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Discussion of the results from solar astrolabes

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Abstract. In this work we investigate on the stability of the results from solar observations obtained with the modified astrolabes of the Observatoire de la Côte d'Azur/France, of the Observatório Abrahão de Moraes/IAG/USP-Brazil and of the Observatório Nacional/Brazil. The mass of data is comprised of 6273 observations of both solar limbs, from 1974 to 1992, and is analyzed to determine the origin of the fundamental reference system and the Earth's orbital parameters. The solution stability was verified by checking against any large variation upon the individual weights, as well as for different sub samples of the whole set of observations. The results were also shown invariant to changes in the adopted model, such as adding unknowns to the refraction algorithm, to the time variation of the obliquity (including the nutation constant) and to the azimuthal anomalies of the refraction. In our analysis we obtain a standard deviation of 0.″6 for the combination of the observations with no bias on its distribution. It is also indicated that the Sun observations at astrolabes do not require any further atmosphere modelling complexity.

Key words: astrometry — reference system — Sun: general — methods: data analysis

1. Introduction

The Sun observation by astrolabe has, nowadays, two distinct goals: solar positional astronomy and heliography. In what follows we are concerned only with the determination of corrections to the placement of the reference system (usually the one in which the astrolabe star observations are done) and corrections to the Earth's orbital parameters. All the observations discussed here are placed on the FK5/J2000 reference system.

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In a previous paper (Penna et al. 1996 - hereafter Paper I) we presented the results from the analyses of the solar observations with astrolabes, which were obtained at Observatoire de La Côte D'Azur, Observatório Abrahão de Moraes (OAM) and Observatório Nacional (ON). The data set contains 6273 individual observations of transits of both the upper and lower solar limbs, spanning 18 orbital revolutions acquired with nearly uniform distribution, due to the geographical location of the observing sites.

The observed zenith distances range from 30° to 70°. The data were divided in 20 subsets, according to the observatory site and observing zenith distance. Also substitutions of the employed prism or filter plate, at a given zenith distance and observatory, demanded the establishment of separate subsets. This last subdivision includes 2 subsets at 30° and 45° each at OCA, 3 subsets at 30° at OAM, and 3 subsets at 30° at ON (Laclare et al. 1981, 1982, 1983; Leister 1989; Penna 1982).

The investigation of the individual sub-campaigns at each zenith distance and at each site has led to devise a scale of weights depending uniquely on the observational zenith distance. To the subsets that include the smaller zenith distances, $z < 45°$, unit weight were assigned. This scheme holds even for the Southern hemisphere results, where the use of transmission prisms in the earlier observations is compensated for by their complementary role in the solution. An intermediate group, with z ranging from 45° to 52°, was given half weight and the 56° to 70° subsets received one fourth weight.

The results of the Standard Weighted Global Solution presented a 0.″63 standard deviation for an observation of unit weight. There were 26 unknowns to be solved for. The typical formal error for them is 0.″05 and the pairwise correlations among them are inferior to 0.65 in absolute value, except for the pair mean longitude and equinox corrections, where it attains 0.96. These enabled us to assert the significance of the solution. However, the large corrections found to the obliquity of the ecliptic (0.″49 ± 0.″05)

Table 1. Standard weighted global solution

ΔA	$-0''.01 \pm 0''.04$	$\longrightarrow$	correction to the equator or declination origin
ΔE	$0''.09 \pm 0''.06$	$\longrightarrow$	correction to the equinox or right ascencion origin
$\Delta\varepsilon$	$0''.49 \pm 0''.05$	$\longrightarrow$	correction to the obliquity of the ecliptic
ΔL	$-0''.03 \pm 0''.05$	$\longrightarrow$	correcction to the mean longitude of the Sun
Δe	$0''.10 \pm 0''.01$	$\longrightarrow$	correction to the eccentricity
$e\Delta\varpi$	$0''.01 \pm 0''.01$	$\longrightarrow$	correction to the longitude of the perihelion
$\sigma_{\rm G}$	$0''.63$	$\longrightarrow$	general standard deviation for unit weight
N	6028	$\longrightarrow$	number of transits of both solar limbs.

and to the eccentricity ($0''.10 \pm 0''.01$), in comparison to their formal errors, deserve a more detailed discussion.

In Paper I a global analysis of the observations was presented. Because of the observational conditions, however, the distribution of the observations is not regular, and, moreover, the atmospheric conditions span a large range. In this paper, thus, we discuss the effects of such irregularities. Additional nutation terms are also investigated in order to better analyze the obliquity of ecliptic results.

