ASTRONOMY & ASTROPHYSICS
SUPPLEMENT SERIES

NOVEMBER II **1997**, PAGE 113

Astron. Astrophys. Suppl. Ser. **126,** 113-119 (1997)

Instrumental polarization caused by telescope optics during wide field imaging

A.K. Sen[1,⋆] **and M. Kakati**[2]

Centre of Plasma Physics, Dispur, Guwahati 781 006, India
e-mail: 1. aks@ipprghy.ren.nic.in 2. mayur@ipprghy.ren.nic.in

Received November 19; accepted December 19, 1996

Abstract. When astronomical observations are made for a celestial object, with non -zero field angle (wrt telescope axis), the beam of parallel rays from the celestial object strikes the telescope mirror obliquely. Each unpolarized ray of light when it strikes the metal coated mirror surface gets polarized due to reflection. On the contrary, when the field angle is zero, these reflected rays for a field star, combine together to produce an instrumental polarization effect. A 100% linearly polarized star when observed even at zero field angle, exhibits depolarization due to the above effect. A detailed procedure has been worked out here to estimate such polarization effects at the prime and Cassegrain focii, considering the case for linear polarizations only.

Also to find the typical values of such polarization, a 2.3 m telescope having beam sizes f/3.23 and f/13 at the prime and Cassegrain focii, has been considered. The instrumental polarization values as calculated at these two focii are 0.000092 and 0.016104% at the field angles 300 and 90 arcsec, respectively. Furthermore, a 100% polarized star when observed at the above two focii will appear to be 99.9999 and 99.9983% polarized respectively due to depolarization.

Key words: techniques: polarimetric techniques — telescopes — polarization

1. Introduction

Polarimetry is a powerful technique to study the ongoing astrophysical processes in celestial objects. When polarimetry is conducted for a single point object, we normally place the object on the axis of the telescope (i.e. the field angle of the object is zero). As each unpolarized ray of light falls on the metal coated mirror surface, it gets polarized due to oblique incidence. The ray after being reflected from the primary mirror also gets reflected at the secondary mirror and thereby the polarization state of the ray gets complicated with a mixture of linear and circular polarization. But if the object is on the axis of the telescope, we have all the rays incident on primary parallel to the telescope axis. Thereby, in case of prime focus (or Cassegrain focus), we will have a total circular symmetry for these rays and the net polarization effect for all the rays considered together will be zero. Thus the instrumental polarization will always be zero for an on-axis object point due to the above mechanism.

However, for an off-axis object point, the field angle will produce some finite value of instrumental polarization. When the field angle is different from zero, the Stokes parameters Q, U and V for the rays for an unpolarized star, add up to give a non-zero instrumental polarization effect. Actually compared to the other sources of instrumental polarization, this effect produces polarization values too small to be detected by any polarimeter. Also objects are normally not observed off-axis and thereby such effects are normally ignored.

As discussed in detail by Serkowski (1974), in polarimetry there are mainly two kinds of errors: (i) the uncertainty (or noise) (δp) in the estimated values of polarization due to photon count statistics, which is in general $\sim \delta I/I$ (Sen et al. 1990); where δI is the uncertainty in measured intensity (I) and (ii) "instrumental polarization" arising due to the polarimeter optics, which is mostly a systematic error. However, such an instrumental polarization can also arise due to the telescope optics itself and will be discussed in detail in this paper. The "instrumental polarization" due to the polarimeter can be caused by the chromaticity and incidence angle dependent performance of the optical components (like polarizers, analyzers etc.) and also unnecessary reflection from such components (Serkowski 1974). In addition, errors in polarization measurements can also be due to the varying sky background. Unless very bright objects are observed, during

Send offprint requests to: A.K. Sen
⋆ *Present address*: Department of Physics, Assam University, Silchar 788 015, India

polarimetry one normally gets photon noise limited polarization values. Typically a present day polarimeter (Sen & Tandon 1994; Rampraksh et al. 1996) can give $\delta p \sim 0.01p$ due to the polarimeter optics and $\delta p \sim 0.3\%$ due to photon noise, when a 17.5 mag (arcsec)$^{-2}$ source is observed for 1000 s.

These days, imaging polarimetry (or area scan polarimetry) is emerging as a new area of observational astronomy. Quite a good amount of work has been done in this area over the last two decades. The Durham University group with their imaging polarimeter sometimes cover a field angle up to 1 arcmin (Scarrott et al. 1983, 1991). Similarly, astronomers from MPIK, Germany have used their polarimeter to cover field angles as large as 1 arcmin (Röser 1981; Röser & Meisenheimer 1986; Schlötelburg et al. 1988). Other imaging polarimetry works can also be mentioned in these connections. Renard et al. (1992) have covered a field angle up to 30 arcsec in their measurement of comet Levy. Sen et al. (1990) in their imaging polarimetry work on comet P/Halley have covered a field angle as large as 10 arcmin. Also astronomers are increasingly using larger formats for their CCDs. So if we take a CCD of 20 mm dimension and assume a plate scale 30 arcsec/mm for prime focus and 10 arcsec/mm for Cassegrain focus; the maximum field angle that an off-axis star will cover will be 300 and 100 arcsec, respectively.

