

Orthogonal operator calculation of E1, M1 and E2 transition probabilities in Ti III and V IV

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Abstract. Transition probabilities in the iron group spectra Ti III and V IV are obtained from an accurate description of the 8 lowest even and 4 lowest odd configurations of the atomic system. Results are compared with experiment and current databases.

Key words: atomic data — Sun: photosphere — stars: abundances

1. Introduction

Fortunately, there is a continuing increase and improvement of astrophysical observational possibilities, both by satellite and by ground based observatories. Technically advanced telescopes are used to search for faint objects and to detect the emitted light with highly dispersive equipment such as the Hubble Space Telescope (HST) and the International Ultraviolet Explorer (IUE). The vacuum ultra-violet region will be observed by FUSE-LYMAN at high resolution down to the Lyman continuum at 912 Å.

At the same time, this progress underlines the need for more and more accurate atomic and molecular transition probabilities. Most stellar spectra are affected with broad features or blends. Therefore, to investigate stellar objects in a state of the art way, transition probabilities (either from theoretical calculations or from laboratory observations) should necessarily be accurate to about 10% or better.

Because of their nuclear stability, iron group elements (especially the even- Z ones without nuclear spin) are frequently encountered in stellar spectra. The lower (I and II) ionization stages can be observed in the photosphere, while higher ionization is seen in the emission lines of the chromosphere with its steep rise of ionic temperature; A-type stars constitute important examples of this.

To start our calculations of transition probabilities, we choose Titanium and Vanadium which are cases of intermediate complexity and allow for comparison with other

work; for the near future, the study of iron and nickel which are of even greater astrophysical importance, is planned.

Ti and V are observed in a variety of stellar objects. In the sun, Ti figures in third place in terms of numbers of lines (Jaschek & Jaschek 1995). Hundreds of Ti I and Ti II lines are observed in the solar photospheric spectrum (Moore et al. 1966; Biémont 1976). Ti II appears in mid-B-type stars, slowly increases towards F-type, and persists up to type M. Ti has been found overabundant (with respect to Fe) in extreme halo dwarfs (Magain 1989). Due to the lack of accurate transition probabilities of higher ionization, one was forced here to use the weaker Ti I lines to determine the abundance. Ti II is used to determine the radial velocity of A-type shell stars (Levato et al. 1995). Ti III is less frequently observed but has been seen in Ap stars (Bidelman 1966) as well as by the IUE in β Orionis. (Rogerson & Upson 1982).

Similarly, V I and V II show several hundreds of lines in the solar photospheric spectrum. Knowledge of accurate transition probabilities is therefore vital for the determination of the solar abundance of V and refining the chemical composition of the sun (Whaling et al. 1985). V II has been seen in late B-type stars and is also observed in A stars (Jaschek & Jaschek 1995). In non-normal stars, V shows a behaviour similar to that of Fe and consequently is considered as “typical metal”. V is slightly overabundant with respect to iron in globular cluster stars (Wheeler et al. 1989). V IV is seen in the balloon UV spectrum of the sun (Samain 1995).

Although the orthogonal operator method has been used for more than a decade now to describe energy levels of iron group elements, the present work is the first to calculate transition probabilities in this framework. It has been shown (Raassen & Uylings 1996) to give reliable results for complex atoms, i.e. atoms with $Z > 20$ and more than one electron outside closed shells. What are the advantages and disadvantages of the new orthogonal operator method? In Sect. 2, a short introduction of the method itself in relation to other current methods of producing gf - or A -values is given; similarities as well as

differences are discussed. In Sect. 3, details of the actual calculation as well as the numerical results are given.

2. Orthogonal operators and other methods to calculate complex spectra

2.1. Some current methods to calculate transition probabilities

Reliable absolute intensities can be obtained in quite a number of ways, but today's successful methods are almost exclusively based on intensive computing, often containing massive Configuration Interaction (CI) calculations. Pure ab initio calculations, like Multi-Configuration Hartree-Fock (MCHF, Froese Fischer 1978) and the analogous relativistic Multi-Configuration Dirac-Fock (MCDF, Parpia et al. 1996; Desclaux 1975) are most suited to produce a few, but accurate results. MCHF is able to include a large amount of electrostatic correlation and is therefore, like the Configuration Interaction Version 3 (CIV 3) code (Hibbert 1975), successful in the field of "lighter", i.e. $Z < 20$, atoms. MCDF, on the other hand, is obviously better equipped for the heavier atoms. However, it can at the moment only handle a limited amount of electrostatic correlation, which means that fine structure is generally better described than absolute energies.

For the production of large quantities of reliable data, Cowan's RCN/RCG suite of programs (Cowan 1981) has gained widespread acceptance. Here, fitting to experimental energy levels with Slater-Condon parameters is vital to raise the accuracy, especially in complex energy structures with several d- or f-electrons outside closed shells where there are many levels of the same J -value in a relatively small energy interval.

The orthogonal operator method can be seen as a continuation of the Slater-Condon theory of atomic spectra. In the Slater-Condon theory, the Coulomb interaction between the electrons is described by means of the Slater F- and G-integrals, while the magnetic interaction due to the motion of the nucleus in the rest frame of an electron is translated into the well known spin-orbit interaction. These interactions, however, are not orthogonal which implies that adding another interaction to the system would influence the values of all previous interaction constants.

2.2. Orthogonal operators

Roughly a decade ago (Judd et al. 1982; Hansen et al. 1988a), this system was orthogonalized. As the parameters in the new operator set are as independent as possible, the fits are highly stabilized which offers the possibility to include new operators that account for small interactions like higher order or pure relativistic effects. As a result, the mean error σ of the fit is reduced by an order of magnitude in a physically significant way.

Although the method is semi-empirical in character, ab initio calculations do constitute an important part of

the procedure, especially to describe relativistic, e.g. fine structure effects. More specifically in connection with this work, two-particle (like dd, dp or ds) magnetic effects are relevant. Due to the small number of experimental levels in the d^2 and dp configuration it is not possible to fit these parameters. Therefore they were fixed at their MCDF ab initio calculated values. In spite of the sometimes large number of parameters, the number of parameters actually *varied* is in many cases about the same as in the traditional Slater-Condon method. Even without varying its additional parameters, the orthogonal operator method still provides an improved description of complex spectra (Raassen & Uylings 1996).

With respect to the classical Slater-Condon type calculations of transition probabilities, the differences are twofold: First, as the level positions and thereby the spacing between them are described in more detail, one may suppose that the corresponding eigenvectors are more accurate. These eigenvectors are used to transform the transition probabilities in pure SL-coupling to the real intermediate coupling scheme. Second, the transition integrals calculated by means of MCDF are corrected for the effect of core-polarization. We find that this correction, which is commonly a reduction of the transition integral, improves the agreement with absolute transition probabilities from experiment.

Of course there are difficulties as well, otherwise it would be hard to explain that not everyone involved in this field is using orthogonal operators. Compared to Cowan's programs, our procedure is less automatic. Parameter values, especially if they are meant to be fixed, have to be obtained either from extrapolation (which requires some experience with the region under study) or from ab initio calculation. Programs to translate Slater F- and G-integrals to their orthogonal counterparts are available. We adapted the GRASP2 package (Parpia et al. 1996) to calculate two-electron magnetic effects; a request to obtain this program should be directed to Froese Fischer and the present authors. However, effective (i.e. higher order) electrostatic parameters may pose a problem if they can not be fitted directly, in which case one should apply extrapolation or second order perturbation theory (Uylings et al. 1993). Yet, we hope that this situation is temporary. Overview articles containing numerous parameter values for d^N (Hansen et al. 1988b), $d^{N-1}s$ (van het Hof et al. 1991) and $d^{N-1}p$ (Uylings & Raassen 1996) have been published and can be used to find starting values for the parameters. Everybody interested in working with orthogonal operators is invited to contact the authors or to visit our Internet address <ftp://nucleus.phys.uva.nl> in the directory `pub/orth`.

3. Transition probabilities

3.1. The need

With efficient filters for cascade effects, one can accurately determine individual transition probabilities from lifetimes derived in observing the fluorescence following laser excitation. In iron group or other complex spectra with their hundreds or thousands of lines, this method is not practicable and the astrophysicist has to take recourse to Internet databases containing bulk data. However, many values available so far and collected e.g. in NIST compilations (Martin et al. 1988) and databases from NIST, Kurucz and Kelly, are accurate to about 50%. Naturally, the data in these databases are not independent. Sometimes the NIST compilation contains experimental data but in many cases semi-empirical calculations are employed. It is hard to give error bars on this type of calculation, and if they are there at all, they are hard to substantiate.

