are located near a large proper motion star on the plate. To obtain a good result and make the absolute proper motion of the frame as small as possible, we should choose as many stars common to all plate pairs as possible, and stars with extraordinarily large proper motions and those in the crowded central region should be discarded. At the same time, the distribution of stars on the plates and the distribution of their magnitudes should also be chosen to be homogeneous. There are 618 stars in all the plate pairs in our study, 503 of which are chosen to be reference stars based on the above principles. One can see the reference frame adopted in the present study in Fig. 4 of the vector point diagram in the next section.

There are two ways used for reduction of relative proper motions. One is the central overlap technique, another the plate constant technique. In the present study, the central overlap technique is used. This approach has been adopted in our group since 1982. First, the error equations are limited only to first-order coordinate and magnitude terms. Second, the solutions must consist of quadratic coordinate and magnitude terms. Because accuracies are functions of time baselines of different pairs, the final results of relative proper motions must be weighted by the time baselines, as mentioned by Zhao et al. (1993) and Zhao & He (1988).

It is important to estimate the accuracies of proper motions for individual stars. In the early examples of such work, only the total accuracy of relative proper motions was given. As we now know, it is not enough to discuss only on the measurements, because the accuracies for individual stars depend on time baselines, number of pairs, exposure time, zenith distance, weather conditions and plate washing. The detailed description can be found in Zhao & He (1990, 1988) and Zhao et al. (1993).

Table 1. Details of M 11 plates (SL: slightly elongated; G: good)

Pair No.	*Plates*	*Epoch* (1900+)	*Exp. time* (min)	*Hour angle* (a.m)	*Quality*	*Baselines* (year)	*Star No.*
1	435	16.72	90	−13	SL	70.00	832
	86020	86.72	30	4	G		
2	459	17.72	132	8	SL	69.02	864
	86030	86.74	25	19	G		
3	457	17.71	44	−10	G	69.02	787
	86028	86.73	20	21	G		
4	456	17.70	130	14	G	69.02	723
	86025	86.72	20	18	G		
5	518	23.71	100	12	G	63.01	703
	86021	86.72	20	14	G		
6	519	23.73	70	4	G	63.01	793
	86023	86.74	20	10	G		
7	64033	64.74	30	14	G	21.99	804
	86024	86.73	20	−17	G		
8	64029	64.70	20	5	G	17.00	828
	81005	81.70	20	−14	G		
9	64025	64.70	20	−7	G	15.97	831
	80017	80.67	25	18	G		
10	64028	64.70	35	−15	G	15.97	824
	80018	80.67	25	11	G		

Figure 1 shows the distribution of accuracies of the relative proper motions of stars in the M 11 region. Most of the stars (85%) have proper motion accuracies better than 1 mas/yr. We also list total accuracies for stars appearing in different pairs in Table 2. It clearly shows that stars appearing in fewer pairs have poorer accuracies.

Table 2. Accuracies $(\varepsilon_x, \varepsilon_y)$ of relative proper motions(in unit of mas/yr) for the stars with different pair numbers in the M 11 region

pair number	*star number*	ε_x	ε_y	ε
2	2	14.01	12.39	13.22
3	2	13.11	11.68	12.42
4	2	11.59	9.72	10.69
5	4	8.70	7.39	8.08
6	13	4.18	4.23	4.21
7	28	3.67	3.34	3.51
8	50	2.79	2.58	2.69
9	117	1.49	1.25	1.37
10	618	0.76	0.81	0.79
total	836	1.11	1.16	1.13

The proper motion errors for 618 stars available in all ten plate pairs in different magnitude ranges are shown in Table 3. The method for obtaining magnitudes will be mentioned in next subsection. From Table 3, the accuracies of proper motions for bright stars are generally better than those for faint stars.

Table 3. Errors $(\varepsilon_x, \varepsilon_y)$ of relative proper motions (in unit of mas/yr) for the stars with all 10 plate pairs available in different magnitude ranges

$m_{\rm b}$	*star number*	ε_x	ε_y	ε
< 12.5	64	0.80	0.89	0.84
12.5-13.0	86	0.44	0.48	0.46
13.0-13.5	164	0.56	0.54	0.55
13.5-14.0	151	0.65	0.67	0.66
14.0-14.5	114	0.88	1.01	0.95
≥ 14.5	39	1.39	1.60	1.50

The histograms of the relative proper motions in X and Y directions are plotted in Fig. 2. It can be seen that the average values in both X and Y directions are all close to zero. This means that the reference frame used here is quite good.

2.3. Magnitudes

For the V band, we can obtain values from McNamara et al. (1977) after careful cross-identification between our catalogue and theirs. There are 435 common stars in the two catalogues, i.e., 435 stars in our sample have V magnitude values.