To further probe on the stability of the results, we analyzed the distribution of the observations and the completeness of the set of unknowns relevant to the problem. Therefore, initially a number of strongly biased sets of data were tried, in order to directly verify the degree of dependence among the results. Then we added up to the complexity of the problem description, incorporating, by steps, terms to account for corrections to 1) the refraction constant; 2) the time variation of the obliquity; 3) azimuthal anomalies of the refraction; 4) a correction of the nutation constant.

This paper brings the results of such investigations, that can be extended and contribute to the effort of combining astrolabe observations in general.

2. Data sampling

The solar observations with astrolabes are per force a routine procedure. Eventhough each observation is carefully performed to avoid introducing systematic errors, and the relevant effects are thoroughly modelled, as discussed in item (4.), the seasonal characteristics are bound to bias the distribution of observations at a certain zenith distance on a certain site. We took a direct approach to the problem, by producing exageratedly biased subsets of the actual data and investigating the outcome from these hypothetical distributions.

Table 1 presents the results obtained in Paper I for the Standard Weighted Global Solution. There, they are compared with several independent investigations and found in overal agreement. The value for the correction to the obliquity of the ecliptic is however outstanding and shall be focused upon in the analysis that follows.

Firstly the observations, inside each original subset, were divided in two groups. Queueing up the observations by order, they were sorted out into the “even group” or into the “odd group”.

The main results are shown in Table 2. They show complete agreement with the Standard Weighted Global Solution, after allowance for the halved sample size, both in regard with the derived corrections and standard deviations.

Table 2. Solution for the “even” and “odd” subsets

	EVEN	ODD
ΔA	$-0''.10 \pm 0''.06$	$0''.08 \pm 0''.06$
ΔE	$0''.11 \pm 0''.08$	$0''.12 \pm 0''.08$
$\Delta\varepsilon$	$0''.57 \pm 0''.07$	$0''.45 \pm 0''.07$
ΔL	$-0''.03 \pm 0''.08$	$-0''.02 \pm 0''.08$
Δe	$0''.11 \pm 0''.02$	$0''.11 \pm 0''.02$
$e\Delta\varpi$	$0''.00 \pm 0''.01$	$0''.02 \pm 0''.02$
$\sigma_{\rm G}$	$0''.62$	$0''.64$
N	3002	3026

As counterproof for the biasing introduced by the previous arrangement of the data, we next produced five trials where 50% of the data were picked up, regardless of the original subsets distribution. The results so obtained are presented in Table 3. The outcome reinforces the previously obtained inference, namely that the results are essentially independent from any “fair” data sampling. ΔA, ΔE, ΔL and $\Delta\varpi$ vary within reasonable bounds with respect to the formal standard desviation. $\Delta\varepsilon$ and Δe keep the tendency shown in the Standard Weighted Global Solution.

The above conclusion does not hold in the case of intentionally biased data sampling. Such would be the case of selecting only northern or southern hemisphere data points. The advantages gained by grouping complementary data sites is well discussed elsewhere (Paper I; Leister 1979, 1989; Chollet 1981; Penna 1982; Bougeard et al.

Table 3. Solutions for the subsets including 50% of the observations, chosen at random

	solution 1	solution 2	solution 3	solution 4	solution 5
ΔA	$-0.''01 \pm 0.''06$	$0.''15 \pm 0.''06$	$0.''10 \pm 0.''06$	$-0.''01 \pm 0.''06$	$0.''15 \pm 0.''06$
ΔE	$0.''06 \pm 0.''08$	$-0.''03 \pm 0.''08$	$0.''04 \pm 0.''08$	$0.''17 \pm 0.''08$	$0.''16 \pm 0.''08$
$\Delta \varepsilon$	$0.''47 \pm 0.''07$	$0.''36 \pm 0.''07$	$0.''42 \pm 0.''07$	$0.''46 \pm 0.''07$	$0.''51 \pm 0.''07$
ΔL	$-0.''06 \pm 0.''08$	$-0.''11 \pm 0.''08$	$-0.''09 \pm 0.''08$	$0.''03 \pm 0.''08$	$0.''05 \pm 0.''08$
Δe	$0.''11 \pm 0.''02$	$0.''10 \pm 0.''02$	$0.''13 \pm 0.''02$	$0.''10 \pm 0.''02$	$0.''11 \pm 0.''02$
$e\Delta\varpi$	$0.''01 \pm 0.''02$	$-0.''01 \pm 0.''01$	$0.''02 \pm 0.''02$	$0.''02 \pm 0.''02$	$0.''00 \pm 0.''01$
$\sigma_{\rm G}$	$0.''63$	$0.''62$	$0.''63$	$0.''62$	$0.''62$
N	3104	3044	3030	2942	3019

1983; Journet 1986 and Poppe 1994) and do not need further consideration.