In view of the above, it is necessary that more data become available to enable the experimentalist to decide on the real accuracy of the methods.

3.2. The present case

Our study of transition probabilities in the iron sequence using orthogonal operators is started with Ti III and V IV. These ions were chosen because the ground configuration contains two d-electrons, which means that they are the first ions with two particle (dd and dp) magnetic effects. In the III- and IV-spectra the configurations are relatively well separated in comparison with the I- and II-spectra in which the energies of the 3d- and 4s-electron are competitive. On the other hand the odd system is not yet perturbed by the open 3p-shell, giving rise to a $3p^5 3d^3$ configuration. This effect occurs in the VI-spectra and is probably already influencing the V-spectra of the iron group elements.

To calculate the $(3d^2 + 3d4s) \rightarrow 3d4p$ transitions properly, it was necessary to take into account the interaction with neighbouring configurations. For that reason the even system was built from $(3d^2 + 3d4s + 4s^2 + 3d5s + 3d6s + 3d4d + 3d5d + 4p^2)$ and the odd system from $(3d4p + 4s4p + 3d5p + 3d4f)$. Interactions with other (far-lying) configurations are taken into account by means of so-called effective operators. These sets of configurations are almost the same as the ones used by Wyart (1975). However he used the conventional method and his investigation was limited to energy values without the calculation of transition probabilities.

3.3. Method

After completing the fitting procedure, eigenvalues and eigenvectors are obtained from the diagonalization of the Hamiltonian. The eigenvectors yield the composition of

Table 1. Values for the electric dipole transition integrals in Ti III calculated by means of MCDF including core polarization

	3d4p	4s4p	3d5p	3d4f
3d ²	.999	-	.231	.617
3d4s	-2.880	.788	.150	-
4s ²	-	-2.700	-	-
3d5s	1.710	-	-5.830	-
3d6s	.501	-	3.550	-
3d4d	-3.270	-	3.790	-4.980
3d5d	-.625	-	-5.690	3.640
4p ²	.780	-2.670	-	-

Table 2. Values for the electric dipole transition integrals in V IV calculated by means of MCDF including core polarization

	3d4p	4s4p	3d5p	3d4f
3d ²	.699	-	.185	.577
3d4s	-2.464	.590	.204	-
4s ²	-	-2.343	-	-
3d5s	1.326	-	-4.835	-
3d6s	.398	-	2.764	-
3d4d	-2.892	-	2.589	-3.770
3d5d	-.317	-	-4.943	2.338
4p ²	.587	-2.325	-	-

the actual state in terms of pure SL-coupling, i.e. the SL-term fractions. They are used to transform the pure SL-transition matrix into the factual transition probabilities in intermediate coupling. The angular part of the SL-transition matrix is calculated from straightforward Racah algebra, the radial transition integral is obtained from the MCDF program (Parpia et al. 1996). The values of these integrals corrected for core polarization are given in Table 1 for Ti III and Table 2 for V IV.

3.4. E1 results

In the Tables 3 and 4 the $\log(gf)$ values for the $(3d^2 + 3d4s) \rightarrow 3d4p$ transitions are given for Ti III and V IV, respectively. This system is selected by cutting off the higher energy values of both the even and the odd system in the final printing procedure (not at an earlier stage of the calculation, as this would have influenced the results). The first column of these tables shows the wavelength obtained from the energy differences between the experimental level values. Wavelengths below 2000 Å are given as vacuum wavelengths and above 2000 Å as air wavelengths. The second column gives the $\log(gf)$ values followed by the J -value, energy value and the name of the level of the lower (even) level. The first character of the level name designates the configuration number: for the even levels “1” refers to 3d² and “2” to 3d4s; for the odd levels “1” refers to 3d4p. An “*” after the energy value indicates

that the level is known, in which case (all cases here) the experimental level value is given. When unknown, the calculated energy value is given and used to calculate the wavelength. In the latter case, the wavelengths could deviate and one should be very cautious to use them for stellar spectroscopic assignments.

The electric dipole intensities are compared to those in Kurucz's database (Kurucz 1993) which is at present the most extended data base for transition probabilities. Although in many cases the results from Kurucz are comparable to ours, there are also a number of cases where large discrepancies (some orders of magnitude for the transition probabilities) occur. Striking examples of these differences are given in Tables 5 and 6 for transitions in the $3d4s \rightarrow 3d4p$ and $3d^2 \rightarrow 3d4p$ transition array respectively. In these tables not only the differences between Kurucz's and our work are given but also the intensity numbers from the original analysis by Edlén & Swensson (1975). These relative intensity numbers are based on visual estimates on a scale 0 – 25 and comparable (on a different scale) to the intensity numbers given in the NBS compilation (Kelly 1987). From the tables it seems that our values fit the experimental data better.

3.5. Forbidden lines

Apart from allowed electric dipole (E1) transitions, many stellar objects show spectral lines due to magnetic dipole (M1) or electric quadrupole (E2) "forbidden" transitions that occur within complexes of the same parity. These spectral lines can only be observed when other means of decay, like E1 or non-radiative collisional transitions are unlikely, implying a low excitation temperature. Therefore, only forbidden lines in the ground state (even) system are relevant.

For the magnetic dipole transitions only an angular part, that depends on the S -, L - and J -values of the initial and final states, exists. For the electric quadrupole transition, however, the transition probability contains, apart from the angular part, also a radial part. Just as in the case of the electric dipole transitions this radial part was calculated from the MCDF wavefunctions. In Tables 7 and 8 the radial integrals for the electric quadrupole transitions are given. The tables are given in the form of a symmetric matrix since the $3d^2 - 3d4s$ integral is the same as the $3d4s - 3d^2$ integral (in both cases a $3d - 4s$ transition is calculated). The table is somewhat complicated by the fact that for E2-transitions within the $3d4d$ (or $3d5d$) configurations, there are two non-zero operators, one for the $3d - 3d$ and one for the $4d - 4d$ (or $5d - 5d$) transition. For these cases, there are two rows in the table: the upper one gives the $3d - 3d$ integral and the lower the $4d - 4d$ (or $5d - 5d$) transition integral.

Tables 9 and 10 give the A -values for the forbidden lines. Just as for the allowed lines only transitions involving $3d^2$ - and $3d4s$ levels are included. The listing is in

wavelength order. The level with the lower J -value is given first in the designation of the transition.

4. Conclusion

Although the orthogonal operator method is known to describe energy levels of complex spectra more accurately (by an order of magnitude) than earlier methods, a similar claim for transition probabilities has yet to be proved. However, the transition probabilities of the present pilot cases Ti III and V IV sustain the comparison with the available experimental data successfully. Global agreement with the database of Kurucz (1993) has been found, while discrepancies seem to be in favour of the present calculation.

To avoid typing errors, all tables containing transition probabilities are computer processed. The only stage in which data typing by hand occurs is in the input of the energy fitting program. However, due to the low mean error σ of the fit with orthogonal operators, the levels values are scrutinized almost automatically.

Complete results of the fits of both the odd and even energy system, as well as the corresponding complete transition arrays can be found in our database (anonymous ftp) at <ftp://nucleus.phys.uva.nl> in the directory `pub/orth/ti3`.