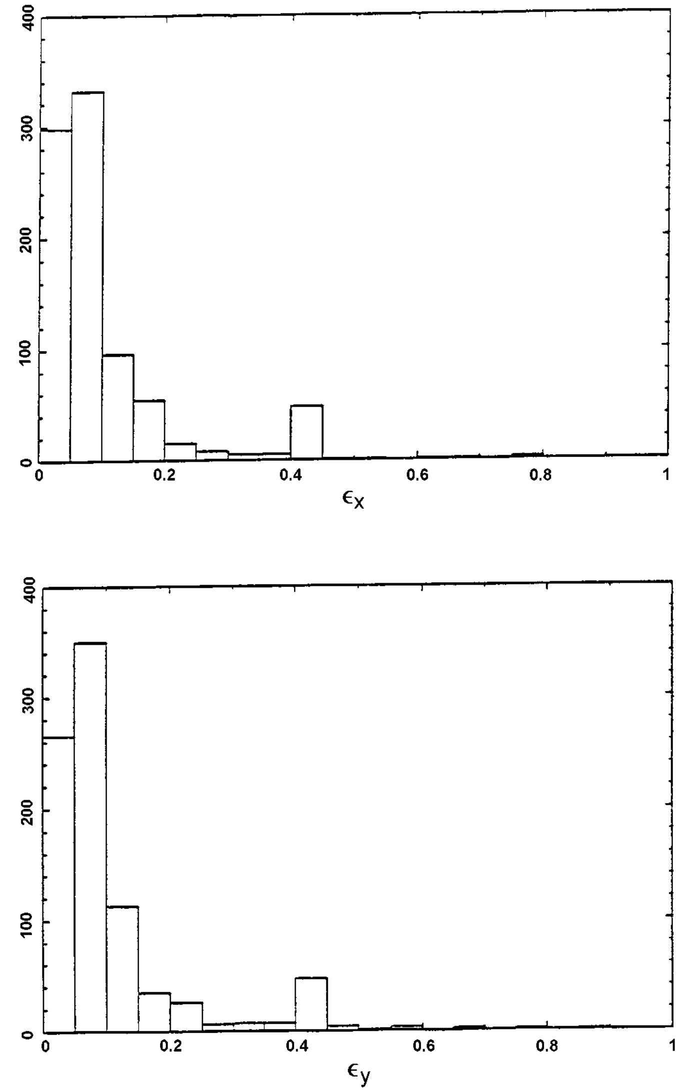

Fig. 1. The histograms of accuracies (in unit of as/100 yrs) of proper motions in X and Y directions for stars in the M 11 region respectively

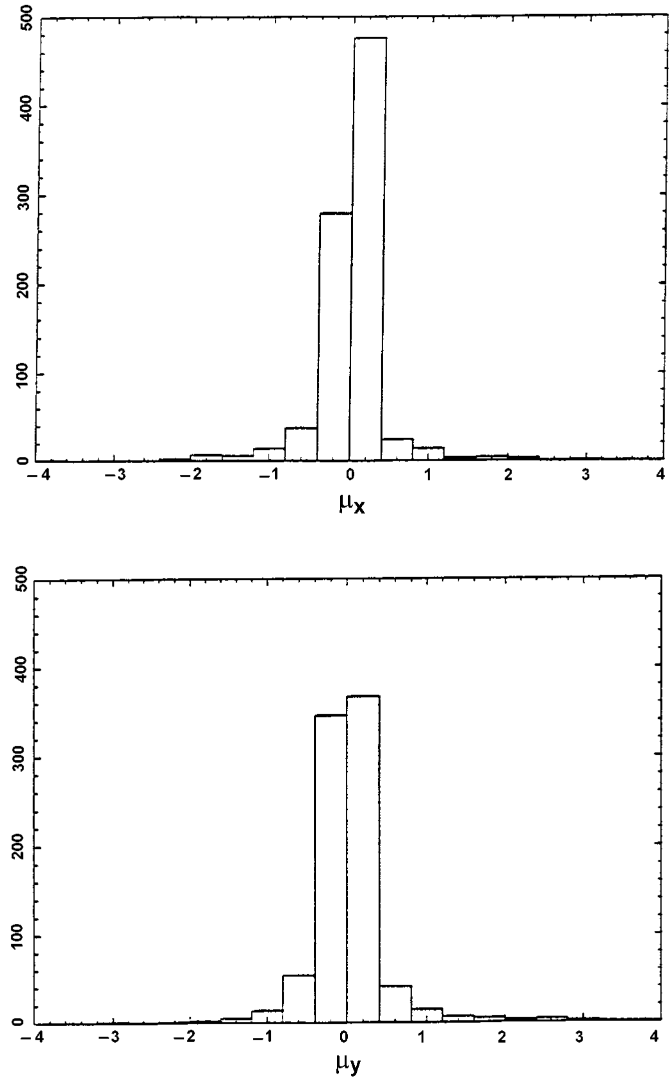

Fig. 2. The histograms of proper motions (in unit of as/100 yrs) in X and Y directions for stars in the M 11 region respectively

Because our plates in the 1980s (Table 1) were taken in the B band, we can use them to reduce B magnitudes of individual stars according to the density and size for each star obtained in the PDS measurements. Since no standard stars were observed at that time, we use the reduction method suggested by Stetson (1979) to calculate B magnitudes, in which the number of stars on the plates with known magnitudes is required to be large enough for statistical analysis. In the present study, we use 72 stars common to the study of Mathieu (1984) to do this, in which all stars have highly accurate magnitude values through CCD photometry.

