On the use of Bowen compensators for polarimetry

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Abstract. — For high precision polarimetry it is very desirable to observe with a telescope that is free of instrumental polarization. Yet it is often inevitable to introduce optical devices which generate crosstalk between the Stokes Q, U components and the V component due to phase retardations upon reflection at mirror surfaces. We show that Bowen compensators are well suited to compensate for these phase changes. They consist of 2 rotatable $\lambda/8$ phase retarder plates, and it can be shown that they act on polarized light either by a rotation of the axes of the polarization ellipse followed by a prescribed, desired phase change or by a phase change followed by rotation of the ellipse. We present applications of Bowen compensators in the Gregory Coudé Telescope at the Observatorio del Teide/Tenerife.

Key words: instrumentation: polarimeters — techniques: polarimetric

1. Introduction

We report here on the application of Bowen compensators to eliminate instrumental polarization in the solar Gregory Coudé Telescope (GCT) at the Observatorio del Teide on Tenerife. Already in the early seventies, Wiehr (1971) and Wiehr & Rossbach (1974) had implemented a Bowen compensator at the GCT to cope with the instrumental polarization of the telescope alone. Martínez Pillet & Sánchez Almeida (1991) have suggested recently to make this telescope polarization free year round, i.e. for any declination, by introducing a properly oriented $\lambda/2$ retarder plate between the two plane 45° mirrors of the optical system that cause, upon reflection, linear polarization and crosstalk between the components (Q, U) and the V component of the Stokes vector $(I, Q, U, V)^{\mathrm{T}}$ (Chandrasekhar 1960; Shurcliff 1962). Here, I is the total intensity, Q and U describe the linearly polarized part of the light beam, V the circularly polarized part, and the symbol '$^{\mathrm{T}}$' denotes the transposed. Sánchez Almeida et al. (1995) demonstrated that the suggested device does indeed work in praxis. This makes the GCT an ideal instrument for solar polarization measurements with analyzers mounted in its focal plane, or close to it.

As the word is saying: "Opportunity makes the thief". Evolving observational requirements, e.g. to have in a coudé system a non-rotating image at the focal plane for time sequences or to implement image scanners at low expenditure for spectroscopy of two-dimensional fields of view, involves the use of further optical elements, essentially plain mirrors which introduce again instrumental polarization.

The most serious effect upon reflection at mirror surfaces is the difference of the change of phase between the electric vectors parallel and perpendicular to the plane of incidence. This produces crosstalk between the Stokes U and V parameters, i.e. it transforms linearly polarized light into circularly polarized light and vice versa. The crosstalk may be determined in the laboratory and the observations may be corrected for it during the data reduction. Yet then both parameters U and V must be measured, in most cases simultaneously, which is difficult. Often only Stokes I and V are of interest. We have therefore chosen to eliminate, as much as possible, the polarization introduced by additional optics by means of Bowen compensators.

Bowen compensators may be used in any optical device whose $U \leftrightarrow V$ crosstalk is to be eliminated. They have been recommended by Babcock (1962) and Jäger & Oetken (1983). But they are not much in use, possibly because they appear intricate. Therefore we shall give in the following Sect. 2 a description of the properties of Bowen compensators, using the formalism of Mueller matrices. These transform the incoming Stokes vector into the outgoing vector. The outcome of this will be that Bowen compensators are actually easy to handle. Section 3, then, deals with specific applications and Sect. 4 concludes the paper.

2. The Bowen compensator

A Bowen compensator consists of two linear phase retarder plates, each possessing a retardation of $\lambda/8$, or possibly little more. (If the retardation is much larger, e.g. $\lambda/4$, the adjustment of the plates becomes simply more critical.) For the ease of notation we will restrict the discussion essentially to exact $\lambda/8$ plates. The extreme phase shifts achievable are then $\pm 90°$ by turning the fast axes of the retarders parallel. For large instrumental shifts between $-180°$ and $-90°$ or between $+90°$ and $+180°$ one will adjust the Bowen compensators to give a total shift (of instrument plus compensator) of $\pm 180°$.