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Table 3. Calculated $\log(gf)$ values for the $(3d^2 + 3d4s) - 3d4p$ transition array of Ti III

$\lambda(\text{\AA})$	$\log(gf)$	J_f	$E_f(\text{cm}^{-1})$	even	J_i	$E_i(\text{cm}^{-1})$	odd
2984.744	.152	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	2.0	75198.21*	$1\frac{2}{2}D^1D$
2832.352	-3.584	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	1.0	77000.23*	$1\frac{2}{2}D^3D$
2818.998	-1.620	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	2.0	77167.43*	$1\frac{2}{2}D^3D$
2798.916	-1.770	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	2.0	77421.86*	$1\frac{2}{2}D^3F$
2798.713	-2.527	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	3.0	77424.45*	$1\frac{2}{2}D^3D$
2773.709	-2.406	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	3.0	77746.44*	$1\frac{2}{2}D^3F$
2718.639	-3.533	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	2.0	75198.21*	$1\frac{2}{2}D^1D$
2701.955	-1.466	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	2.0	75198.21*	$1\frac{2}{2}D^1D$
2692.161	-1.874	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	2.0	75198.21*	$1\frac{2}{2}D^1D$
2580.443	-.526	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	2.0	77167.43*	$1\frac{2}{2}D^3D$
2576.463	-.561	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	1.0	77000.23*	$1\frac{2}{2}D^3D$
2567.556	-.137	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	1.0	77000.23*	$1\frac{2}{2}D^3D$
2565.408	.128	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	2.0	77167.43*	$1\frac{2}{2}D^3D$
2563.606	-1.904	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	2.0	77421.86*	$1\frac{2}{2}D^3F$
2563.435	.368	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	3.0	77424.45*	$1\frac{2}{2}D^3D$
2556.576	-2.549	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	2.0	77167.43*	$1\frac{2}{2}D^3D$
2548.765	-2.279	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	2.0	77421.86*	$1\frac{2}{2}D^3F$
2548.597	-2.336	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	3.0	77424.45*	$1\frac{2}{2}D^3D$
2547.985	-2.611	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	1.0	80939.19*	$1\frac{2}{2}D^3P$
2542.459	-2.149	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	2.0	81024.47*	$1\frac{2}{2}D^3P$
2542.442	-2.077	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	3.0	77746.44*	$1\frac{2}{2}D^3F$
2540.048	.204	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	2.0	77421.86*	$1\frac{2}{2}D^3F$
2527.845	.360	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	3.0	77746.44*	$1\frac{2}{2}D^3F$
2516.066	.471	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	4.0	78158.61*	$1\frac{2}{2}D^3F$
2413.987	.373	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	3.0	83116.93*	$1\frac{2}{2}D^1F$
2374.990	-.016	2.0	41704.27*	$2\frac{3}{2}D$) ¹ D	1.0	83796.86*	$1\frac{2}{2}D^1P$
2346.783	.132	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	2.0	81024.47*	$1\frac{2}{2}D^3P$
2338.999	-.140	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	1.0	80939.19*	$1\frac{2}{2}D^3P$
2334.340	-.540	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	2.0	81024.47*	$1\frac{2}{2}D^3P$
2331.655	-.554	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	1.0	80939.19*	$1\frac{2}{2}D^3P$
2331.346	-.473	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	.0	80944.87*	$1\frac{2}{2}D^3P$
2327.026	-1.678	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	2.0	81024.47*	$1\frac{2}{2}D^3P$
2245.246	-3.595	.0	32475.50*	$1\frac{1}{0}S$)	1.0	77000.23*	$1\frac{2}{2}D^3D$
2236.895	-2.452	3.0	38425.99*	$2\frac{3}{2}D$) ³ D	3.0	83116.93*	$1\frac{2}{2}D^1F$
2225.588	-2.287	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	3.0	83116.93*	$1\frac{2}{2}D^1F$
2192.398	-2.461	2.0	38198.95*	$2\frac{3}{2}D$) ³ D	1.0	83796.86*	$1\frac{2}{2}D^1P$
2185.944	-4.026	1.0	38064.35*	$2\frac{3}{2}D$) ³ D	1.0	83796.86*	$1\frac{2}{2}D^1P$
2062.741	-2.903	.0	32475.50*	$1\frac{1}{0}S$)	1.0	80939.19*	$1\frac{2}{2}D^3P$
1948.506	-.770	.0	32475.50*	$1\frac{1}{0}S$)	1.0	83796.86*	$1\frac{2}{2}D^1P$
1586.625	-2.869	4.0	14397.60*	$1\frac{1}{2}G$)	3.0	77424.45*	$1\frac{2}{2}D^3D$
1578.561	-4.381	4.0	14397.60*	$1\frac{1}{2}G$)	3.0	77746.44*	$1\frac{2}{2}D^3F$
1568.357	-4.507	4.0	14397.60*	$1\frac{1}{2}G$)	4.0	78158.61*	$1\frac{2}{2}D^3F$
1550.940	-3.225	2.0	10721.20*	$1\frac{3}{2}P$)	2.0	75198.21*	$1\frac{2}{2}D^1D$
1548.117	-4.573	1.0	10603.60*	$1\frac{3}{2}P$)	2.0	75198.21*	$1\frac{2}{2}D^1D$
1508.773	-2.947	2.0	10721.20*	$1\frac{3}{2}P$)	1.0	77000.23*	$1\frac{2}{2}D^3D$
1506.101	-1.587	1.0	10603.60*	$1\frac{3}{2}P$)	1.0	77000.23*	$1\frac{2}{2}D^3D$
1504.976	-1.718	2.0	10721.20*	$1\frac{3}{2}P$)	2.0	77167.43*	$1\frac{2}{2}D^3D$
1504.623	-1.375	.0	10538.40*	$1\frac{3}{2}P$)	1.0	77000.23*	$1\frac{2}{2}D^3D$
1502.317	-1.081	1.0	10603.60*	$1\frac{3}{2}P$)	2.0	77167.43*	$1\frac{2}{2}D^3D$
1499.236	-2.596	2.0	10721.20*	$1\frac{3}{2}P$)	2.0	77421.86*	$1\frac{2}{2}D^3F$
1499.177	-.865	2.0	10721.20*	$1\frac{3}{2}P$)	3.0	77424.45*	$1\frac{2}{2}D^3D$
1498.695	-.231	2.0	8473.50*	$1\frac{1}{2}D$)	2.0	75198.21*	$1\frac{2}{2}D^1D$
1496.597	-1.932	1.0	10603.60*	$1\frac{3}{2}P$)	2.0	77421.86*	$1\frac{2}{2}D^3F$
1491.975	-1.625	2.0	10721.20*	$1\frac{3}{2}P$)	3.0	77746.44*	$1\frac{2}{2}D^3F$
1459.285	-4.557	2.0	8473.50*	$1\frac{1}{2}D$)	1.0	77000.23*	$1\frac{2}{2}D^3D$
1455.733	-2.445	2.0	8473.50*	$1\frac{1}{2}D$)	2.0	77167.43*	$1\frac{2}{2}D^3D$