The equation for the reduction is as follows:

$$B_i = b_0 - \log(d_i R_i^2) \qquad (1)$$

where for the ith, B_i is its B magnitude, R_i radius of its image, d_i its density and b_0 a constant. Using our 72 stars, we obtain $b_0 = 20.5 \pm 0.4$. Thus we have a B magnitude for each star based on Eq. (1).

The final results of relative proper motions and magnitudes for stars in the M 11 region are listed in Table 4 (available in electric form). The first column is the star number, among which with asterisk"*" are reference stars. The second is the star number in the catalogue of McNamara et al. (1977). The total number of the stars common with theirs is 435. The third and fourth denote X and Y coordinates (cm); the 5th and 6th are B and V magnitudes; the 7th and 8th are relative proper motions in X and Y respectivel Y, and their corresponding uncertainties are in the 9th and 10th (all in unit of as/100 yr);

the 11th is the number of plate pairs on which individual stars are available.

There are still 36 stars that are available on only one plate pair. So, we can only reduce their proper motion values but not their accuracies. These stars are all fainter than $m_b < 14.5$. In the present paper, the average accuracy for stars fainter than $m_b < 14.5$ available in at least two plate pairs is estimated as the accuracy of these stars.

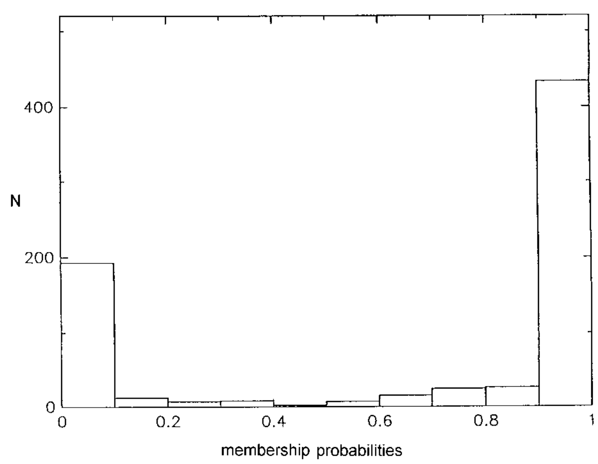

Fig. 3. The histogram of membership probabilities of M 11

3. Membership determination

The firststep in astrophysical research of an open cluster is to make a reasonable membership determination, as mentioned in Sect. 1. The popular methods can be summed up in two aspects: photometric and kinematics. But, as pointed out by Mathieu (1984), the uncertainty for photometric determination is quite large especially for binaries. Although Cabrera-Canio & Alfaro (1990) suggested another method, the computation is very complicated.

The most popular way to distinguish cluster stars from field stars now is based on their kinematical data, especially on proper motions. The fundamental work is suggested by Vasilevskis et al. (1958) and Sanders (1971) using the maximum likelihood principle. Zhao & He (1990) improved the method to be used for the condition of different accuracies of proper motions for individual stars.

It must be pointed out that there still are two shortcomings in these studies. On the one hand, the space distribution of cluster stars is not considered. The results obtained from this method must have biases to stars in the outer part of the cluster, i.e., the outer stars will have larger membership probabilities than they should have. In general, fainter stars are in the outer region, so that their probabilities will be overestimated. On the other hand, the distribution parameters are dependent upon magnitudes of stars, but only the average magnitude is concerned in their method of membership determination. There are more faint stars than bright stars in an open cluster. This will also lead to biases to fainter stars. These two aspects will enlarge the uncertainty in membership determination, especially for a cluster whose age is sufficient for dynamical relaxation.

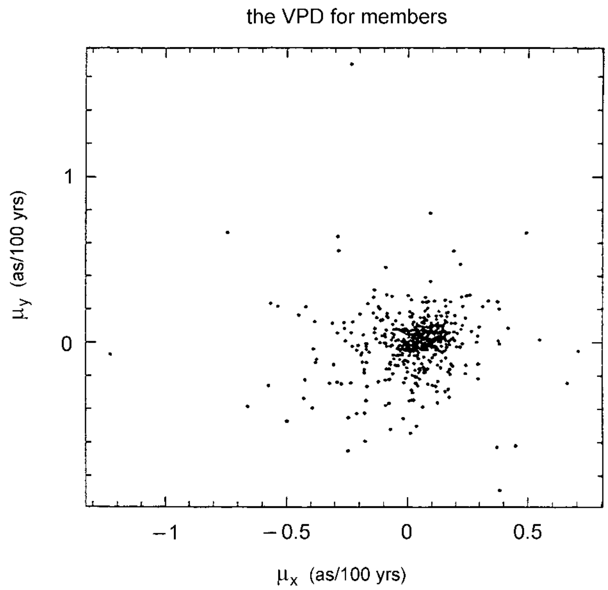

Fig. 4. The vector-point diagram (VPD) of 541 member stars of M 11

Jones & Walker (1988) developed some improvements in this field. While the two factors mentioned above are considered in the distribution function of cluster stars, the influence for field stars has not been taken into account reasonably (See Eq. (8) and Eq. (9) in their paper). Su et al. (1995) made some corrections for them, used successfully for the open cluster M 67. In the present study, we will use the method of Su et al. (1995) to do membership determination. A brief introduction is given below.