Thus, it is important to estimate such instrumental polarization values and to understand their nature, however small their effects may be at the present stage. In this paper we derive a procedure for estimating such instrumental polarization values and also calculate them in the actual case of a 2.3 m telescope having beam sizes f/3.23 and f/13 at the prime and Cassegrain focii respectively. For the simplicity of calculations we shall limit our discussions to the effects on linear polarization only.

2. Geometry of rays incident on primary mirror

The primary mirror of the telescope is in general a concave paraboloid. We consider a paraboloid, having its axis coinciding with the Z-axis and the vertex coinciding with the origin of the co-ordinate system. We consider one incident ray in the ZX plane, making an angle η with Z axis (and $(90^{\mathrm{o}} - \eta)$ with X axis) and passing through the origin. The Y axis will be perpendicular to this ray. The direction cosine (henceforth d.c.) of the incident ray can be written as:

$$(pl_{\mathrm{i}}, pm_{\mathrm{i}}, pn_{\mathrm{i}}) = (\sin\eta, 0, \cos\eta). \tag{1}$$

Now we consider a beam of rays which are parallel to this above ray and coming from a celestial object having a field angle η with respect to the telescope axis. The rays which lie in the outer periphery of this beam, will be incident on the parabolloid mirror at points defining a circle, with radius $h1$ and the centre of the circle lying on the Z axis. (where $2 \times h1$ is the diameter of the paraboloid mirror). The plane defined by this circle will have Z axis perpendicular to it. The (x, y) co-ordinates of any point on this circle can be expressed as $(h1 \times \cos\theta, h1 \times \sin\theta)$. In this case θ is the azimuthal angle of the ray. For example the rays which are contained in the ZX plane will have $\theta = 0, 180$. We further assume that $f1$ is the f-number of the primary mirror, which is nothing but the ratio of focal length to the diameter.

Now the equation of the above paraboloid can be expressed as:

$$x^2 + y^2 = (8 \times h1 \times f1) \times z. \tag{2}$$

Substituting the values $(x, y) = (h1 \times \cos\theta, h1 \times \sin\theta)$, we get the Z coordinate of the point of incidence as $h1/(8 \times f1)$. Now the d.c. of the normal at this point $(h1 \times \cos\theta, h1 \times \sin\theta, h1/(8 \times f1))$ can be expressed as:

$$(pl_{\mathrm{n}}, pm_{\mathrm{n}}, pn_{\mathrm{n}})) = \left(\frac{-\cos\theta}{\sqrt{1 + 16 \times f1^2}}, \frac{-\sin\theta}{\sqrt{1 + 16 \times f1^2}}, \frac{4 \times f1}{\sqrt{1 + 16 \times f1^2}} \right). \tag{3}$$

The angle of incidence (pi) between the normal and incident ray can be expressed as:

$$\cos pi = \frac{-\cos\theta \times \sin\eta + 4 \times f1 \times \cos\eta}{\sqrt{1 + 16 \times f1^2}}. \tag{4}$$

The d.c of the reflected ray $(pl_{\mathrm{r}}, pm_{\mathrm{r}}, pn_{\mathrm{r}})$, which makes an angle pi with the normal and $2 \times pi$ with the incident ray can be expressed as:

$$(pl_{\mathrm{r}}, pm_{\mathrm{r}}, pn_{\mathrm{r}}) = \left(\frac{-2 \times \cos\theta \times \cos pi}{\sqrt{1 + 16 \times f1^2}} - \sin\eta, \right. \\ \left. \frac{-2 \times \sin\theta \times \cos pi}{\sqrt{1 + 16 \times f1^2}}, \frac{8 \times f1 \times \cos pi}{\sqrt{1 + 16 \times f1^2}} - \cos\eta \right). \tag{5}$$

Now we shall find the d.c. of the other vectors connected to the incident plane. The electric field vectors of the incident and reflected rays can be resolved in two directions, one perpendicular to the plane of incidence (s-direction) and the other orthogonal to the s-direction (we call it p-direction). Actually there are two p-directions, one corresponds to the incident ray (pi-direction) and the other corresponds to the reflected ray (pr-direction). While discussing reflection on primary mirror, we refer to these directions as ps, ppi and ppr.

While considering the reflection on the secondary mirror, we shall denote the corresponding quantities as ss, spi and spr vectors.