We proceed now to assert whether the weighting system was determinant for the solution of the unknowns. The applied test consists on varying the weight of each subset by ± 0.1. In this way the variation ranges from 10%, for the best quality, less numerically dense, small zenith distance subsets, to 40%, for the large zenith distance, numerically denser, worst observational quality subsets. The test is applied upon each subset in turn.

It is then possible to obtain numerically the partial derivative of each unknown with respect to the weight of each subset. The results are presented in Table 4. The only important values are the ones relative to the unknowns ΔE and ΔL, for the subsets corresponding to large zenith distances ($z > 49°$). The response of these two unknowns is, non-surprisingly, quite similar. Notwithstanding their difference is well determined and independent from the weighting scheme. The corrections to the eccentricity and to the longitude of the perihelion do not vary, their derivative being practically null. Nearly the same outcome happens in relation to the equator correction, which exhibits a very small variation. As for the correction to the obliquity of the ecliptic, it presents a significant partial derivative only relatively to the 70° zenith distance subset, that is numerically sparse and has the lowest weight in the Standard Weighted Global Solution.

The test allows us to conclude that the values obtained for the unknowns show very small dependence on the weighting system. For all subsets corresponding to zenith distances smaller than 45° the variation as function of weight is truly negligible.

3. Deep probing the residuals

As seen in the previous item, the quantity of data is overabundant, therefore it is possible to examine in detail the standing of the hypotheses implicitly adopted in building the Standard Weighted Global Solution. That is, the residuals from the solution behave as white noise, of null average and uniform variance.

Paper I brings the histograms of the residuals inside each subset. No important anomaly (skewness or curtosis) is apparent on the plots. In general, the central portion of the histograms depicts a normal distribution of zero average and uniform standard deviation.

There is, however, in the most critical cases - OCA at 65° and 70° of zenith distance - a significant tail of negative residuals. The fact corroborates the need to assign smaller weights to these subsets.

A possible time evolution of the residuals was also studied. Figures 1 to 14 show the plotting of the residuals against the Julian date of observation, for each subset. Altough some trends might be suggested, they do not repeat for neighbouring zenith distances nor for similar epochs. Their amplitude is allways smaller than the scattering inside a campaign (about 1.5 arcsec), thus, even if they were real their cause could hardly be disclosed and their effect is too small to trouble.

One of the most conspicuous of these trends is shown in Fig. 1. The downstepping of the average from 1983 onwards is probably associated with the tightening of the campaigns' scatter. The feature is believed to be due to an improvement of the observational routine and apparatus, leading to higher quality of the results. Figures 3 and 8 show a smooth downwards trend. Altough barely significant, vis-a-vis each campaign scatter, as discussed above, they might indicate a slow deformation of the corresponding prisms (37° and 56°). A longer time interval would be required to confirm the feature.

In some of the plots, for $z \geq 41°$, specially in Figs. 5 and 7, it could be distinguished an upside loop for the residuals spanning the years 1985 to 1987 at the OCA. The changing in the method employed to measure the atmospheric parameters used for calculate the refraction could be a cause of such effect.

To check upon this possibility, we prepared a run of the Standard Weighted Global Solution, including as additional unknown an additive constant to the zenith distance. This new unknown applies for all the OCA subsets, between 1985 and 1987, for which $z \geq 41°$. The results are presented in Table 5. There are no important