Table 3. continued

$\lambda(\text{\AA})$	$\log(\text{gf})$	J_f	$E_f(\text{cm}^{-1})$	even	J_i	$E_i(\text{cm}^{-1})$	odd
1455.195	.208	4.0	14397.60*	$1\frac{1}{2}\text{G}$	3.0	83116.93*	$1\frac{1}{2}\text{D}^1\text{F}$
1450.361	-2.075	2.0	8473.50*	$1\frac{1}{2}\text{D}$	2.0	77421.86*	$1\frac{1}{2}\text{D}^3\text{F}$
1450.306	-2.890	2.0	8473.50*	$1\frac{1}{2}\text{D}$	3.0	77424.45*	$1\frac{1}{2}\text{D}^3\text{D}$
1443.565	-4.045	2.0	8473.50*	$1\frac{1}{2}\text{D}$	3.0	77746.44*	$1\frac{1}{2}\text{D}^3\text{F}$
1424.136	-.798	2.0	10721.20*	$1\frac{3}{2}\text{P}$	1.0	80939.19*	$1\frac{1}{2}\text{D}^3\text{P}$
1422.409	-.302	2.0	10721.20*	$1\frac{3}{2}\text{P}$	2.0	81024.47*	$1\frac{1}{2}\text{D}^3\text{P}$
1421.755	-.997	1.0	10603.60*	$1\frac{3}{2}\text{P}$	1.0	80939.19*	$1\frac{1}{2}\text{D}^3\text{P}$
1421.641	-.883	1.0	10603.60*	$1\frac{3}{2}\text{P}$.0	80944.87*	$1\frac{1}{2}\text{D}^3\text{P}$
1420.439	-.901	.0	10538.40*	$1\frac{3}{2}\text{P}$	1.0	80939.19*	$1\frac{1}{2}\text{D}^3\text{P}$
1420.034	-.809	1.0	10603.60*	$1\frac{3}{2}\text{P}$	2.0	81024.47*	$1\frac{1}{2}\text{D}^3\text{P}$
1381.297	-3.243	2.0	10721.20*	$1\frac{3}{2}\text{P}$	3.0	83116.93*	$1\frac{1}{2}\text{D}^1\text{F}$
1379.963	-2.194	2.0	8473.50*	$1\frac{1}{2}\text{D}$	1.0	80939.19*	$1\frac{1}{2}\text{D}^3\text{P}$
1378.341	-3.390	2.0	8473.50*	$1\frac{1}{2}\text{D}$	2.0	81024.47*	$1\frac{1}{2}\text{D}^3\text{P}$
1368.445	-2.232	2.0	10721.20*	$1\frac{3}{2}\text{P}$	1.0	83796.86*	$1\frac{1}{2}\text{D}^1\text{P}$
1366.246	-3.256	1.0	10603.60*	$1\frac{3}{2}\text{P}$	1.0	83796.86*	$1\frac{1}{2}\text{D}^1\text{P}$
1365.030	-2.811	.0	10538.40*	$1\frac{3}{2}\text{P}$	1.0	83796.86*	$1\frac{1}{2}\text{D}^1\text{P}$
1339.703	-1.847	2.0	8473.50*	$1\frac{1}{2}\text{D}$	3.0	83116.93*	$1\frac{1}{2}\text{D}^1\text{F}$
1333.097	-3.986	3.0	184.90*	$1\frac{3}{2}\text{F}$	2.0	75198.21*	$1\frac{1}{2}\text{D}^1\text{D}$
1329.819	-2.083	2.0	.00*	$1\frac{3}{2}\text{F}$	2.0	75198.21*	$1\frac{1}{2}\text{D}^1\text{D}$
1327.609	-.601	2.0	8473.50*	$1\frac{1}{2}\text{D}$	1.0	83796.86*	$1\frac{1}{2}\text{D}^1\text{P}$
1298.996	-.280	3.0	184.90*	$1\frac{3}{2}\text{F}$	2.0	77167.43*	$1\frac{1}{2}\text{D}^3\text{D}$
1298.697	-.317	2.0	.00*	$1\frac{3}{2}\text{F}$	1.0	77000.23*	$1\frac{1}{2}\text{D}^3\text{D}$
1298.633	-.121	4.0	420.40*	$1\frac{3}{2}\text{F}$	3.0	77424.45*	$1\frac{1}{2}\text{D}^3\text{D}$
1295.884	-.680	2.0	.00*	$1\frac{3}{2}\text{F}$	2.0	77167.43*	$1\frac{1}{2}\text{D}^3\text{D}$
1294.717	-.653	3.0	184.90*	$1\frac{3}{2}\text{F}$	2.0	77421.86*	$1\frac{1}{2}\text{D}^3\text{F}$
1294.674	-.600	3.0	184.90*	$1\frac{3}{2}\text{F}$	3.0	77424.45*	$1\frac{1}{2}\text{D}^3\text{D}$
1293.225	-.507	4.0	420.40*	$1\frac{3}{2}\text{F}$	3.0	77746.44*	$1\frac{1}{2}\text{D}^3\text{F}$
1291.625	-.878	2.0	.00*	$1\frac{3}{2}\text{F}$	2.0	77421.86*	$1\frac{1}{2}\text{D}^3\text{F}$
1291.582	-1.874	2.0	.00*	$1\frac{3}{2}\text{F}$	3.0	77424.45*	$1\frac{1}{2}\text{D}^3\text{D}$
1289.299	-.737	3.0	184.90*	$1\frac{3}{2}\text{F}$	3.0	77746.44*	$1\frac{1}{2}\text{D}^3\text{F}$
1286.369	-.304	4.0	420.40*	$1\frac{3}{2}\text{F}$	4.0	78158.61*	$1\frac{1}{2}\text{D}^3\text{F}$
1286.233	-1.661	2.0	.00*	$1\frac{3}{2}\text{F}$	3.0	77746.44*	$1\frac{1}{2}\text{D}^3\text{F}$
1282.484	-1.478	3.0	184.90*	$1\frac{3}{2}\text{F}$	4.0	78158.61*	$1\frac{1}{2}\text{D}^3\text{F}$
1237.018	-3.070	3.0	184.90*	$1\frac{3}{2}\text{F}$	2.0	81024.47*	$1\frac{1}{2}\text{D}^3\text{P}$
1235.495	-3.341	2.0	.00*	$1\frac{3}{2}\text{F}$	1.0	80939.19*	$1\frac{1}{2}\text{D}^3\text{P}$
1234.195	-3.910	2.0	.00*	$1\frac{3}{2}\text{F}$	2.0	81024.47*	$1\frac{1}{2}\text{D}^3\text{P}$
1209.241	-3.196	4.0	420.40*	$1\frac{3}{2}\text{F}$	3.0	83116.93*	$1\frac{1}{2}\text{D}^1\text{F}$
1205.807	-3.666	3.0	184.90*	$1\frac{3}{2}\text{F}$	3.0	83116.93*	$1\frac{1}{2}\text{D}^1\text{F}$
1203.124	-4.407	2.0	.00*	$1\frac{3}{2}\text{F}$	3.0	83116.93*	$1\frac{1}{2}\text{D}^1\text{F}$
1193.362	-3.411	2.0	.00*	$1\frac{3}{2}\text{F}$	1.0	83796.86*	$1\frac{1}{2}\text{D}^1\text{P}$

Table 4. Calculated $\log(gf)$ values for the $(3d^2 + 3d4s)$ - $3d4p$ transition array of V IV