The ps-vector is actually perpendicular to the plane containing vectors $(pl_{\mathrm{i}}, pm_{\mathrm{i}}, pn_{\mathrm{i}})$ and $(pl_{\mathrm{n}}, pm_{\mathrm{n}}, pn_{\mathrm{n}})$. Therefore the d.c. of the ps-vector $(pl_{\mathrm{s}}, pm_{\mathrm{s}}, pn_{\mathrm{s}})$ will be proportional to

$$(pm_{\mathrm{i}} \times pn_{\mathrm{n}} - pm_{\mathrm{n}} \times pn_{\mathrm{i}}, pn_{\mathrm{i}} \times pl_{\mathrm{n}} - pn_{\mathrm{n}} \times pl_{\mathrm{i}}, pl_{\mathrm{i}} \times pm_{\mathrm{n}} - pl_{\mathrm{n}} \times pm_{\mathrm{i}}).$$

After proper substitution and normalisation, these values can be determined as:

$$(pl_{\rm s}, pm_{\rm s}, pn_{\rm s}) = \left(\frac{\cos\eta\sin\theta}{\sqrt{\sin^2\theta + (\cos\eta\cos\theta + 4f1\sin\eta)^2}}, \frac{-(\cos\eta\cos\theta + 4f1\sin\eta)}{\sqrt{\sin^2\theta + (\cos\eta\cos\theta + 4f1\sin\eta)^2}}, \frac{-\sin\eta\sin\theta}{\sqrt{\sin^2\theta + (\cos\eta\cos\theta + 4f1\sin\eta)^2}} \right). \tag{6}$$

Similarly the *ppi*-vector with directions (pl_{pi}, pm_{pi}, pn_{pi}), is perpendicular to the directions ($pl_{\rm s}, pm_{\rm s}, pn_{\rm s}$) and ($pl_{\rm i}, pm_{\rm i}, pn_{\rm mi}$) and therefore, one can derive

$$(pl_{pi}, pm_{pi}, pn_{pi}) = (-(\cos\eta\cos\theta + 2f1\sin 2\eta)/G, -\sin\theta/G, \sin\eta(\cos\eta\cos\theta + 4f1\sin\eta)/G)\,. \tag{7}$$

The *ppr*-vector with direction ($pl_{pr}, pm_{pr}, pn_{pr}$), is perpendicular to the directions ($pl_{\rm s}, pm_{\rm s}, pn_{\rm s}$) and ($pl_{\rm r}, pm_{\rm r}, pn_{\rm r}$) and one can similarly derive

$$\begin{aligned}(pl_{pr}, pm_{pr}, pn_{pr}) = \big(&-(F(32\ f1^2\ \sin\eta \\ &+8f1\ \cos\eta\ \cos\theta + 2\ \sin\eta\ \sin^2\theta) \\ &\cos pi - \cos^2\eta\ \cos\theta)/G, \\ &(F(\sin\eta\ \sin 2\theta - 8f1\ \sin\theta\ \cos\eta)\ \cos pi \\ &+\sin\eta + \sin\theta\ \cos^2\eta)/G, \\ &-(F(2\ \cos\eta + 8f1\ \cos\theta\ \sin\eta)\ \cos pi \\ &+4f1\ \sin^2\eta)/G\big)\end{aligned} \tag{8}$$

where we have substituted

$$\textbf{a)}\ F = \frac{1}{\sqrt{1+16f1^2}} \quad \text{and} \quad \textbf{b)}\ G = \sqrt{\sin^2\theta + (\cos\eta\ \cos\theta + 4f1\ \sin\eta)^2}. \tag{9}$$

3. Instrumental polarization at the prime focus

Once the angle of incidence i (in general) for a particular ray is known we can find the reflectivities $r_{\rm s}$ and $r_{\rm p}$ corresponding to the p and s component of the electric vector of the incident ray. These reflectivities are complex numbers and can be expressed as (Born & Wolf 1957):

$$\textbf{a)}\ r_{\rm p} = \frac{\tan(i-r)}{\tan(i+r)}; \quad \textbf{b)}\ r_{\rm s} = -\frac{\sin(i-r)}{\sin(i+r)} \tag{10}$$

where r is the angle of refraction, determined from the relation

$$\sin\ r = \frac{\sin\ i}{n_{\rm c}} \tag{11}$$

$n_{\rm c}$ is the complex refractive index of the telescope surface. In the present case, we shall assume $n_{\rm c} = (1.44 - i \times 5.23)$, the complex refractive index of aluminium at wavelength $\lambda = 5893$ Angström (Sodium D line) (Born & Wolf 1957).

However, when the angle of incidence i is 0, we use the relation

$$\textbf{a)}\ r_{\rm p} = \frac{(n-1)}{(n+1)}; \quad \textbf{b)}\ r_{\rm s} = \frac{(n-1)}{(n+1)}. \tag{12}$$

The amplitudes of the reflected ray in the p and s-directions (denoted by $R_{\rm p}$ and $R_{\rm s}$) are related to the corresponding amplitudes for the incident ray ($E_{\rm p}$ and $E_{\rm s}$) by the following relations:

$$\textbf{a)}\ R_{\rm p} = r_{\rm p} \times E_{\rm p}; \quad \textbf{b)}\ R_{\rm s} = r_{\rm s} \times E_{\rm s} \tag{13}$$

Here it is to be noted that the p-directions of the vector for R and E are not same.