We use the convention that angles are measured with respect to the vertical in the plane perpendicular to the light beam. When looking towards the light source we define a counterclockwise rotation as positive. Figure 1 depicts schematically a Bowen compensator, the orientation of the axes of the retarder plates, and the definition of various angles.
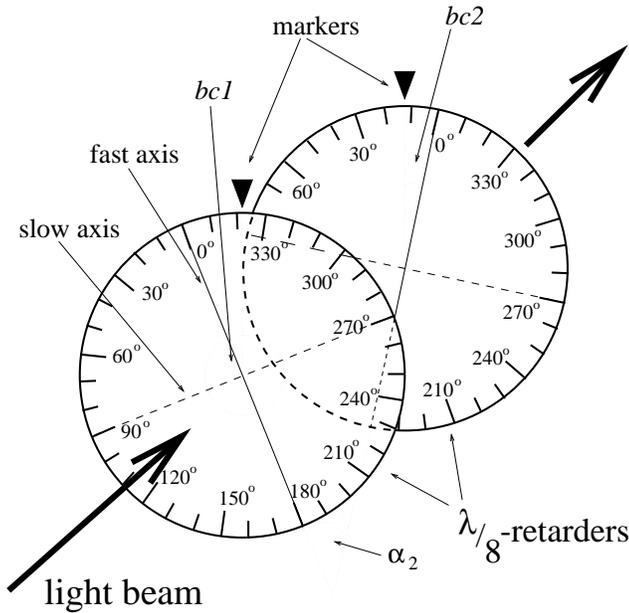

Fig. 1. Bowen compensator, schematically, with definition of angles relative to the vertical direction and with orientation of fast axes

We remind that a rotation of the co-ordinate system by an angle α involves the Mueller matrix

$$R_\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\alpha & \sin 2\alpha & 0 \\ 0 & -\sin 2\alpha & \cos 2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{1}$$

A retarder plate with retardation by an angle ϵ, with the fast axis oriented vertically, is described by the matrix

$$M_\epsilon = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \epsilon & \sin \epsilon \\ 0 & 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix} . \tag{2}$$

Furthermore, we adopt that the absorption by a real retarder plate is independent of the state of polarization of the light.

The effect of a Bowen compensator may then be decomposed into the subsequent operations (cf. Fig. 1):

1. rotation of the co-ordinate system by an angle α_1;
2. retardation by the first $\lambda/8$ plate;
3. rotation by an angle α_2;
4. retardation by the second $\lambda/8$ plate;
5. rotation back to the original co-ordinate system, i.e. by $-(\alpha_1 + \alpha_2)$.

Thus, apart from a constant factor k representing the absorption, the Mueller matrix of a Bowen compensator is

$$M_{\text{BC}} = R_{-(\alpha_1+\alpha_2)} \cdot M_{\lambda/8} \cdot R_{\alpha_2} \cdot M_{\lambda/8} \cdot R_{\alpha_1} . \tag{3}$$

The expressions for the matrix elements are rather lengthy. It may suffice, for further use, to give here few elements:

$$m_{41} = 0 , \tag{4}$$

$$\begin{aligned} m_{22} &= \cos(2\alpha_1 + 2\alpha_2)\cos(2\alpha_1)\cos(2\alpha_2) \\ &\quad + 1/\sqrt{2}\,\sin(2\alpha_1 + 2\alpha_2)\cos(2\alpha_1)\sin(2\alpha_2) \\ &\quad - 1/\sqrt{2}\,\cos(2\alpha_1 + 2\alpha_2)\sin(2\alpha_1)\sin(2\alpha_2) \\ &\quad + 1/2\,\sin(2\alpha_1 + 2\alpha_2)\sin(2\alpha_1)\cos(2\alpha_2) \\ &\quad - 1/2\,\sin(2\alpha_1 + 2\alpha_2)\sin(2\alpha_1) , \end{aligned} \tag{5}$$