$\lambda(\text{\AA})$	$\log(gf)$	J_f	$E_f(\text{cm}^{-1})$	even	J_i	$E_i(\text{cm}^{-1})$	odd
2268.292	.139	2.0	100200.70*	$2\frac{1}{2}D^1D$	2.0	144273.10*	1^2D^1D
2177.161	-3.837	2.0	100200.70*	$2\frac{1}{2}D^1D$	1.0	146117.70*	1^2D^3D
2162.484	-1.442	2.0	100200.70*	$2\frac{1}{2}D^1D$	2.0	146429.30*	1^2D^3D
2142.745	-2.416	2.0	100200.70*	$2\frac{1}{2}D^1D$	3.0	146855.10*	1^2D^3D
2129.956	-1.401	2.0	100200.70*	$2\frac{1}{2}D^1D$	2.0	147135.20*	1^2D^3F
2106.556	-1.927	2.0	100200.70*	$2\frac{1}{2}D^1D$	3.0	147656.50*	1^2D^3F
2105.700	-3.414	3.0	96798.00*	$2\frac{1}{2}D^3D$	2.0	144273.10*	1^2D^1D
2088.719	-1.178	2.0	96412.10*	$2\frac{1}{2}D^3D$	2.0	144273.10*	1^2D^1D
2079.334	-1.685	1.0	96196.10*	$2\frac{1}{2}D^3D$	2.0	144273.10*	1^2D^1D
2014.207	-.513	3.0	96798.00*	$2\frac{1}{2}D^3D$	2.0	146429.30*	1^2D^3D
2011.196	-.573	2.0	96412.10*	$2\frac{1}{2}D^3D$	1.0	146117.70*	1^2D^3D
2002.492	-.169	1.0	96196.10*	$2\frac{1}{2}D^3D$	1.0	146117.70*	1^2D^3D
1999.312	.069	2.0	96412.10*	$2\frac{1}{2}D^3D$	2.0	146429.30*	1^2D^3D
1997.719	.340	3.0	96798.00*	$2\frac{1}{2}D^3D$	3.0	146855.10*	1^2D^3D
1990.715	-1.235	1.0	96196.10*	$2\frac{1}{2}D^3D$	2.0	146429.30*	1^2D^3D
1986.602	-3.257	3.0	96798.00*	$2\frac{1}{2}D^3D$	2.0	147135.20*	1^2D^3F
1982.436	-1.763	2.0	96412.10*	$2\frac{1}{2}D^3D$	3.0	146855.10*	1^2D^3D
1971.488	-1.276	2.0	96412.10*	$2\frac{1}{2}D^3D$	2.0	147135.20*	1^2D^3F
1966.240	-1.931	3.0	96798.00*	$2\frac{1}{2}D^3D$	3.0	147656.50*	1^2D^3F
1963.129	.162	1.0	96196.10*	$2\frac{1}{2}D^3D$	2.0	147135.20*	1^2D^3F
1952.122	-2.792	2.0	100200.70*	$2\frac{1}{2}D^1D$	1.0	151427.00*	1^2D^3P
1951.433	.333	2.0	96412.10*	$2\frac{1}{2}D^3D$	3.0	147656.50*	1^2D^3F
1946.790	-1.849	2.0	100200.70*	$2\frac{1}{2}D^1D$	2.0	151567.30*	1^2D^3P
1939.067	.449	3.0	96798.00*	$2\frac{1}{2}D^3D$	4.0	148369.20*	1^2D^3F
1861.573	.351	2.0	100200.70*	$2\frac{1}{2}D^1D$	3.0	153918.70*	1^2D^1F
1825.840	.108	3.0	96798.00*	$2\frac{1}{2}D^3D$	2.0	151567.30*	1^2D^3P
1817.689	-.164	2.0	96412.10*	$2\frac{1}{2}D^3D$	1.0	151427.00*	1^2D^3P
1813.066	-.544	2.0	96412.10*	$2\frac{1}{2}D^3D$	2.0	151567.30*	1^2D^3P
1810.581	-.556	1.0	96196.10*	$2\frac{1}{2}D^3D$	1.0	151427.00*	1^2D^3P
1809.856	-.491	1.0	96196.10*	$2\frac{1}{2}D^3D$.0	151449.10*	1^2D^3P
1806.202	-.031	2.0	100200.70*	$2\frac{1}{2}D^1D$	1.0	155565.50*	1^2D^1P
1805.993	-1.674	1.0	96196.10*	$2\frac{1}{2}D^3D$	2.0	151567.30*	1^2D^3P
1750.679	-2.185	3.0	96798.00*	$2\frac{1}{2}D^3D$	3.0	153918.70*	1^2D^1F
1738.931	-1.899	2.0	96412.10*	$2\frac{1}{2}D^3D$	3.0	153918.70*	1^2D^1F
1690.520	-2.603	2.0	96412.10*	$2\frac{1}{2}D^3D$	1.0	155565.50*	1^2D^1P
1684.369	-4.146	1.0	96196.10*	$2\frac{1}{2}D^3D$	1.0	155565.50*	1^2D^1P
964.733	-3.446	.0	42462.10*	1^1_0S	1.0	146117.70*	1^2D^3D
917.727	-2.845	.0	42462.10*	1^1_0S	1.0	151427.00*	1^2D^3P
884.147	-.821	.0	42462.10*	1^1_0S	1.0	155565.50*	1^2D^1P
778.429	-2.589	4.0	18391.20*	1^1_2G	3.0	146855.10*	1^2D^3D
773.603	-5.685	4.0	18391.20*	1^1_2G	3.0	147656.50*	1^2D^3F
769.361	-4.204	4.0	18391.20*	1^1_2G	4.0	148369.20*	1^2D^3F
764.439	-2.844	2.0	13458.30*	1^3_2P	2.0	144273.10*	1^2D^1D
763.161	-4.607	1.0	13239.20*	1^3_2P	2.0	144273.10*	1^2D^1D
753.810	-3.036	2.0	13458.30*	1^3_2P	1.0	146117.70*	1^2D^3D
752.567	-1.614	1.0	13239.20*	1^3_2P	1.0	146117.70*	1^2D^3D
752.044	-1.727	2.0	13458.30*	1^3_2P	2.0	146429.30*	1^2D^3D
751.909	-1.371	.0	13122.80*	1^3_2P	1.0	146117.70*	1^2D^3D
750.807	-1.038	1.0	13239.20*	1^3_2P	2.0	146429.30*	1^2D^3D