3.1. Case of an unpolarized star

For unpolarized incident light the electric vector of incident ray does not have any preference for a particular directions. Therefore we assume

$$E_{\rm p} = E_{\rm s} = \frac{1}{\sqrt{2}}$$

so that the two components are equal in magnitude and the total intensity is 1. For any electromagnetic wave (with two orthogonal amplitude vectors), one generally defines a set of four Stokes parameters for the analysis of polarization (Shurcliff 1962). In the present case for $R_{\rm p}$ and $R_{\rm s}$ orthogonal components, the Stokes parameters in the $p-s$ coordinate frame can be written as follows:

$$\begin{array}{ll}\textbf{a)}\ i_{\rm ps} = R_{\rm p}^2 + R_{\rm s}^2; & \textbf{b)}\ q_{\rm ps} = R_{\rm p}^2 - R_{\rm s}^2 \\ \textbf{c)}\ u_{\rm ps} = 2\ R_{\rm p}\ R_{\rm s}\ \cos\ (\delta p - \delta s) & \textbf{d)}\ v_{\rm ps} = 2\ R_{\rm p}\ R_{\rm s}\ \sin\ (\delta p - \delta s)\end{array} \tag{14}$$

where δp and δs as the phase angles of $R_{\rm p}$ and $R_{\rm s}$ components. The fourth Stokes parameter v is related to the circular polarization which will not be considered in the present case.

Further, any set of three stokes parameters (I, Q, U) are related to the degree of linear polarization (P) and position angle of linear polarization (PA) by the following relations (Shurcliff 1962):

$$\textbf{a)}\ P = \frac{\sqrt{Q^2 + U^2}}{I} \quad \text{and} \quad \textbf{b)}\ \text{PA} = \tan^{-1}\frac{U}{Q}. \tag{15}$$

Now in order to find the net instrumental polarization combining all the rays, we should express all the Stokes parameters in a common coordinate frame. The choice obviously is the XY coordinate frame. This is because the polarizers and analyzers of polarimeter are normally placed in a plane perpendicular to the telescope axis (which in our case is the XY plane) and all the polarization measurements are done with respect to that plane.

Thus to transform the individual set of Stokes parameters into the XY frame, one should rotate the $p-s$ frame by an angle θ. However, strictly speaking, this is not correct as the reflected rays are not parallel to the Z axis and accordingly the ps-plane is not parallel to the XY-plane. But in present day polarimeters the optical components (polarizers, analysers etc.) are supposed to take care of this oblique incidence of rays on them. Most of them have a correction facility (up to a few degrees of angle of incidence) and polarization measurements are made by the polarimeter as though all the rays are entering the polarimeter parallel to telescope axis. Therefore we can assume by rotating the ps-frame by an angle θ we can transform the Stokes parameter to an XY-frame. The new set of Stokes parameters under such a rotation can be expressed as (Chandrasekhar 1960)

$$\begin{aligned} &\textbf{a)}\ i_{XY} = i_{\rm ps} \\ &\textbf{b)}\ q_{XY} = q_{\rm ps} \times \cos(2\times\theta) - u_{\rm ps} \times \sin(2\times\theta) \\ &\textbf{c)}\ u_{XY} = q_{\rm ps} \times \sin(2\times\theta) + u_{\rm ps} \times \cos(2\times\theta). \end{aligned} \tag{16}$$

In our case since each ray is unpolarized, we assume there is no systematic phase relation between its p and s components. Accordingly the time average value of $u_{\rm ps}$ can be assumed to be zero. The above i_{XY}, q_{XY} and u_{XY} values are functions of θ and field angle η. These values are determined for each ray individually and the corresponding Stokes parameters are added to get the resultant Stokes parameter values (I, Q, and U) for the entire beam.

From this I, Q, U values we estimate the instrumental polarization and position angle by using relation (15a,b). An expression for instrumental polarization so produced at the prime focus can be written as:

$$P = \frac{\sqrt{Q^2+U^2}}{I}$$

where

$$\begin{aligned} &\textbf{a)}\ I = \textstyle\int_0^{2\pi}(r_{\rm p}^2 + r_{\rm s}^2){\rm d}\theta \\ &\textbf{b)}\ Q = \textstyle\int_0^{2\pi}(r_{\rm p}^2 - r_{\rm s}^2) \times \cos(2\theta){\rm d}\theta \\ &\textbf{c)}\ U = \textstyle\int_0^{2\pi}(r_{\rm p}^2 - r_{\rm s}^2) \times \sin(2\theta){\rm d}\theta. \end{aligned} \tag{17}$$

The expressions for $r_{\rm p}$ and $r_{\rm s}$ are available in (10a,b).