$$\begin{aligned} m_{24} &= 1/\sqrt{2}\,\cos(2\alpha_1 + 2\alpha_2)\sin(2\alpha_2) \\ &\quad - 1/2\,\sin(2\alpha_1 + 2\alpha_2)\cos(2\alpha_2) \\ &\quad - 1/2\,\sin(2\alpha_1 + 2\alpha_2) , \end{aligned} \tag{6}$$

$$m_{44} = 1/2 - 1/2\cos(2\alpha_2) . \tag{7}$$

We have made use of the retardation by a $\lambda/8$ plate, i.e. $\cos\epsilon = \sin\epsilon = 1/\sqrt{2}$ in the Mueller matrix of Eq. (2) above.

It may be shown, though by somewhat lengthy trigonometric manipulations, that the action of a Bowen compensator of Eq. (3) is equivalent to a rotation of the axes of the (partially) elliptically polarized light by an angle $-\beta$ with matrix R_β followed by a phase retardation by an angle ϵ_{BC}, that is, by means of Eqs. (1) and (2)

$$\begin{aligned} M_{\text{BC}} &= M_{\epsilon\text{BC}} \cdot R_\beta \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\beta & \sin 2\beta & 0 \\ 0 & -\sin 2\beta \cos\epsilon_{\text{BC}} & \cos 2\beta \cos\epsilon_{\text{BC}} & \sin\epsilon_{\text{BC}} \\ 0 & \sin 2\beta \sin\epsilon_{\text{BC}} & -\cos 2\beta \sin\epsilon_{\text{BC}} & \cos\epsilon_{\text{BC}} \end{pmatrix} . \end{aligned} \tag{8}$$

The angle $-\beta$ is a physical rotation of the polarization of the incoming light with respect to the fixed co-ordinate system. So, although it is *clockwise* a rotation matrix with the same signs as in Eq. (1) has to be used. Comparing now the matrix elements of (3) and (8) one obtains from (7) for a desired retardation ϵ_{BC}

$$\alpha_2 = 1/2 \arccos(1 - 2\cos\epsilon_{\mathrm{BC}}) \qquad (9)$$

and

$$\alpha_1 = 1/2 \arctan(\sqrt{2}\tan\alpha_2) - \alpha_2 . \qquad (10)$$

Another possible combination is $\alpha_1' = -\alpha_1$ and $\alpha_2' = -\alpha_2$. Inspection of the matrix element m_{33} in (3) shows that then $\beta' = -\beta$. The rotation by $-\beta$ is inevitable. But it is a known angle from the matrix element m_{22} in Eqs. (5) and (8) and it turns out to be small, $|\beta| < 10°$ (see below).

The above representation, Eq. (8), of a Bowen compensator by rotation of the polarization axes of the incoming light followed by a phase retardation is adequate if the compensator is mounted in front of the device whose phase change is to be compensated. (We apply Bowen compensators in this way at the GCT on mechanical grounds.) In cases where the compensator is to be mounted behind the optical device one takes the representation in which the action of the Bowen compensator is equivalent to a desired phase change ϵ_{BC} followed by a rotation of the incoming polarized light by an angle $-\beta$ or

$$M_{\mathrm{BC}} = R_\beta \cdot M_{\epsilon\,\mathrm{BC}}$$
$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\beta & \sin 2\beta \cos\epsilon_{\mathrm{BC}} & \sin 2\beta \sin\epsilon_{\mathrm{BC}} \\ 0 & -\sin 2\beta & \cos 2\beta \cos\epsilon_{\mathrm{BC}} & \cos 2\beta \sin\epsilon_{\mathrm{BC}} \\ 0 & 0 & -\sin\epsilon_{\mathrm{BC}} & \cos\epsilon_{\mathrm{BC}} \end{pmatrix} . \qquad (11)$$

Here, from Eqs. (7), (8), and (11), α_2 is the same as above in Eq. (9), while α_1 becomes

$$\alpha_1 = 1/2 \arctan(1/\sqrt{2}\tan\alpha_2) . \qquad (12)$$

The difference between this α_1 and the one from the other representation, Eq. (10), is small, about $1°$.