750.110	-.330	2.0	10959.30*	1^1_2D	2.0	144273.10*	1^2D^1D
749.643	-.843	2.0	13458.30*	1^3_2P	3.0	146855.10*	1^2D^3D
748.072	-3.253	2.0	13458.30*	1^3_2P	2.0	147135.20*	1^2D^3F
746.848	-2.429	1.0	13239.20*	1^3_2P	2.0	147135.20*	1^2D^3F
745.166	-1.981	2.0	13458.30*	1^3_2P	3.0	147656.50*	1^2D^3F
739.873	-4.297	2.0	10959.30*	1^1_2D	1.0	146117.70*	1^2D^3D
738.171	-2.518	2.0	10959.30*	1^1_2D	2.0	146429.30*	1^2D^3D

Table 4. continued

$\lambda(\text{\AA})$	$\log(gf)$	J_f	$E_f(\text{cm}^{-1})$	even	J_i	$E_i(\text{cm}^{-1})$	odd
737.858	.175	4.0	18391.20*	$1 _{\frac{1}{2}}G$	3.0	153918.70*	$1 ^2D^1F$
735.858	-2.438	2.0	10959.30*	$1 _{\frac{1}{2}}D$	3.0	146855.10*	$1 ^2D^3D$
734.344	-1.853	2.0	10959.30*	$1 _{\frac{1}{2}}D$	2.0	147135.20*	$1 ^2D^3F$
731.544	-4.121	2.0	10959.30*	$1 _{\frac{1}{2}}D$	3.0	147656.50*	$1 ^2D^3F$
724.802	-.834	2.0	13458.30*	$1 _{\frac{3}{2}}P$	1.0	151427.00*	$1 ^2D^3P$
724.066	-.324	2.0	13458.30*	$1 _{\frac{3}{2}}P$	2.0	151567.30*	$1 ^2D^3P$
723.653	-1.015	1.0	13239.20*	$1 _{\frac{3}{2}}P$	1.0	151427.00*	$1 ^2D^3P$
723.537	-.905	1.0	13239.20*	$1 _{\frac{3}{2}}P$.0	151449.10*	$1 ^2D^3P$
723.044	-.931	.0	13122.80*	$1 _{\frac{3}{2}}P$	1.0	151427.00*	$1 ^2D^3P$
722.919	-.841	1.0	13239.20*	$1 _{\frac{3}{2}}P$	2.0	151567.30*	$1 ^2D^3P$
711.944	-2.701	2.0	13458.30*	$1 _{\frac{3}{2}}P$	3.0	153918.70*	$1 ^2D^1F$
711.907	-1.928	2.0	10959.30*	$1 _{\frac{1}{2}}D$	1.0	151427.00*	$1 ^2D^3P$
711.197	-2.730	2.0	10959.30*	$1 _{\frac{1}{2}}D$	2.0	151567.30*	$1 ^2D^3P$
703.694	-1.931	2.0	13458.30*	$1 _{\frac{3}{2}}P$	1.0	155565.50*	$1 ^2D^1P$
702.611	-3.172	1.0	13239.20*	$1 _{\frac{3}{2}}P$	1.0	155565.50*	$1 ^2D^1P$
702.037	-2.706	.0	13122.80*	$1 _{\frac{3}{2}}P$	1.0	155565.50*	$1 ^2D^1P$
699.499	-1.423	2.0	10959.30*	$1 _{\frac{1}{2}}D$	3.0	153918.70*	$1 ^2D^1F$
694.697	-3.471	3.0	325.40*	$1 _{\frac{3}{2}}F$	2.0	144273.10*	$1 ^2D^1D$
693.130	-1.857	2.0	.00*	$1 _{\frac{3}{2}}F$	2.0	144273.10*	$1 ^2D^1D$
691.533	-.560	2.0	10959.30*	$1 _{\frac{1}{2}}D$	1.0	155565.50*	$1 ^2D^1P$
684.444	-.224	3.0	325.40*	$1 _{\frac{3}{2}}F$	2.0	146429.30*	$1 ^2D^3D$
684.380	-.334	2.0	.00*	$1 _{\frac{3}{2}}F$	1.0	146117.70*	$1 ^2D^3D$
684.367	-.075	4.0	734.70*	$1 _{\frac{3}{2}}F$	3.0	146855.10*	$1 ^2D^3D$
682.923	-.828	2.0	.00*	$1 _{\frac{3}{2}}F$	2.0	146429.30*	$1 ^2D^3D$
682.456	-.729	3.0	325.40*	$1 _{\frac{3}{2}}F$	3.0	146855.10*	$1 ^2D^3D$
681.153	-.915	3.0	325.40*	$1 _{\frac{3}{2}}F$	2.0	147135.20*	$1 ^2D^3F$
680.943	-2.062	2.0	.00*	$1 _{\frac{3}{2}}F$	3.0	146855.10*	$1 ^2D^3D$
680.634	-.732	4.0	734.70*	$1 _{\frac{3}{2}}F$	3.0	147656.50*	$1 ^2D^3F$
679.647	-.754	2.0	.00*	$1 _{\frac{3}{2}}F$	2.0	147135.20*	$1 ^2D^3F$
678.743	-.631	3.0	325.40*	$1 _{\frac{3}{2}}F$	3.0	147656.50*	$1 ^2D^3F$
677.348	-.317	4.0	734.70*	$1 _{\frac{3}{2}}F$	4.0	148369.20*	$1 ^2D^3F$
677.248	-1.596	2.0	.00*	$1 _{\frac{3}{2}}F$	3.0	147656.50*	$1 ^2D^3F$
675.476	-1.492	3.0	325.40*	$1 _{\frac{3}{2}}F$	4.0	148369.20*	$1 ^2D^3F$
661.192	-2.842	3.0	325.40*	$1 _{\frac{3}{2}}F$	2.0	151567.30*	$1 ^2D^3P$
660.384	-3.106	2.0	.00*	$1 _{\frac{3}{2}}F$	1.0	151427.00*	$1 ^2D^3P$
659.773	-3.654	2.0	.00*	$1 _{\frac{3}{2}}F$	2.0	151567.30*	$1 ^2D^3P$
652.810	-2.944	4.0	734.70*	$1 _{\frac{3}{2}}F$	3.0	153918.70*	$1 ^2D^1F$
651.070	-3.342	3.0	325.40*	$1 _{\frac{3}{2}}F$	3.0	153918.70*	$1 ^2D^1F$
649.694	-3.943	2.0	.00*	$1 _{\frac{3}{2}}F$	3.0	153918.70*	$1 ^2D^1F$
642.816	-3.339	2.0	.00*	$1 _{\frac{3}{2}}F$	1.0	155565.50*	$1 ^2D^1P$