3.2. Case of a polarized star

With the star on the axis of the telescope (i.e. $\eta = 0$), we assume that the polarization vector (denoted by the corresponding electric vector "$\boldsymbol{E}$") makes an angle α with the X axis. For this ray the d.c. of the electric vector($\boldsymbol{E}$) will be $(\cos\alpha, \sin\alpha, 0)$. Now we assume the position of the star is rotated from the z axis and moved towards the x axis in the xz plane by an angle η. In other words, if the field angle of the star is now η, the new d.c. of the electric vector ($\boldsymbol{E}$) will be now $(l_{\rm e}, \sin\alpha, n_{\rm e})$. Here, $l_{\rm e}$ and $n_{\rm e}$ are the two unknown parameters to be determined. Since the incident ray and the electric vector($\boldsymbol{E}$), are orthogonal to each other we should have

$$l_{\rm e} \times \sin\eta + n_{\rm e} \times \cos\eta = 0. \tag{18}$$

Again for the electric vector ($\boldsymbol{E}$) we should have

$$l_{\rm e} \times l_{\rm e} + m_{\rm e} \times m_{\rm e} + n_{\rm e} \times n_{\rm e} = 1. \tag{19}$$

Combining Eqs. (18) and (19), we finally get

$$(l_{\rm e}, m_{\rm e}, n_{\rm e}) = (\cos\alpha \times \cos\eta, \sin\alpha, \cos\alpha \times \sin\eta)\,. \tag{20}$$

Thus knowing the position angle of polarization vector we can find its d.c. $(l_{\rm e}, m_{\rm e}, n_{\rm e})$. With $E_{\rm p}$ and $E_{\rm s}$, as the components of $\boldsymbol{E}$-vector in the directions $(pl_{pi}, pm_{pi}, pn_{pi})$ and $(pl_{\rm s}, pm_{\rm s}, pn_{\rm s})$, they can be expressed by the relations:

$$\begin{aligned} &\textbf{a)}\ E_{\rm p} = l_{\rm e} \times pl_{pi} + m_{\rm e} \times pm_{pi} + n_{\rm e} \times pn_{pi}; \\ &\textbf{b)}\ E_{\rm s} = l_{\rm e} \times pl_{\rm s} + m_{\rm e} \times pm_{\rm s} + n_{\rm e} \times pn_{\rm s}. \end{aligned} \tag{21}$$

Now substituting these values of $E_{\rm p}$ and $E_{\rm s}$ in (13a,b) we can get the values of $R_{\rm p}$ and $R_{\rm s}$, which can in turn be used to calculate the Stokes parameters from relation (14a,b,c). But in the present case we shall not assume the Stokes parameter $u_{\rm ps}$ to be zero as the light is already polarized. Finally we can calculate the polarization values observed for a 100% polarized star, as a measure of depolarization effect. When the field angle (η) is zero, for any value of α such observed polarization value (P) can be expressed as:

$$P = \left(\frac{1}{2} + \frac{r_{\rm p} \times r_{\rm s}}{r_{\rm p}^2 + r_{\rm s}^2}\cos\phi\right) \tag{22}$$

where ϕ is the phase difference between the reflectivities $r_{\rm p}$ and $r_{\rm s}$.

4. Instrumental polarization at the Cassegrain focus

4.1. Reflection geometry for hyperbolic secondary

The secondary mirror is a convex hyperboloid, which can be represented by the following equation:

$$ap \times x^2 + ap \times y^2 + cp \times (z-\tau)^2 = 1. \tag{23}$$

The above equation represents two branches of hyperboloid along the Z-axis. However, the one towards the positive direction of Z-axis will represent the secondary mirror in our case. In Eq. (23)

$$ap = \frac{1}{c^2 \times (1-e^2)}; \quad cp = \frac{1}{c^2}; \quad \tau = dd - c \tag{24}$$

where e is the eccentricity of the hyperboloid, $2 \times c$ is the distance between the two branches of hyperboloid (vertex-vertex), dd is the distance of the vertex of secondary mirror from the origin (which is the vertex of paraboloid also).

The quantities c and dd can be found out easily once we know the diameters of two mirrors ($h1$ and $h2$), beam sizes ($f1$ and $f2$) at the two focii and eccentricity of the hyperbolic surfaces (e).