Table 4. Partial derivative of the unknowns relatively to the weight, in arcseconds

station/z	central/weight	$\frac{\Delta A}{\Delta p}$	$\frac{\Delta E}{\Delta p}$	$\frac{\Delta \varepsilon}{\Delta p}$	$\frac{\Delta L}{\Delta p}$	$\frac{\Delta e}{\Delta p}$	$\frac{e\Delta \varpi}{\Delta p}$
OCA30°	0.9	0.05	0.00	0.00	0.00	0.00	0.00
OCA34°	0.9	0.05	0.00	0.00	0.00	0.00	0.00
OCA37°	0.9	0.00	−0.10	0.00	−0.05	0.00	0.05
OCA41°	0.9	−0.05	0.10	0.00	0.10	0.00	0.05
OCA45°	0.7	0.05	−0.05	0.00	0.00	0.00	0.05
OCA49°	0.7	0.10	0.35	0.00	0.35	0.00	0.00
OCA52°	0.7	−0.05	−0.20	−0.05	−0.15	0.00	0.00
OCA56°	0.5	0.00	−0.10	0.00	−0.10	0.00	−0.05
OCA60°	0.5	0.05	−0.45	−0.05	−0.50	0.00	−0.05
OCA65°	0.5	0.10	−0.35	0.05	−0.40	0.00	−0.05
OCA70°	0.5	0.00	−0.50	0.20	−0.50	0.00	−0.05
OAM30°	0.9	−0.10	0.10	0.05	0.25	−0.05	−0.05
OAM45°	0.7	0.10	0.25	−0.05	0.15	0.00	0.05
ON30°	0.9	0.05	0.10	−0.10	0.05	0.00	0.05