Table 5. Examples of differences for some 3d4s – 3d4p transitions in Ti III between log gf data obtained by our method and Kurucz data base compared to experimental intensity numbers by Edlén

$\lambda(\text{\AA})$	Kurucz	Edlén	Our work
2798.916	-2.92	8	-1.77
2798.713	-2.08	5	-2.53
2773.709	-3.52	4	-2.41
2718.639	-3.54	0	-3.53
2567.556	-0.14	22	-0.14
2565.408	-0.04	23	0.13
2556.576	-0.30	5	-2.55
2548.765	-0.38	5	-2.28
2548.597	-0.19	6	-2.34

Table 9. Calculated A -values for the $(3d^2 + 3d4s) - (3d^2 + 3d4s)$ M1 and E2 transition arrays of Ti III; the notation $x(y)$ means $x \times 10^y$

$\lambda(\text{\AA})$	$A_{M1}(s^{-1})$	$A_{E2}(s^{-1})$	J_f	$E_f(\text{cm}^{-1})$	name	J_i	$E_i(\text{cm}^{-1})$	name
1533324.348	5.01(-6)	-	.0	10538.40*	$1 _{3/2}^3P$	1.0	10603.60*	$1 _{3/2}^3P$
850108.397	2.19(-5)	2.66(-12)	1.0	10603.60*	$1 _{3/2}^3P$	2.0	10721.20*	$1 _{3/2}^3P$
742739.580	5.93(-5)	1.97(-12)	1.0	38064.35*	$2 _{1/2}^2D^3D$	2.0	38198.95*	$2 _{1/2}^2D^3D$
546896.868	-	1.07(-11)	.0	10538.40*	$1 _{3/2}^3P$	2.0	10721.20*	$1 _{3/2}^3P$
540685.492	1.63(-4)	1.61(-12)	2.0	.00*	$1 _{3/2}^3F$	3.0	184.90*	$1 _{3/2}^3F$
440330.988	2.11(-4)	2.20(-11)	2.0	38198.95*	$2 _{1/2}^2D^3D$	3.0	38425.99*	$2 _{1/2}^2D^3D$
424512.728	2.65(-4)	4.35(-12)	3.0	184.90*	$1 _{3/2}^3F$	4.0	420.40*	$1 _{3/2}^3F$
276442.726	-	2.26(-11)	1.0	38064.35*	$2 _{1/2}^2D^3D$	3.0	38425.99*	$2 _{1/2}^2D^3D$
237803.871	-	3.53(-12)	2.0	.00*	$1 _{3/2}^3F$	4.0	420.40*	$1 _{3/2}^3F$
48415.294	-	5.20(-8)	.0	10538.40*	$1 _{3/2}^3P$	2.0	8473.50*	$1 _{3/2}^3P$
46933.356	1.31(-3)	7.61(-8)	1.0	10603.60*	$1 _{3/2}^3P$	2.0	8473.50*	$1 _{3/2}^3P$
44477.795	2.76(-3)	1.32(-10)	2.0	8473.50*	$1 _{3/2}^3D$	2.0	10721.20*	$1 _{3/2}^3P$
30495.483	2.41(-3)	5.13(-8)	2.0	41704.27*	$2 _{1/2}^2D^1D$	3.0	38425.99*	$2 _{1/2}^2D^3D$
28520.286	5.25(-4)	8.70(-8)	2.0	38198.95*	$2 _{1/2}^2D^3D$	2.0	41704.27*	$2 _{1/2}^2D^1D$
27465.639	3.18(-3)	7.63(-8)	1.0	38064.35*	$2 _{1/2}^2D^3D$	2.0	41704.27*	$2 _{1/2}^2D^1D$
27193.104	-	2.52(-7)	2.0	10721.20*	$1 _{3/2}^3P$	4.0	14397.60*	$1 _{3/2}^3G$
17887.884	5.18(-16)	-	.0	32475.50*	$1 _{3/2}^3S$	1.0	38064.35*	$2 _{1/2}^2D^3D$
17467.209	-	8.65(-7)	.0	32475.50*	$1 _{3/2}^3S$	2.0	38198.95*	$2 _{1/2}^2D^3D$
16875.592	-	3.22(-4)	2.0	8473.50*	$1 _{3/2}^3D$	4.0	14397.60*	$1 _{3/2}^3G$
12414.182	-	5.66(-5)	2.0	8473.50*	$1 _{3/2}^3D$	4.0	420.40*	$1 _{3/2}^3F$
12061.463	1.02(-2)	1.17(-5)	2.0	8473.50*	$1 _{3/2}^3D$	3.0	184.90*	$1 _{3/2}^3F$
11798.270	5.46(-3)	1.18(-5)	2.0	.00*	$1 _{3/2}^3F$	2.0	8473.50*	$1 _{3/2}^3D$
10832.713	-	6.18(-3)	.0	32475.50*	$1 _{3/2}^3S$	2.0	41704.27*	$2 _{1/2}^2D^1D$
9705.322	-	2.77(-2)	2.0	10721.20*	$1 _{3/2}^3P$	4.0	420.40*	$1 _{3/2}^3F$
9595.495	-	2.55(-2)	1.0	10603.60*	$1 _{3/2}^3P$	3.0	184.90*	$1 _{3/2}^3F$
9488.395	7.99(-5)	8.05(-3)	2.0	10721.20*	$1 _{3/2}^3P$	3.0	184.90*	$1 _{3/2}^3F$
9486.504	-	4.05(-2)	.0	10538.40*	$1 _{3/2}^3P$	2.0	.00*	$1 _{3/2}^3F$
9428.173	3.41(-6)	1.39(-2)	1.0	10603.60*	$1 _{3/2}^3P$	2.0	.00*	$1 _{3/2}^3F$
9324.756	1.58(-5)	1.24(-3)	2.0	.00*	$1 _{3/2}^3F$	2.0	10721.20*	$1 _{3/2}^3P$
7152.537	7.13(-3)	9.32(-6)	4.0	420.40*	$1 _{3/2}^3F$	4.0	14397.60*	$1 _{3/2}^3G$
7034.021	4.50(-3)	2.65(-7)	3.0	184.90*	$1 _{3/2}^3F$	4.0	14397.60*	$1 _{3/2}^3G$
6943.687	-	1.56(-5)	2.0	.00*	$1 _{3/2}^3F$	4.0	14397.60*	$1 _{3/2}^3G$
4595.505	-	2.23(-2)	.0	32475.50*	$1 _{3/2}^3S$	2.0	10721.20*	$1 _{3/2}^3P$
4570.795	8.87(-2)	-	.0	32475.50*	$1 _{3/2}^3S$	1.0	10603.60*	$1 _{3/2}^3P$
4200.259	-	1.76(-2)	2.0	38198.95*	$2 _{1/2}^2D^3D$	4.0	14397.60*	$1 _{3/2}^3G$
4165.145	-	4.95(0)	.0	32475.50*	$1 _{3/2}^3S$	2.0	8473.50*	$1 _{3/2}^3D$
4160.571	2.48(-14)	3.39(-4)	3.0	38425.99*	$2 _{1/2}^2D^3D$	4.0	14397.60*	$1 _{3/2}^3G$
3661.066	-	1.48(1)	2.0	41704.27*	$2 _{1/2}^2D^1D$	4.0	14397.60*	$1 _{3/2}^3G$
3656.181	6.25(-7)	1.19(0)	1.0	38064.35*	$2 _{1/2}^2D^3D$	2.0	10721.20*	$1 _{3/2}^3P$
3640.524	6.41(-10)	3.69(0)	1.0	10603.60*	$1 _{3/2}^3P$	1.0	38064.35*	$2 _{1/2}^2D^3D$
3638.271	6.21(-8)	2.82(0)	2.0	10721.20*	$1 _{3/2}^3P$	2.0	38198.95*	$2 _{1/2}^2D^3D$
3631.900	2.15(-10)	-	.0	10538.40*	$1 _{3/2}^3P$	1.0	38064.35*	$2 _{1/2}^2D^3D$
3622.766	9.85(-8)	4.19(-1)	1.0	10603.60*	$1 _{3/2}^3P$	2.0	38198.95*	$2 _{1/2}^2D^3D$
3614.226	-	1.70(0)	.0	10538.40*	$1 _{3/2}^3P$	2.0	38198.95*	$2 _{1/2}^2D^3D$
3608.455	2.77(-7)	3.40(0)	2.0	10721.20*	$1 _{3/2}^3P$	3.0	38425.99*	$2 _{1/2}^2D^3D$
3593.202	-	1.75(0)	1.0	10603.60*	$1 _{3/2}^3P$	3.0	38425.99*	$2 _{1/2}^2D^3D$
3378.453	1.00(-4)	3.12(-2)	1.0	38064.35*	$2 _{1/2}^2D^3D$	2.0	8473.50*	$1 _{3/2}^3D$
3363.154	1.16(-5)	7.99(-2)	2.0	8473.50*	$1 _{3/2}^3D$	2.0	38198.95*	$2 _{1/2}^2D^3D$
3337.661	4.63(-5)	3.54(-2)	2.0	8473.50*	$1 _{3/2}^3D$	3.0	38425.99*	$2 _{1/2}^2D^3D$
3226.638	9.67(-5)	1.62(-1)	2.0	10721.20*	$1 _{3/2}^3P$	2.0	41704.27*	$2 _{1/2}^2D^1D$
3214.436	3.27(-5)	2.09(-3)	1.0	10603.60*	$1 _{3/2}^3P$	2.0	41704.27*	$2 _{1/2}^2D^1D$
3207.711	-	5.00(-3)	.0	10538.40*	$1 _{3/2}^3P$	2.0	41704.27*	$2 _{1/2}^2D^1D$
3078.350	-	4.17(-3)	.0	32475.50*	$1 _{3/2}^3S$	2.0	.00*	$1 _{3/2}^3F$
3008.382	5.97(-7)	1.89(1)	2.0	8473.50*	$1 _{3/2}^3D$	2.0	41704.27*	$2 _{1/2}^2D^1D$
2646.217	-	1.18(1)	2.0	38198.95*	$2 _{1/2}^2D^3D$	4.0	420.40*	$1 _{3/2}^3F$
2639.167	-	1.86(1)	1.0	38064.35*	$2 _{1/2}^2D^3D$	3.0	184.90*	$1 _{3/2}^3F$
2630.408	1.52(-9)	4.35(1)	3.0	38425.99*	$2 _{1/2}^2D^3D$	4.0	420.40*	$1 _{3/2}^3F$
2629.822	4.14(-7)	2.84(1)	2.0	38198.95*	$2 _{1/2}^2D^3D$	3.0	184.90*	$1 _{3/2}^3F$
2626.347	5.95(-8)	3.82(1)	1.0	38064.35*	$2 _{1/2}^2D^3D$	2.0	.00*	$1 _{3/2}^3F$
2617.092	1.22(-7)	1.67(1)	2.0	.00*	$1 _{3/2}^3F$	2.0	38198.95*	$2 _{1/2}^2D^3D$
2614.208	1.20(-9)	1.26(1)	3.0	184.90*	$1 _{3/2}^3F$	3.0	38425.99*	$2 _{1/2}^2D^3D$
2601.628	1.58(-8)	1.22(0)	2.0	.00*	$1 _{3/2}^3F$	3.0	38425.99*	$2 _{1/2}^2D^3D$
2421.518	-	1.68(-2)	2.0	41704.27*	$2 _{1/2}^2D^1D$	4.0	420.40*	$1 _{3/2}^3F$
2407.782	1.33(-4)	1.19(-1)	2.0	41704.27*	$2 _{1/2}^2D^1D$	3.0	184.90*	$1 _{3/2}^3F$
2397.106	6.75(-5)	8.33(-3)	2.0	.00*	$1 _{3/2}^3F$	2.0	41704.27*	$2 _{1/2}^2D^1D$