Now the equation of the ray reflected from the primary mirror (which is same as the ray incident on the secondary mirror) can be written as

$$\frac{x - h1 \times \cos(\theta)}{pl_\mathrm{r}} = \frac{y - h1 \times \sin(\theta)}{pm_\mathrm{r}} = \frac{z - h1/(8 \times f1)}{pn_\mathrm{r}} = r. \tag{25}$$

The expressions for $(pl_\mathrm{r}, pm_\mathrm{r}, pn_\mathrm{r})$ are given in Eq. (5). Now substituting $(x, y, z) = (h1 \times \cos\theta + pl_r \times r, h1 \times \sin\theta + pm_\mathrm{r} \times r, h1/(8 \times f1) + pn_\mathrm{r} \times r)$ in (23) we get

$$\begin{aligned} &r^2 \times (ap \times (pl_\mathrm{r}^2 + pm_\mathrm{r}^2) + cp \times pn_\mathrm{r}^2) + r \\ &\times(2 \times ap \times h1 \times (pl_\mathrm{r} \times \cos\theta + pm_\mathrm{r} \\ &\times \sin\theta) + 2 \times n_\mathrm{r} \times cp \\ &\times(h1/(8 \times f1) - \tau)) \\ &+(ap \times h1^2 + cp \times (h1/(8 \times f1) - \tau)^2 - 1) = 0. \end{aligned} \tag{26}$$

This Eq. (26) has two roots for r and we shall accept only that root (say $r = rt$) which corresponds to the hyperboloid of our choice (i.e. the secondary mirror).

In this process we determine the point of incidence (xs, ys, zs) on the secondary mirror as the following:

$$\begin{aligned} &(xs, ys, zs) = (h1 \times \cos\theta + pl_\mathrm{r} \times rt, h1 \\ &\times \sin\theta + pm_\mathrm{r} \times rt, h1/(8 \times f1) + pn_\mathrm{r} \times rt). \end{aligned} \tag{27}$$

Now the d.c. of the normal at the above point of incidence on to the secondary can be given by

$$(sl_\mathrm{n}, sm_\mathrm{n}, sn_\mathrm{n}) = (xs \times ap, ys \times ap, (zs - \tau) \times cp). \tag{28}$$

Now, as we derived in the case of primary paraboloid earlier, in the present the angle of incidence si, d.c. of reflected ray $(sl_\mathrm{r}, sm_\mathrm{r}, sn_\mathrm{r})$, the d.c. of the vectors ss $(sl_\mathrm{s}, sm_\mathrm{s}, sn_\mathrm{s})$, spi $(sl_{pi}, sm_{pi}, sn_{pi})$, spr $(sl_{pr}, sm_{pr}, sn_{pr})$ can also be derived as (pl. see Sect. 2):

$$si = \cos^{-1}(pl_\mathrm{r} \times sl_\mathrm{n} + pm_\mathrm{r} \times sm_\mathrm{n} + pn_\mathrm{r} \times sn_\mathrm{n}) \tag{29}$$

$$\begin{aligned} &(sl_\mathrm{r}, sm_\mathrm{r}, sn_\mathrm{r}) = (2 \times \cos si \times sl_\mathrm{n} - pl_\mathrm{r}, \\ &2 \times \cos si \times sm_\mathrm{n} - pm_\mathrm{r}, 2 \times \cos si \times sn_\mathrm{n} - pn_\mathrm{r}) \end{aligned} \tag{30}$$

$$(sl_\mathrm{s}, sm_\mathrm{s}, sn_\mathrm{s}) = \frac{(pm_\mathrm{r} \times sn_\mathrm{n} - sm_\mathrm{n} \times pn_\mathrm{r}, pn_\mathrm{r} \times sl_\mathrm{n} - sn_\mathrm{n} \times pl_\mathrm{r}, pl_\mathrm{r} \times sm_\mathrm{n} - sl_\mathrm{n} \times pm_\mathrm{r})}{\text{normalising const.}} \tag{31}$$

$$(sl_{pi}, sm_{pi}, sn_{pi}) = \frac{(pm_\mathrm{r} \times sn_\mathrm{s} - sm_\mathrm{s} \times pn_\mathrm{r}, pn_\mathrm{r} \times sl_\mathrm{s} - sn_\mathrm{s} \times pl_\mathrm{r}, pl_\mathrm{r} \times sm_\mathrm{s} - sl_\mathrm{s} \times pm_\mathrm{r})}{\text{normalising const.}} \tag{32}$$

$$(sl_{pr}, sm_{pr}, sn_{pr}) = \frac{(sm_\mathrm{r} \times sn_\mathrm{s} - sm_\mathrm{s} \times sn_\mathrm{r}, sn_\mathrm{r} \times sl_\mathrm{s} - sn_\mathrm{s} \times sl_\mathrm{r}, sl_\mathrm{r} \times sm_\mathrm{s} - sl_\mathrm{s} \times sm_\mathrm{r})}{\text{normalising const.}}. \tag{33}$$

We are now in a position to calculate the reflectivities for the p and s components of the electric vector (r_p and r_s) for reflection on the secondary mirror. For the ray reflected from the primary mirror we have the R_p and R_s amplitude components. Again taking their components in the $(sl_{pi}, sm_{pi}, sn_{pi})$ and (sl_s, sm_s, sn_s) directions, we get the amplitudes (SE_p, SE_s) for the incident ray on the secondary in the p and s directions. Multiplying them by the reflectivities for the secondary mirror, we get the reflected amplitudes SR_p and SR_s for the secondary mirror. Now to calculate instrumental polarization, we shall consider two cases (i) when rays incident on the primary are completely unpolarized and (ii) when rays incident on the primary are 100% polarized.