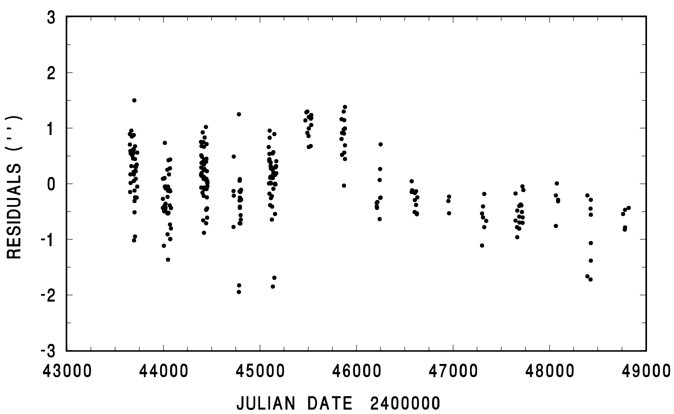

Fig. 1. Residuals Distribution $z = 30°$ / OCA

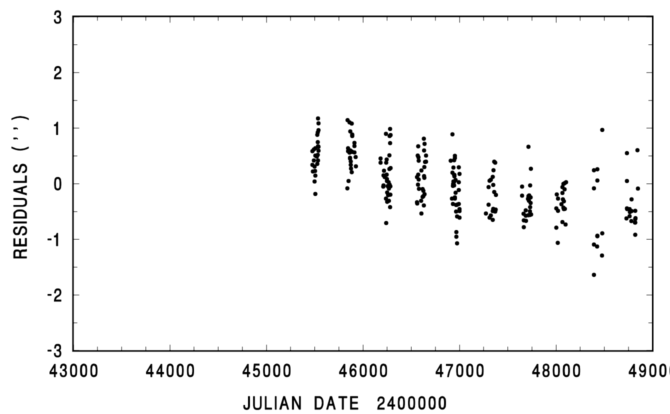

Fig. 3. Residuals Distribution $z = 37°$ / OCA

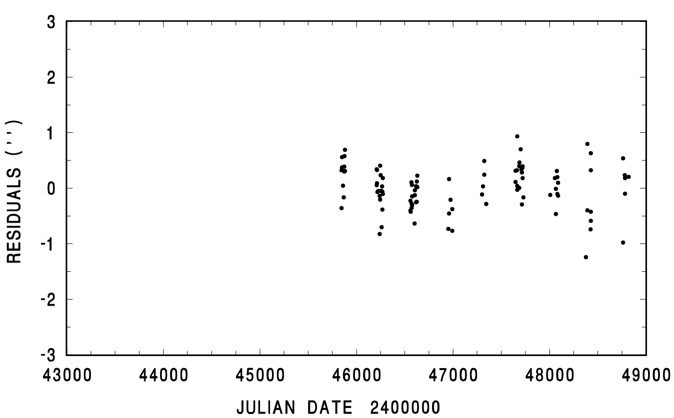

Fig. 2. Residuals Distribution $z = 34°$ / OCA

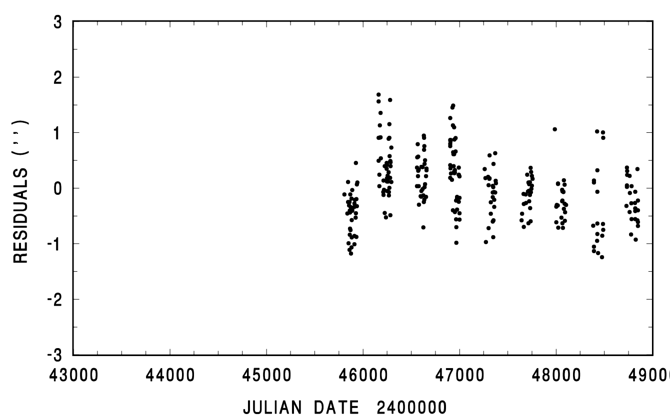

Fig. 4. Residuals Distribution $z = 41°$ / OCA

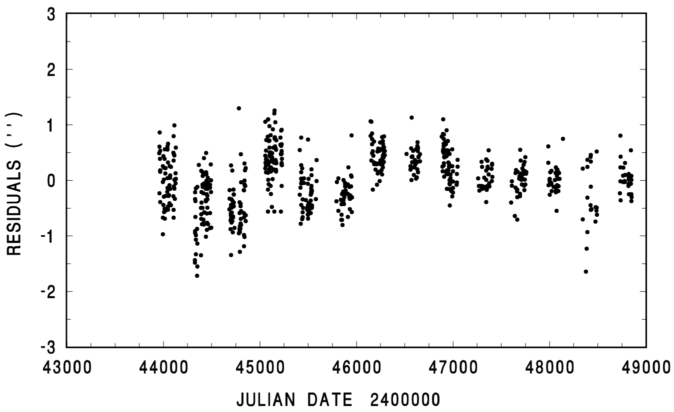

Fig. 5. Residuals Distribution $z = 45°$ / OCA

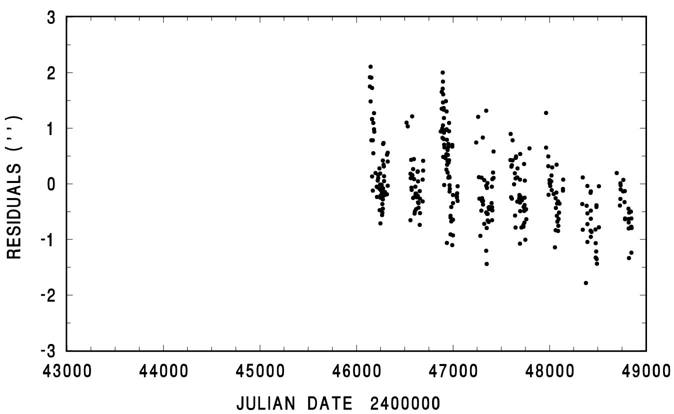

Fig. 6. Residuals Distribution $z = 49°$ / OCA

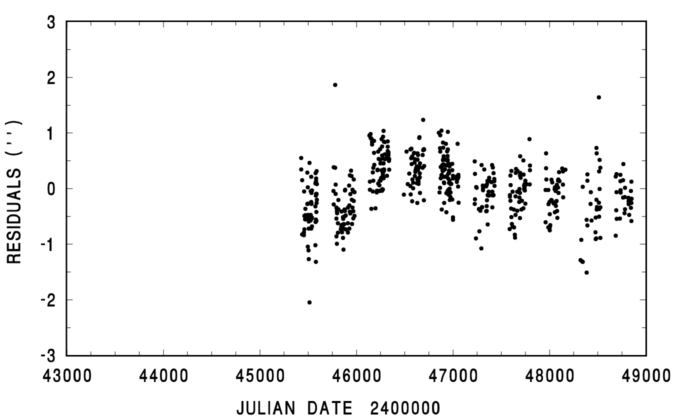

Fig. 7. Residuals Distribution $z = 52°$ / OCA

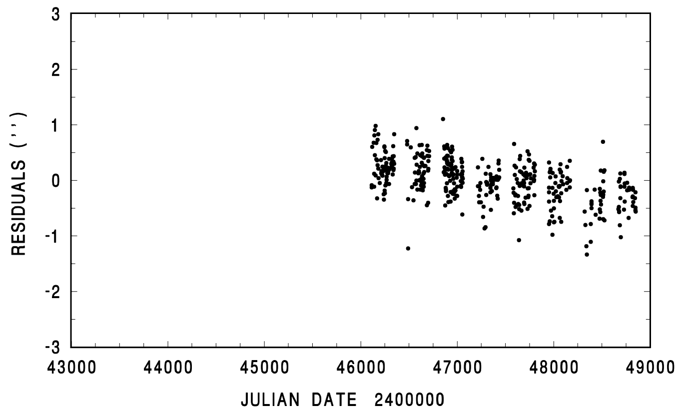

Fig. 8. Residuals Distribution $z = 56°$ / OCA

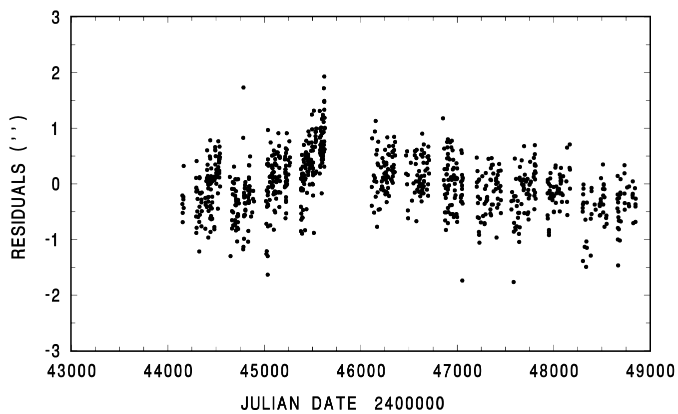

Fig. 9. Residuals Distribution $z = 60°$ / OCA

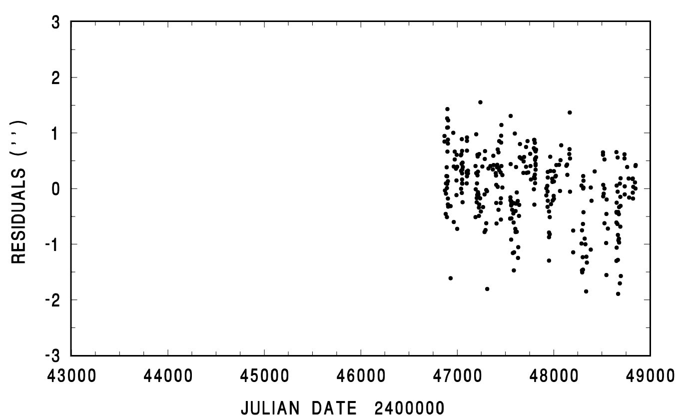

Fig. 10. Residuals Distribution $z = 65°$ / OCA

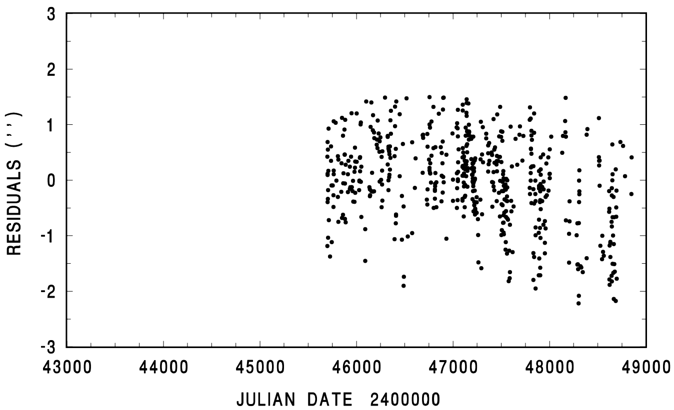

Fig. 11. Residuals Distribution $z = 70°$ / OCA

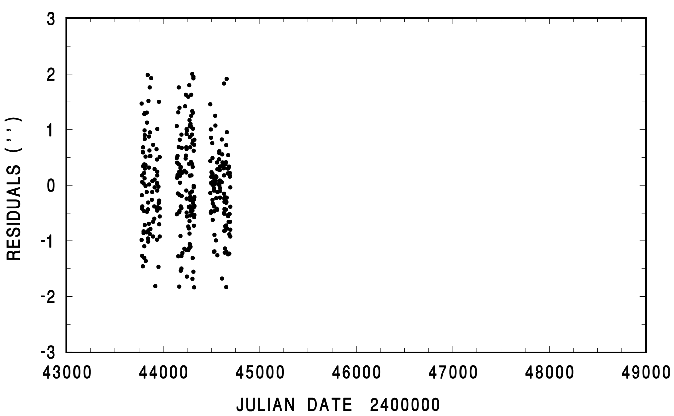

Fig. 12. Residuals Distribution $z = 30°$ / ON