According to van den Bergh & Sher (1960) and Francic (1989), the surface number density distribution for cluster stars can be assumed as

$$\rho_c = \rho_0 e^{-\alpha r} = \rho_0 e^{-r/r_0} \qquad (2)$$

and the surface density distribution for field stars

$$\rho_f = f. \qquad (3)$$

Where ρ_0 is the central surface density of the cluster, r_0 the characteristic radius of the cluster, r the distance of

Table 5. Subsamples for the stars in different B magnitude ranges in the M 11 region

group No.	1	2	3	4	5	6	7	8
$m_{\rm b}$	<12.6	12.6−13.0	13.0−13.4	13.4−13.8	13.8−14.2	14.2−14.6	14.6−15.0	>15.0
star No.	77	62	116	119	122	114	85	90

Table 6. Distribution parameters for the stars in different magnitude ranges of M 11

group No.	1	2	3	4
ρ_0/f	12.38 ± 1.01	23.38 ± 1.57	19.91 ± 1.46	24.14 ± 2.57
r_0	0.758 ± 0.080	0.855 ± 0.124	0.870 ± 0.083	0.909 ± 0.149
$\mu_{x\rm f}$	-0.193 ± 0.021	-0.123 ± 0.014	-0.059 ± 0.017	-0.098 ± 0.007
$\mu_{y\rm f}$	0.034 ± 0.011	0.121 ± 0.007	0.247 ± 0.015	-0.198 ± 0.011
$\mu_{x\rm c}$	0.089 ± 0.006	0.068 ± 0.009	0.071 ± 0.011	0.053 ± 0.005
$\mu_{y\rm c}$	0.037 ± 0.009	0.022 ± 0.008	0.023 ± 0.006	0.028 ± 0.007
σ_x	0.750 ± 0.081	0.501 ± 0.107	0.758 ± 0.110	0.525 ± 0.085
σ_y	0.747 ± 0.087	0.679 ± 0.094	0.812 ± 0.107	0.769 ± 0.066
σ_0	0.021 ± 0.004	0.019 ± 0.003	0.014 ± 0.003	0.021 ± 0.002
group No.	5	6	7	8
ρ_0/f	27.44 ± 2.79	20.34 ± 1.73	26.62 ± 2.37	9.82 ± 1.14
r_0	0.935 ± 0.079	1.299 ± 0.236	2.778 ± 0.926	2.857 ± 1.061
$\mu_{x\rm f}$	-0.028 ± 0.026	-0.183 ± 0.014	-0.109 ± 0.006	-0.006 ± 0.014
$\mu_{y\rm f}$	-0.054 ± 0.009	-0.021 ± 0.011	0.012 ± 0.021	0.039 ± 0.023
$\mu_{x\rm c}$	0.028 ± 0.008	0.001 ± 0.007	-0.044 ± 0.008	-0.145 ± 0.013
$\mu_{y\rm c}$	-0.005 ± 0.011	-0.005 ± 0.012	-0.007 ± 0.012	-0.029 ± 0.014
σ_x	0.279 ± 0.061	0.553 ± 0.063	0.509 ± 0.104	0.753 ± 0.114
σ_y	0.451 ± 0.057	0.637 ± 0.089	0.790 ± 0.116	0.815 ± 0.128
σ_0	0.042 ± 0.003	0.040 ± 0.005	0.048 ± 0.007	0.063 ± 0.011

individual stars from the cluster center, f only depending upon magnitudes. Now, the normalized factor for cluster stars and field ones are

$$\frac{\rho_{\rm c}}{\rho_{\rm c}+\rho_{\rm f}} = \frac{1}{1+f/\rho_0\cdot {\rm e}^{-r/r_0}} \tag{4}$$

and

$$\frac{\rho_{\rm f}}{\rho_{\rm c}+\rho_{\rm f}} = \frac{1}{1+\rho_0\cdot {\rm e}^{-r/r_0}/f} \tag{5}$$

respectively.