4.2. Case of unpolarized rays incident on primary

Each individual ray, which is initially unpolarized after reflection from primary will get polarized (say, with degree of polarzation pp). This ray is now incident on the secondary and it consists of two parts unpolarized and polarized maintaining a ratio $(1 - pp) : pp$, in their intensities. For the unpolarized part to calculate the stokes parameters we follow the same proceedure as was done in case of primary mirror (assuming $u_\mathrm{ps} = 0$). For polarized part we calculate the Stokes parameters in a similar way but the Stokes parameter u_ps will no longer be assumed to be zero.

The two sets of Stokes parameters will now be added to calculate the instrumental polarization for a particular ray (reflection geometry) and then transformed into the XY-frame. Integrating all these Stokes parameter values for different rays (within range $\theta = 0 - 2\pi$), we get the resultant Stokes parameter values. These values will finally help us to calculate instrumental polarization by using Eqs. (15a,b) at the Cassegrain focus, for unpolarized light incident on primary.

4.3. Case of 100% polarized rays incident on primary

When light is totally polarized and incident on primary mirror, we first calculate the reflected components R_p and R_s on the primary. We take their components in the directions $(sl_{pi}, sm_{pi}, sn_{pi})$ and $(sl_\mathrm{s}, sm_\mathrm{s}, sn_\mathrm{s})$ to get the p and s components of incident ray on the secondary. From this we get the p and s components of reflected amplitudes (SR_p and SR_s) on the secondary. These amplitudes help us to calculate the Stokes parameters in the local $(p - s)$ frame of secondary and these are later transformed into the $(X - Y)$ frame. These set of Stokes parameter values are used to calculate the observed polarization values, as was done in Sect. 3.2.

5. Results and discussions

In order to find some typical estimates of such instrumental polarization, we shall consider the case of 2.3 m

Vainu Bappu telescope of Indian Institute of Astrophysics, Kavalur. This telescope has beam sizes $f/3.23$ and $f/13$ at the prime and Cassegrain focii. The diameters of the primary and secondary mirrors are 2.32 and 0.63 m. The hyperbolic secondary has an eccentricity of 2.7776.

We consider a bunch of unpolarized parallel rays incident on the primary of such a telescope and calculate the instrumental polarization at the prime focus. The bunch of parallel rays are coming from a direction making a field angle η with the telescope axis. We consider different values of η within range $0 - 300$ arcsec and corresponding instrumental polarization values as calculated are reproduced in Table 1. At 300 arcsec field angle, the amount of instrumental polarization is of the order of 0.0001%. This value is too low and no present day astronomical polarimeter can measure such a small value.

Table 1. The instrumental polarization values (in percent) introduced at f/3.23 prime focus for an unpolarized star observed at a given field angle (η in arcsec) are shown in Col. 2. Whereas in Col. 3 the depolarized values of a 100% polarized star (with position angle = 45 degrees) are shown

FLD (sec) η	POL (unpol)	Pol (for 100% pol. str.)
0	0.000000	99.999886
20	0.000001	99.984192
40	0.000002	99.937111
60	0.000005	99.858650
80	0.000007	99.748764
100	0.000010	99.607513
120	0.000015	99.434906
140	0.000020	99.230873
160	0.000026	98.995458
180	0.000033	98.728645
200	0.000041	98.430466
220	0.000049	98.100891
240	0.000059	97.739960
260	0.000069	97.347595
280	0.000079	96.923820
300	0.000092	96.468719

We consider the above rays coming from a star instead of being unpolarized, are now 100% polarized with the polarization vector making an angle 45 degrees with the reference X-axis. Here we want to see the extent of instrumental polarization effect. At zero field angle the depolarized value of stellar polarization is 99.999886%. Also as we can see from Table 1, the degree of polarization reduces to 96.5% from 100% at a field angle 300 arcsec. However, the polarization vector remains fixed at 45 degrees (not shown in Table 1).

It can be seen from the detailed calculations described in previous sections, that such instrumental polarization values increase with the fastness of prime focus beam and obviously with the field angle.

Now we consider the instrumental polarization observed at the Cassegrain focus for the above two cases, (i) unpolarized light and (ii) 100% polarized light incident on the primary. In the first case we can see from Table 2 that the polarization values are definitely higher compared to the prime focus. At 90 arcsec one gets a value of 0.016% polarization. With a high precision polarimeter one can make attempts to measure such a value. In the second case, we observe that at the Cassegrain focus there is considerable depolarization. The depolarized value of polarization at zero field angle is 99.998334%, slightly higher than the corresponding value at prime focus. At a field angle of 90 arcsec, as can be seen from Table 2, a 100% polarized star will show 99.998184% polarization.