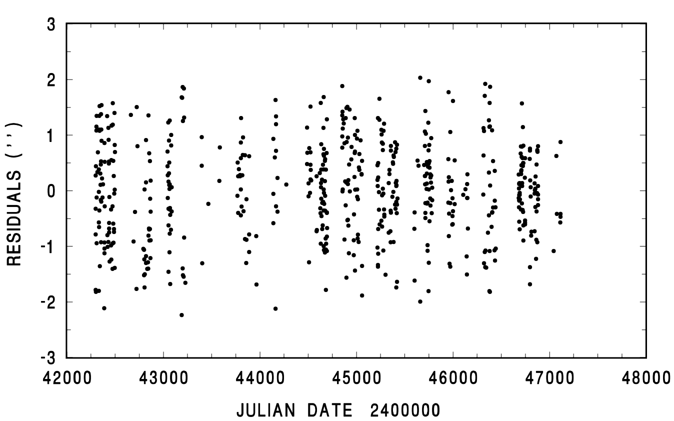

Fig. 13. Residuals Distribution $z = 30°$ / OAM

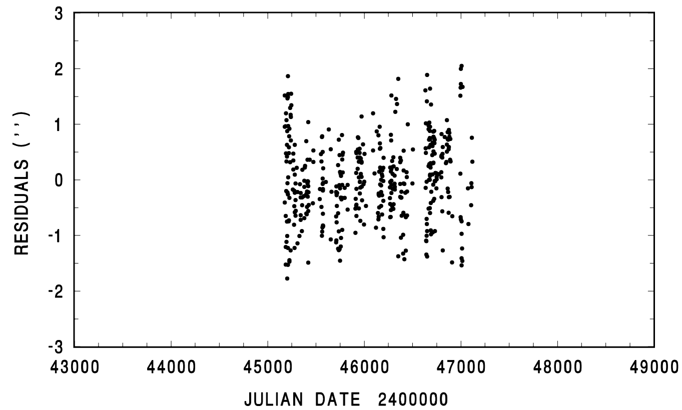

Fig. 14. Residuals Distribution $z = 45°$ / OAM

differences on the results compared to the Standard Solution displayed in Table 1. The general standard deviation, however, drops from 0.″63 to 0.″61, which is marginally significant and seems to confirm the effect. The additional unknown (ΔR) takes the value of 0.″70 with standard deviation 0.″03, i.e. it is hightly significant. Even so, as the reference system and Earth's orbit orientation parameters are not changed by the inclusion of the additional unknown and as its definition is empirical, we choose to rather not include it in the Standard Weighted Global Solution.

Table 5. Global Solution $+R(z > 41°)$

ΔA	$-0.''05 \pm 0.''04$
ΔE	$0.''11 \pm 0.''06$
$\Delta\varepsilon$	$0.''44 \pm 0.''05$
ΔL	$-0.''02 \pm 0.''05$
Δe	$0.''10 \pm 0.''01$
$e\Delta\varpi$	$0.''00 \pm 0.''01$
ΔR	$0.''70 \pm 0.''03$
σ_G	$0.''61$
N	6032

4. Model refinement

As discussed in the previous item, the model for the Standard Weighted Global Solution might favourably admit additional unknowns. Here, thus, we consider additional unknowns, to better account for the treatment of the refraction and to investigate the correction to the obliquity of the ecliptic. These are:

(i) - A correction to the constant of refraction.

(ii) - A time variation of the correction to the obliquity of the ecliptic.

Table 6. The output from the modified models of the Standard Weighted Global Solution. The additional unknowns (U1, U2, U3, U4, see text) enter in each solution as follows: 1 - None, standard model. 2 - Unknown (i), constant of refraction for the three sites. 3 - Unknown (i), only for OCA. 4 - Unknown (ii) 5 - Unknown (iii), harmonic of azimuthal frequency $2Z$. 6 - Unknown (iii), as solution 5, but only for OCA and z $\geq 60°$. 7 - Unknown (iii), harmonic of azimuthal frequency $3Z$. 8 - Unknown (iii), harmonic of azimuthal frequencies $2Z$ and $3Z$. 9 - Unknown (iii), as solution 8, but only for OCA and $z \geq 60°$. 10 - Unknown (iv)