So the frequency functions of proper motions for cluster members and field stars, considering the space distribution and magnitudes of stars can be written as

$$\Psi_{{\rm c}i} = \frac{1}{1+f/\rho_0\cdot {\rm e}^{-r/r_0}}\cdot\frac{1}{2\pi(\sigma_0^2+\varepsilon_i^2)} \times\ \exp\left\{-\frac{1}{2}\left[\frac{(\mu_{xi}-\mu_{x\rm c})^2}{\sigma_0^2+\varepsilon_i^2}+\frac{(\mu_{yi}-\mu_{y\rm c})^2}{\sigma_0^2+\varepsilon_i^2}\right]\right\} \tag{6}$$

and

$$\Psi_{{\rm f}i} = \frac{1}{1+\rho_0\cdot {\rm e}^{-r/r_0}/f}\cdot\frac{1}{2\pi(\sigma_x^2+\varepsilon_i^2)^{1/2}}\cdot\frac{1}{(\sigma_y^2+\varepsilon_i^2)^{1/2}} \times\ \exp\left\{-\frac{1}{2}\left[\frac{(\mu_{xi}-\mu_{x\rm f})^2}{\sigma_x^2+\varepsilon_i^2}+\frac{(\mu_{yi}-\mu_{y\rm f})^2}{\sigma_y^2+\varepsilon_i^2}\right]\right\} \tag{7}$$

respectively. In Eq. (6) and (7), we have nine distribution parameters to be solved by means of the maximum likelihood method. They are β_j $(j = 1,9) \equiv (\rho_0/f, r_0, \mu_{x\rm c}, \mu_{y\rm c}, \mu_{x\rm f}, \mu_{y\rm f}, \sigma_0, \sigma_x, \sigma_y)$, where ρ_0/f is the ratio of central surface density for cluster stars to that for field stars; r_0 the characteristic radius; $\mu_{x\rm c}, \mu_{y\rm c}$ and $\mu_{x\rm f}, \mu_{y\rm f}$ the proper motion centers for cluster members and field stars respectively; σ_0 the intrinsic proper motion dispersion for members; σ_x, σ_y the intrinsic proper motion dispersions for field stars in X, Y directions and ε_i the error of the proper motion of ith individual star. It must be kept in mind that all these nine parameters are functions of magnitude.

In the present study, stars with radial distances within 25 arcminutes centred to M 11 are chosen for membership determination. The number of these stars is 785. Because the distribution parameters now are functions of magnitude, we must divide our sample into several subsamples with different magnitude ranges. The principle of grouping the stars is that there should be roughly the same number of stars in each subsample and that the number of stars in each subsample should be large enough for statistical analysis.

The 785 stars in the M 11 region are divided into eight subsamples based on their magnitudes in the B band, which are shown in Table 5. By means of the maximum likelihood method, nine unknown distribution parameters for each subsample are determined. The results and the corresponding uncertainties are listed in Table 6, in units of arcmin and arcsec/century. The membership probabilities for individual stars can be calculated as follows

$$P_i = \frac{\Psi_{ci}}{\Psi_i} = \frac{\Psi_{ci}}{\Psi_{ci} + \Psi_{fi}} \quad (8)$$

where P_i is the membership probability for the ith star. In Table 4, the membership probabilities for individual stars in the M 11 region are listed in Col. 12. The histogram of the membership probabilities for the 785 stars is shown in Fig. 3. It can be found that the separation for cluster stars and field stars is very good. The total integrated membership probabilities of these 785 stars is 547 and the number of the stars with membership probabilities higher than 0.7 is 541. From the point of statistics, there will be about 1% field star contamination if these 541 stars are treated as the members of M 11. That is to say, it is a good sample of M 11 for detailed astrophysical researches. In Fig. 4, we also plot the vector-point-diagram of these 541 stars. From this figure, we can see that the average motion of these 541 member stars is close to zero. Thus the reference frame we chose has no unwanted distortions.

4. Comparison and discussion

4.1. The method of membership determination

In order to show clearly the advantage of the method used here for membership determination, we also calculated the result based on the method suggested by Zhao & He (1990). The histogram of the membership probabilities for this result is plotted in Fig. 5. Comparing Fig. 3 and Fig. 5, one sees that the separation for cluster stars and field stars from the improved method used in the present paper is much better than that from the old method. According to the efficiency for distinguishing cluster members from field members suggested by Shao & Zhao (1996), the efficiencies for these two kinds of membership determinations are 0.85 and 0.67, respectively. This also gives strong support to the improved method.

Furthermore, we can compare the surface density distributions for cluster stars and field stars to check which method is better for membership determination. The surface densities for cluster stars and field stars can be defined

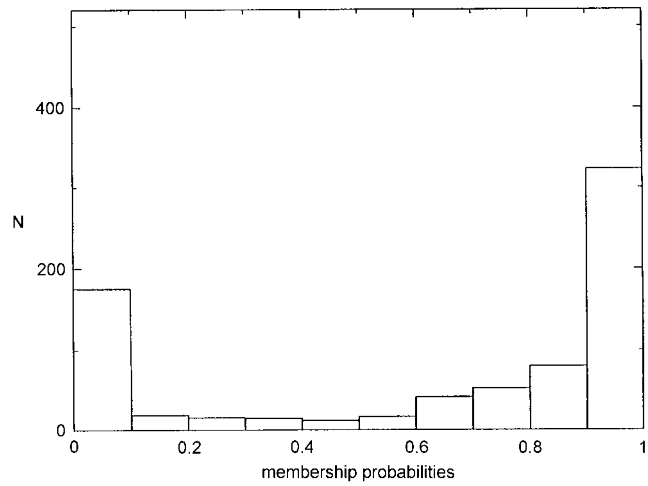

Fig. 5. The histogram of membership probabilities of M 11, which the membership determination is based on Zhao & He (1990)