Table 10. Calculated A -values for the $(3d^2 + 3d4s) - (3d^2 + 3d4s)$ M1 and E2 transition arrays of V IV; the notation $x(y)$ means $x \times 10^y$

$\lambda(\text{\AA})$	$A_{M1}(s^{-1})$	$A_{E2}(s^{-1})$	J_f	$E_f(\text{cm}^{-1})$	name	J_i	$E_i(\text{cm}^{-1})$	name
858872.401	2.85(-5)	-	.0	13122.80*	$1\frac{3}{2}P$	1.0	13239.20*	$1\frac{3}{2}P$
462836.794	2.44(-4)	1.04(-11)	1.0	96196.10*	$2\frac{3}{2}D^3D$	2.0	96412.10*	$2\frac{3}{2}D^3D$
456288.213	1.40(-4)	2.72(-11)	1.0	13239.20*	$1\frac{3}{2}P$	2.0	13458.30*	$1\frac{3}{2}P$
307230.324	8.88(-4)	1.13(-11)	2.0	.00*	$1\frac{3}{2}F$	3.0	325.40*	$1\frac{3}{2}F$
297981.363	-	1.02(-10)	.0	13122.80*	$1\frac{3}{2}P$	2.0	13458.30*	$1\frac{3}{2}P$
259063.870	1.03(-3)	1.54(-10)	2.0	96412.10*	$2\frac{3}{2}D^3D$	3.0	96798.00*	$2\frac{3}{2}D^3D$
244252.986	1.39(-3)	2.88(-11)	3.0	325.40*	$1\frac{3}{2}F$	4.0	734.70*	$1\frac{3}{2}F$
166095.277	-	1.43(-10)	1.0	96196.10*	$2\frac{3}{2}D^3D$	3.0	96798.00*	$2\frac{3}{2}D^3D$
136072.882	-	2.35(-11)	2.0	.00*	$1\frac{3}{2}F$	4.0	734.70*	$1\frac{3}{2}F$
46208.801	-	8.40(-8)	.0	13122.80*	$1\frac{3}{2}P$	2.0	10959.30*	$1\frac{1}{2}D$
43849.616	4.09(-3)	1.18(-7)	1.0	13239.20*	$1\frac{3}{2}P$	2.0	10959.30*	$1\frac{1}{2}D$
40005.097	9.55(-3)	5.34(-9)	2.0	10959.30*	$1\frac{1}{2}D$	2.0	13458.30*	$1\frac{3}{2}P$
29380.413	6.32(-3)	7.34(-8)	2.0	100200.70*	$2\frac{3}{2}D^1D$	3.0	96798.00*	$2\frac{3}{2}D^3D$
26387.775	1.55(-3)	8.30(-8)	2.0	96412.10*	$2\frac{3}{2}D^3D$	2.0	100200.70*	$2\frac{3}{2}D^1D$
24964.472	9.93(-3)	1.45(-7)	1.0	96196.10*	$2\frac{3}{2}D^3D$	2.0	100200.70*	$2\frac{3}{2}D^1D$
20266.519	-	1.61(-6)	2.0	13458.30*	$1\frac{3}{2}P$	4.0	18391.20*	$1\frac{1}{2}G$
13451.831	-	6.17(-4)	2.0	10959.30*	$1\frac{1}{2}D$	4.0	18391.20*	$1\frac{1}{2}G$
9777.652	-	2.09(-4)	2.0	10959.30*	$1\frac{1}{2}D$	4.0	734.70*	$1\frac{3}{2}F$
9401.308	3.92(-2)	4.64(-5)	2.0	10959.30*	$1\frac{1}{2}D$	3.0	325.40*	$1\frac{3}{2}F$
9122.167	2.16(-2)	3.39(-5)	2.0	.00*	$1\frac{3}{2}F$	2.0	10959.30*	$1\frac{1}{2}D$
7857.249	-	3.46(-2)	2.0	13458.30*	$1\frac{3}{2}P$	4.0	734.70*	$1\frac{3}{2}F$
7741.524	-	3.28(-2)	1.0	13239.20*	$1\frac{3}{2}P$	3.0	325.40*	$1\frac{3}{2}F$
7618.228	-	5.34(-2)	.0	13122.80*	$1\frac{3}{2}P$	2.0	.00*	$1\frac{3}{2}F$
7612.369	7.64(-4)	1.06(-2)	2.0	13458.30*	$1\frac{3}{2}P$	3.0	325.40*	$1\frac{3}{2}F$
7551.247	2.41(-5)	1.85(-2)	1.0	13239.20*	$1\frac{3}{2}P$	2.0	.00*	$1\frac{3}{2}F$
7428.313	1.84(-4)	1.66(-3)	2.0	.00*	$1\frac{3}{2}F$	2.0	13458.30*	$1\frac{3}{2}P$
5662.065	2.76(-2)	2.88(-5)	4.0	734.70*	$1\frac{3}{2}F$	4.0	18391.20*	$1\frac{1}{2}G$
5533.784	1.78(-2)	7.14(-7)	3.0	325.40*	$1\frac{3}{2}F$	4.0	18391.20*	$1\frac{1}{2}G$
5435.872	-	5.61(-5)	2.0	.00*	$1\frac{3}{2}F$	4.0	18391.20*	$1\frac{1}{2}G$
3446.836	-	1.32(-1)	.0	42462.10*	$1\frac{1}{2}S$	2.0	13458.30*	$1\frac{3}{2}P$
3420.993	3.87(-1)	-	.0	42462.10*	$1\frac{1}{2}S$	1.0	13239.20*	$1\frac{3}{2}P$
3173.403	-	1.11(1)	.0	42462.10*	$1\frac{1}{2}S$	2.0	10959.30*	$1\frac{1}{2}D$
2354.321	-	1.96(-2)	.0	42462.10*	$1\frac{1}{2}S$	2.0	.00*	$1\frac{3}{2}F$
1861.019	4.59(-14)	-	.0	42462.10*	$1\frac{1}{2}S$	1.0	96196.10*	$2\frac{3}{2}D^3D$
1853.568	-	5.84(-2)	.0	42462.10*	$1\frac{1}{2}S$	2.0	96412.10*	$2\frac{3}{2}D^3D$
1731.944	-	2.16(1)	.0	42462.10*	$1\frac{1}{2}S$	2.0	100200.70*	$2\frac{3}{2}D^1D$
1281.708	-	5.73(0)	2.0	96412.10*	$2\frac{3}{2}D^3D$	4.0	18391.20*	$1\frac{1}{2}G$
1275.400	1.15(-13)	9.60(-2)	3.0	96798.00*	$2\frac{3}{2}D^3D$	4.0	18391.20*	$1\frac{1}{2}G$
1222.352	-	1.30(3)	2.0	100200.70*	$2\frac{3}{2}D^1D$	4.0	18391.20*	$1\frac{1}{2}G$
1208.637	1.43(-7)	1.11(2)	1.0	96196.10*	$2\frac{3}{2}D^3D$	2.0	13458.30*	$1\frac{3}{2}P$
1205.490	1.19(-8)	2.59(2)	2.0	13458.30*	$1\frac{3}{2}P$	2.0	96412.10*	$2\frac{3}{2}D^3D$
1205.445	7.30(-10)	3.49(2)	1.0	13239.20*	$1\frac{3}{2}P$	1.0	96196.10*	$2\frac{3}{2}D^3D$
1203.756	2.44(-10)	-	.0	13122.80*	$1\frac{3}{2}P$	1.0	96196.10*	$2\frac{3}{2}D^3D$
1202.315	2.33(-8)	3.91(1)	1.0	13239.20*	$1\frac{3}{2}P$	2.0	96412.10*	$2\frac{3}{2}D^3D$
1200.634	-	1.58(2)	.0	13122.80*	$1\frac{3}{2}P$	2.0	96412.10*	$2\frac{3}{2}D^3D$
1199.908	5.94(-8)	3.12(2)	2.0	13458.30*	$1\frac{3}{2}P$	3.0	96798.00*	$2\frac{3}{2}D^3D$
1196.762	-	1.61(2)	1.0	13239.20*	$1\frac{3}{2}P$	3.0	96798.00*	$2\frac{3}{2}D^3D$
1173.202	9.67(-6)	5.32(0)	1.0	96196.10*	$2\frac{3}{2}D^3D$	2.0	10959.30*	$1\frac{1}{2}D$
1170.237	1.15(-6)	1.60(1)	2.0	10959.30*	$1\frac{1}{2}D$	2.0	96412.10*	$2\frac{3}{2}D^3D$
1164.976	4.40(-6)	6.39(0)	2.0	10959.30*	$1\frac{1}{2}D$	3.0	96798.00*	$2\frac{3}{2}D^3D$
1152.839	1.03(-5)	2.85(1)	2.0	13458.30*	$1\frac{3}{2}P$	2.0	100200.70*	$2\frac{3}{2}D^1D$
1149.934	3.50(-6)	3.12(-1)	1.0	13239.20*	$1\frac{3}{2}P$	2.0	100200.70*	$2\frac{3}{2}D^1D$
1148.397	-	8.10(-1)	.0	13122.80*	$1\frac{3}{2}P$	2.0	100200.70*	$2\frac{3}{2}D^1D$
1120.556	1.53(-7)	1.11(3)	2.0	10959.30*	$1\frac{1}{2}D$	2.0	100200.70*	$2\frac{3}{2}D^1D$
1045.179	-	4.92(2)	2.0	96412.10*	$2\frac{3}{2}D^3D$	4.0	734.70*	$1\frac{3}{2}F$
1043.072	-	7.74(2)	1.0	96196.10*	$2\frac{3}{2}D^3D$	3.0	325.40*	$1\frac{3}{2}F$
1040.980	1.80(-9)	1.79(3)	3.0	96798.00*	$2\frac{3}{2}D^3D$	4.0	734.70*	$1\frac{3}{2}F$
1040.727	8.19(-8)	1.17(3)	2.0	96412.10*	$2\frac{3}{2}D^3D$	3.0	325.40*	$1\frac{3}{2}F$
1039.543	1.79(-8)	1.57(3)	1.0	96196.10*	$2\frac{3}{2}D^3D$	2.0	.00*	$1\frac{3}{2}F$
1037.214	1.66(-8)	6.83(2)	2.0	.00*	$1\frac{3}{2}F$	2.0	96412.10*	$2\frac{3}{2}D^3D$
1036.564	1.42(-9)	5.13(2)	3.0	325.40*	$1\frac{3}{2}F$	3.0	96798.00*	$2\frac{3}{2}D^3D$
1033.079	2.25(-9)	4.94(1)	2.0	.00*	$1\frac{3}{2}F$	3.0	96798.00*	$2\frac{3}{2}D^3D$
1005.369	-	1.52(0)	2.0	100200.70*	$2\frac{3}{2}D^1D$	4.0	734.70*	$1\frac{3}{2}F$
1001.249	1.22(-5)	9.02(0)	2.0	100200.70*	$2\frac{3}{2}D^1D$	3.0	325.40*	$1\frac{3}{2}F$
997.997	6.19(-6)	8.69(-1)	2.0	.00*	$1\frac{3}{2}F$	2.0	100200.70*	$2\frac{3}{2}D^1D$