SOL./ σ_G	ΔA	ΔE	$\Delta\varepsilon$	ΔL	Δe	$e\Delta\varpi$	U1	U2	U3	U4
1	−0.″01	0.″09	0.″49	−0.″03	0.″10	0.″01				
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01				
2	−0.″02	0.″05	0.″49	−0.″07	0.″10	0.″01	−0.″41			
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″02			
3	−0.″01	0.″08	0.″48	−0.″04	0.″10	0.″01	−0.″45			
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″02			
4	0.″00	0.″09	0.″37	−0.″04	0.″10	0.″01	−0.″03			
0.″62	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″01			
5	−0.″09	0.″11	0.″57	−0.″04	0.″09	−0.″04	0.″08	0.″16		
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″04	±0.″02		
6	−0.″09	0.″12	0.″34	−0.″02	0.″10	0.″02	−0.″69	0.″33		
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″14	±0.″09		
7	0.″18	0.″02	0.″34	−0.″05	0.″10	0.″01	−0.″17	−0.″10		
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″03	±0.″02		
8	−0.″01	0.″09	0.″49	−0.″03	0.″10	0.″01	0.″26	0.″29	−0.″11	−0.″10
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″02	±0.″03	±0.″01	±0.″03
9	−0.″10	0.″04	0.″30	−0.″06	0.″11	0.″04	0.″39	0.″76	−0.″23	−0.″55
0.″62	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″03	±0.″10	±0.″05	±0.″05
10	0.″02	0.″09	0.″50	−0.″03	0.″10	0.″01	0.″03			
0.″63	±0.″04	±0.″06	±0.″05	±0.″05	±0.″01	±0.″01	±0.″03			

(iii) - Corrections to azimuthal anomalies of the refraction.

(iv) - A correction to the constant of nutation.

With the corrections defined as above a number of trials of the Standard Weighted Global Solution were run, admitting some or all of the additional unknowns. The results are expressed in Table 6.

Solutions 2 and 3 include a correction to the constant of refraction. The first one refers to the bulk of the observations while the second one corrects only the OCA data. In both cases small corrections, at the level of −0.7% upon the constant of refraction are obtained, without any major modification upon the other unknowns.

Solution 4 includes the coefficient for the time derivative of the obliquity of the ecliptic. The value of the correction to the obliquity of the ecliptic turned out to be 0.″37, for the epoch 1990.0 (the approximate average epoch of the observations), with an additional yearly rate of −0.″03. This result must be taken cautiosly, since the time interval is small and, to this purpose, the Northern and Southern data may lack in sincronicity. Taken at its face value, the negative sign of the time derivative would weaken the hypothesis that the high value found for the obliquity of the ecliptic itself might be due to an increasing trend not yet recognized.

Solutions 5 to 9 include unknowns of the type $\cos(PZ)$ and $\sin(PZ)$, for P egual to 2 and 3, where Z is the azimuth. These terms represent azimuthal anomalies of refractive origin, that may arise from the characteristics of each site. Solutions 6 and 9 refer only to the high zenith distance subsets ($z \geq 60°$, hence for the OCA only), that are more prone to present such refraction anomalies. In the other solutions of this kind (that is, 5, 7 and 9) the same azimuth dependent unknowns are applied to all the observations. They are probably less realistic, since they do not account for the local characteristics. Only the correction to the equator suffers changes, eventhough at a level smaller than 2σ.

Solution 10 includes a correction to the constant of nutation in obliquity. It takes advantage of the fact that the observational period (about 19 years) is comensurable to the period of retrogradation of the nodes of the lunar orbit. The correction obtainded is negligible and thus do not apport any change on the value of the correction to the obliquity of the ecliptic.

In general, it is seen that the modification introduced in the model did not affect but marginally the results obtained in the Standard Weighted Global Solution. The unknowns introduced are usually significant, thus well modelled, without casting new light upon the parameters originally researched.

5. Conclusions

The corrections to the origin of the fundamental reference system and the Earth's orbital parameters (Paper I) display a puzzling value for the correction to the obliquity of the ecliptic. The large and good quality set of solar transits, observed visually with modified Danjon astrolabes, that had been used to derive the results, was submitted to a series of tests to assert the homogeneity of the data and its dependence on the adopted modelling. The results displayed a remarkable resistance to significant changes with respect to the attempted tests. However, this does not rule out the possibility of the presence of unsuspected systematic errors of quite special nature. As a conclusion we may say that while the results of Paper I are the best that can be afforded by the available data, the effort to observe the Sun's transits need to be pursued and would gain enourmously by the adherance of other observing groups around the world.

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