by the following formulae:

$$D_c = \frac{\sum P_i}{\Delta S} \quad (9)$$

and

$$D_f = \frac{\sum(1 - P_i)}{\Delta S} \quad (10)$$

respectively, where the sum is done for stars within the area ΔS and P_i the membership probability for ith star. Figure 6 and Fig. 7 are the surface density distributions for cluster stars and field stars obtained from the improved method used in the present study and the old method suggested by Zhao & He (1990), respectively. It can be seen clearly that the density for field stars in Fig. 6 is more homogeneous than that in Fig. 7. This gives a further support of our method. The reasons for the distribution of field stars obtained from the old method, for which the density increases significantly to the cluster center, are: (1) as mentioned in Sect. 3, i.e., the dependencies of the space distribution for cluster stars and magnitude are not taken into account in the old method for membership determination; (2) the stellar density is higher in the core and therefore more blended images are present on the plates. Blends are hard to measure since the centroid is very sensitive to exposure time, seeing, etc. They normally produce large, incorrect proper motions that are computationally assigned as field stars.

We must also point out that the surface density model assumed in Eq. (2) in the present study is only a simple approach. The empirical density distribution law of King (1962) can fit star clusters much better than this one. But

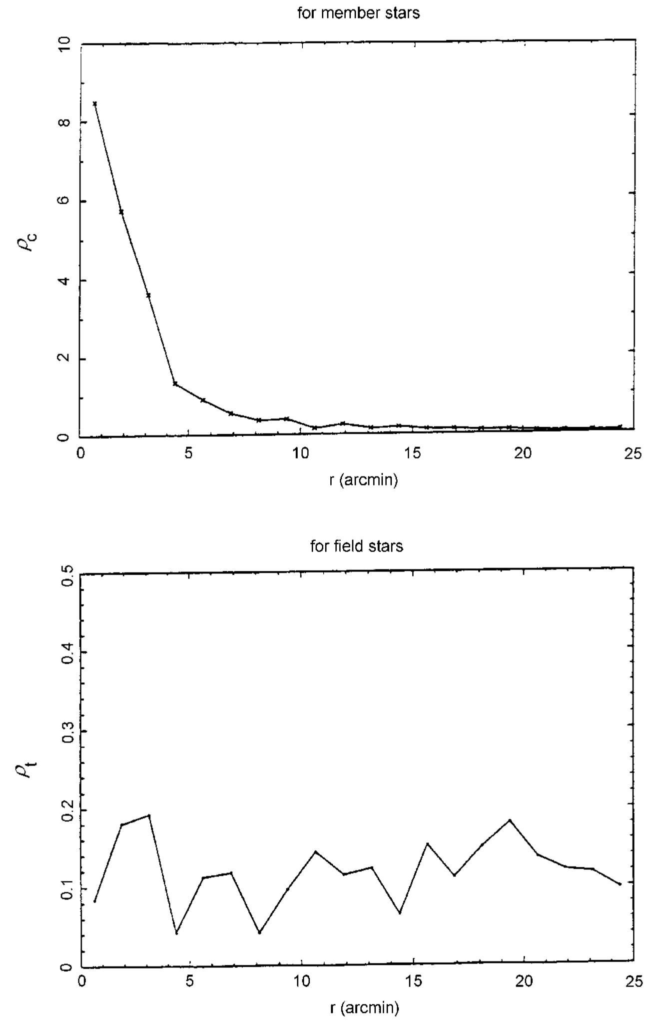

Fig. 6. The surface density distributions (in unit of /arcmin^2) for cluster stars and field stars in the M 11 region: **a)** cluster stars; **b)** field stars