Table 2. The instrumental polarization values (in percent) introduced at f/13 Cassegrain focus for an unpolarized star observed at a given field angle (η in arcsec) are shown in Col. 2. Whereas in Col. 3 the depolarized values of a 100% polarized star (with position angle = 45 degrees) are shown

FLD (sec) η	POL (unpol)	Pol (for 100% pol. str.)
0	0.000000	99.998334
10	0.001774	99.998329
20	0.003552	99.998322
30	0.005328	99.998314
40	0.007114	99.998299
50	0.008901	99.998283
60	0.010693	99.998253
70	0.012491	99.998230
80	0.014294	99.998207
90	0.016104	99.998184

The instrumental polarization at the Cassegrain focus depends upon the diameters of two mirrors, the corresponding beam sizes and eccentricity of the hyperboloid. For unpolarized light we normally expect a higher instrumental polarization value at the Cassegrain focus as compared to the prime focus. In case of primary mirror the light is incident symmetrically over the entire mirror surface for any field angle. However, for secondary mirror the rays are incident asymmetrically and this asymmetry increases with the field angle. This should cause higher values of instrumental polarization for the Cassegrain focus. In our calculations we have considered only those rays which are reflected from the periphery of the primary mirror. However, in actual case we should consider reflections from the entire surface of the primary mirror (down upto the Cassegrain hole) and then integrate the Stokes parameter values. The inner region of the mirror will exhibit a lower instrumental polarization value as the beam becomes slower there. Therefore the instrumental polarization values that we have calculated can be considered only as the upper limit of such effects.

It has been already discussed in Sect. 1 (also Sen & Tandon 1994), that a typical present day polarimeter can measure linear polarization with an accuracy $\delta p \sim 0.01\%$ (barring the case of very bright objects like moon, planets etc.). Under such a condition one may question our attempts to quote up to six places after the decimal, the percent polarization values. This we have done to understand the nature of field angle dependence of such polarization. Higher accuracy in polarization measurements, helps one to understand the ongoing astrophysical processes in a better way. For example, way back in 1974, Clarke & Mclean (1974) had discussed about the polarization occurring within the stellar line profile and recommended the need for measuring polarization with a detectability $p \sim 0.001$. One also requires high accuracy on the estimated p values in order to study the nature of its wavelength dependence, where the polarization is caused by synchrotron emission or dust scattering.

During imaging polarimetry, while we try to improve polarimetric accuracies with better telescope aperture and instrument, we should also keep in mind the limitation put by the telescope optics itself.

Acknowledgements. We are thankful to Prof. S. Bujarbarua, Director, Centre of Plasma Physics, for his encouragement to this work and Prof. S.N. Tandon, Inter University Centre of Astronomy and Astrophysics, for useful discussions on the problem.

References

Born M., Wolf E., 1959, Chapter XIII, Principles of Optics, Sixth Edition. Pergamon Press, Oxford

Chandrasekhar S., 1960, Chapter 1, Radiative Transfer. Dover Publications, Inc., New York

Clarke D., McLean I.S., 1974, “Polarization measurements within stellar line profiles”, Planets, stars and Nebulae studied with photopolarimetry, Gehrels T. (eds.). The University of Arizona Press, Tucson, Arizona, pp. 752-761

Renard J.B., Levasseur-Regourd A.C., Dollfus A., 1992, Ann. Geophys. 10, 288

Röser H.-J., 1981, A&A 103, p. 374

Röser H.-J., Meisenheimer K., 1986, A&A 154, 15

Ramapraksh A.N., Gupta R., Sen A.K., Tandon S.N., 1996, A&A (in press)

Scarrott S.M., Warren-Smith R.F., Pallister W.S., Axon D.J., Bingham R.G., 1983, MNRAS 204, 1163

Scarrott S.M., Eaton N., Axon D.J., 1991, MNRAS 252, 12

Schlötelberg M., Meisenheimer K., Röser H.-J., 1988, A&A 202, L23

Sen A.K., Joshi U.C., Deshpande M.R., Debi Prasad C., 1990, Icarus 86, 248

Sen A.K., Tandon S.N., 1994, Proceedings SPIE- The International Society of Optical Engineering, Instrumentation in Astronomy VIII, 2198, p. 264

Serkowski K., “Polarimeters for optical astronomy” Planets, stars and Nebulae studied with photopolarimetry, Gehrels T. (eds.). The University of Arizona Press, Tucson, Arizona, 1974, pp. 135-174

Shurcliff W.A., 1962, Polarized light-production and use Harvard University Press, Cambridge