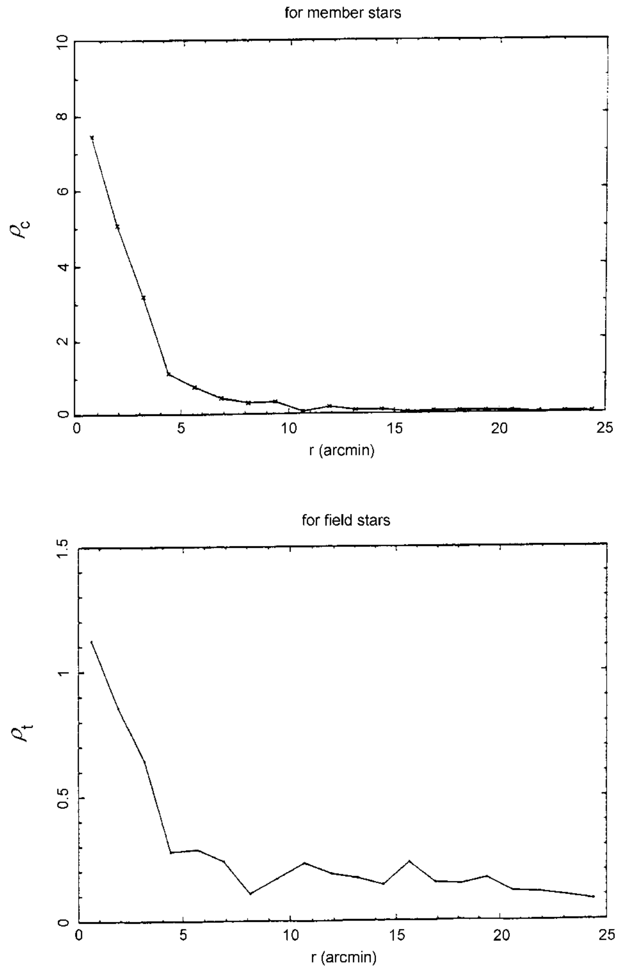

Fig. 7. The surface density distributions (in unit of /arcmin^2) for cluster stars and field stars in the M 11 region, for which the membership determination is based on Zhao & He (1990). **a)** cluster stars; **b)** field stars

there are more parameters (central density ρ_0, core radius r_c and tidal radius r_t) in the King formulae, and it is too complicated to solve with the maximal likelihood method. In the inner part of a cluster, Eq. (2) is quite the same as the King formulae. The major difference appears in the outer part. In Fig. 6, we see that the density distribution for cluster stars plays a role of exponential. So, Eq. (2) can be adopted reasonably here.

4.2. The dynamics

McNamara & Sanders (1977) analysed the internal motion of M 11 based on the sample of McNamara et al.(1977). They found that the cluster radius increases with decreasing stellar mass after calculating the densities of cluster members as functions of distance from the cluster center and magnitude. They also found that there is a vast extensive halo of relatively low-mass stars for M 11. Dynamically, they suggested that equipartition of kinetic energy does not exist in M 11, but the relaxation of its core has been intense, which means that mass segregation exists.

From Table 6 of the present study, the characteristic radius for cluster stars, which is one of the nine distribution parameters, increases with magnitude, i.e. the concentration to the cluster center for bright members is higher than that for faint members. There obviously exists a space mass segregation effect in M 11. The proper motion dispersion for members has a tendency to increase with magnitude also, but not as clearly as for the characteristic radius. Thus there exists velocity mass segregation to some extent. These two conclusions are consistente with McNamara & Sanders (1977) and also supported by Su (1994), Ying et al. (1996) and Su et al. (1997) by means of other statistic methods.

Table 7. Proper motion errors (in unit of mas/yr) versus magnitudes

m_b	*star number*	ε_x	ε_y	ε
< 13.0	155	0.66	0.55	0.61
13.0−13.5	165	0.61	0.63	0.62
14.5−14.0	168	0.71	0.77	0.74
15.0−14.5	153	0.78	0.83	0.81
> 14.5	231	1.29	1.31	1.30

Here is a brief explanation for these two effects: There are two important dynamical time scales for an open cluster. One is the crossing time scale and the other a relaxation time scale. The crossing time scale is defined as the time spent for a typical member star to cross through the cluster, and the relaxation time scale is defined as the time for a cluster undergoing dynamical relaxation to approach equilibrium. Based on the distance of 1.6 kpc (Mathieu 1984; Su et al. 1994), the tidal radius of 24.5 pc (Su 1994; Su et al. 1997) for M 11 and the proper motion dispersion obtained above, we estimate the crossing time scale on the order of 1 10^7 yrs. For the relaxation time scale, we use the method suggested by Spitzer & Hart (1958) and Su (1994) to obtain 2 10^7 yrs. Given an age of 2 10^8 yrs (Mathieu 1984; Su 1994; Su et al. 1997), then M 11 is some 10 and 20 times older than its crossing time scale and relaxation time scale, respectively. Dynamical relaxation of M 11 must be well advanced. Mass segregation effects should have appeared.

Table 8. The variation of proper motion dispersions (km/s) with different threshold membership probabilities for M 11 members in different magnitude ranges

m_b	< 12.4	12.4−13.0	13.0−13.6	> 13.6
$P > 0.7$	1.91	1.96	2.10	2.23
$P > 0.8$	1.75	1.80	1.83	1.86
$P > 0.9$	1.68	1.71	1.75	1.80
$P > 0.95$	1.66	1.70	1.73	1.79

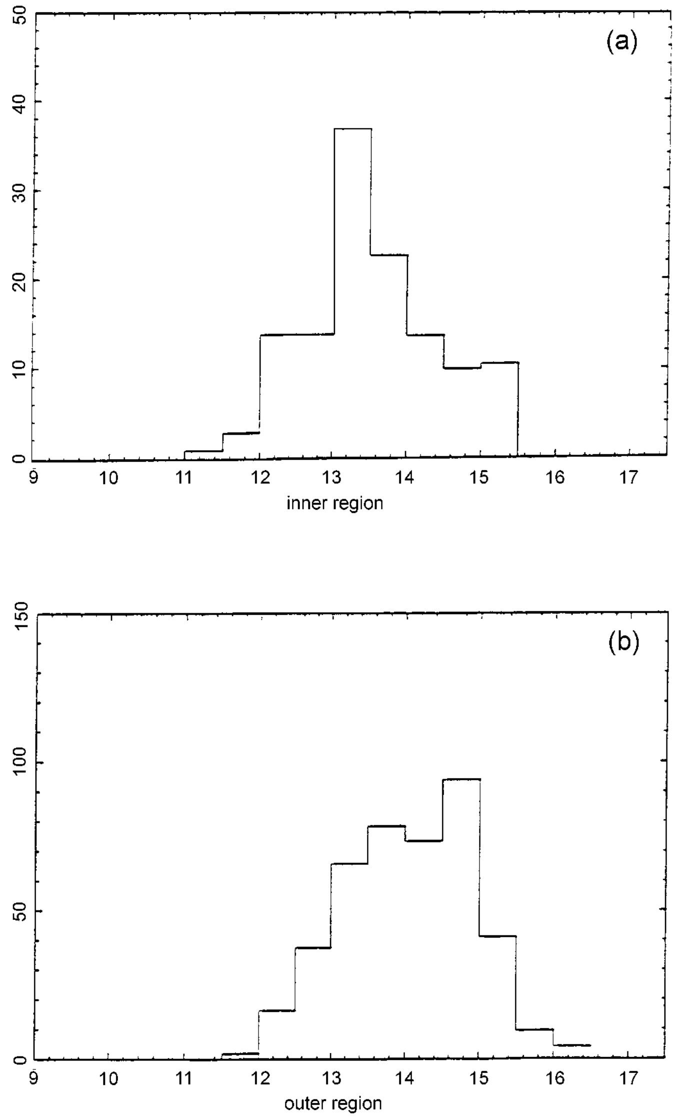

Fig. 8. Luminosity functions for M 11 in different region. **a)** inner region ($r \leq 3'$); **b)** outer region($r > 3'$)

The reason that its velocity-mass segregation is unclear may be due to low accuracies of proper motion measurements for faint stars. This can be seen from Table 7, where the proper motion errors versus magnitudes are listed. Fainter stars have lower accuracies.

To make these two effects for M 11 more certain, the luminosity functions for member stars in different regions and the normalized cumulative surface density distributions for different magnitude ranges of cluster stars are plotted in Fig. 8 and Fig. 9, respectively. In Fig. 8, the luminosity function in the central region is flatter than that in the outer region for M 11, i.e. there are relatively fewer faint stars in the central region. From Fig. 9, it can be seen that the central concentration for bright stars is higher than that for faint stars. Thus the space-mass segregation for M 11 is very clear. In another aspect, the variation of the proper motion dispersions with different threshold membership probabilities for members in different magnitude ranges are listed in Table 8. One sees that a velocity-mass segregation effect still exists to some extent, although the values of dispersions decrease and the differences among the values decrease with threshold probability increasing. This probably reflects contamination by field stars.

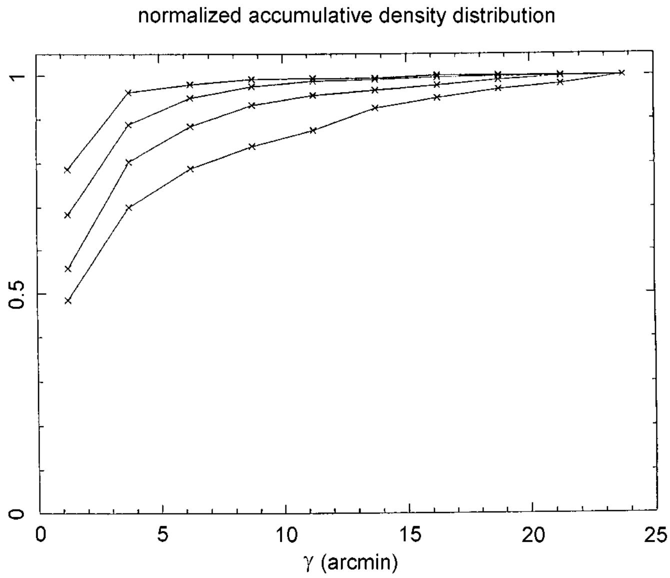

Fig. 9. Normalized cumulative density distribution for M 11 clusters stars. From top to bottom, the curves denote magnitude ranges of $m_b < 12.5$, 12.5−13.5, 13.5−14.5 and > 14.5 respectively

5. Summary

In the present paper, we use 10 plate pairs, for which the time baselines are from 16 to 70 years, to obtain the relative proper motions of 872 stars in the open cluster M 11 region. The accuracy of our sample is relative high, the average being some 1.1 mas/yr and 85% of the data better than 1 mas/yr.

Using an improved method, we finish the membership determination of 785 stars within the central 25′ region of M 11 and membership probabilities for individual stars are given. The results are significantly good. The sample is very useful for further astrophysical researches of M 11. Some comparisons between our method and the old one are made, showing that the improved method is much better than the old method.

During the membership determination, we find that there exist both space- and velocity-mass segregation effects for M 11. This conclusion is also supported by other authors. A brief explanation is presented in the paper.

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