A possible realisation of a Bowen compensator is sketched in Fig. 1. The view here is along the light path. The markers are oriented vertically. To achieve a retardation ϵ_{BC} one sets the angle $bc1$ of the fast axis of the first $\lambda/8$ plate with respect to the marker to $bc1 = \alpha_1$ and the angle $bc2$ of the second $\lambda/8$ plate to $bc2 = \alpha_2 + \alpha_1$. The angle α_2 of the second fast axis with respect to the first is thus $bc2 - \alpha_1 = \alpha_2$.

Figure 2 shows in the upper panel settings of $bc1$ and $bc2$ for required phase retardations ϵ_{BC} from $0°$ to $90°$. In the example of Fig. 1 we have $bc1 \approx 338° \,\widehat{=}\, -22°$ and $bc2 = 12°$, thus $\alpha_2 = 34°$. The retardation is, from Fig. 2, $\epsilon_{\mathrm{BC}} = 72°$. The lower panel of Fig. 2 gives the rotation angle β. In the same example we have $\beta \approx 9.7°$. For $\epsilon_{\mathrm{BC}} = 0°$ it suffices to orient the fast axes of the $\lambda/8$

plates perpendicularly to each other, $\alpha_2 = 90°$ for any α_1, and $\beta = 0°$ results from Eq. (5), which was expected. Likewise, for $\epsilon_{\mathrm{BC}} = 90°$ one has $\alpha_1 = \alpha_2 = 0°$ from (9) and (10) and $\beta = 0°$ from (5).
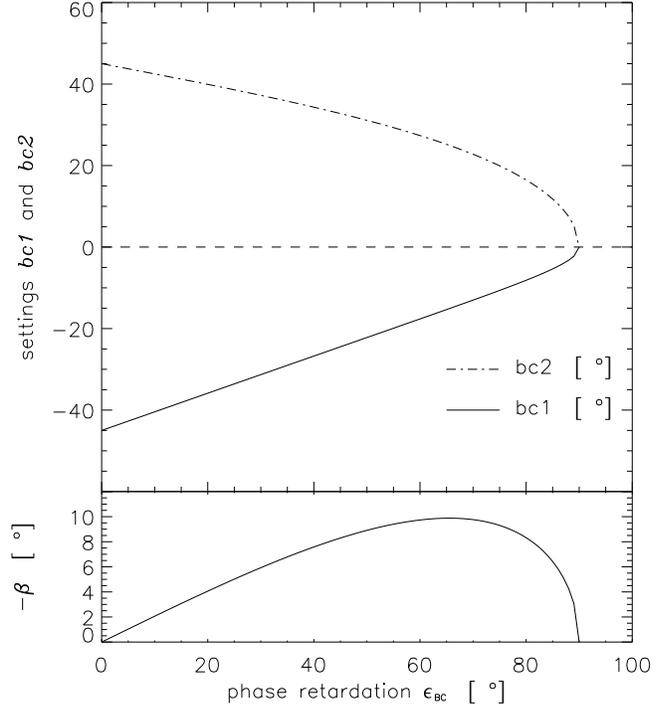

Fig. 2. Orientations $bc1$ and $bc2$ of the $\lambda/8$ plates for a prescribed phase retardation ϵ_{BC} (upper panel) and corresponding rotation $-\beta$ of the polarization of the incoming light (lower panel)

To produce negative phase shifts, $-90° \leqq \epsilon_{\mathrm{BC}} < 0°$, one rotates both $\lambda/8$ plates by $90°$, i.e. $\alpha_1(-\epsilon_{\mathrm{BC}}) = \alpha_1(\epsilon_{\mathrm{BC}}) + 90°$. This interchanges their fast and slow axes. Inspection of the matrix elements in (3) shows that m_{34}, m_{42}, and m_{43} in Eq. (8) change sign, while all others remain unchanged, thus under this operation $\epsilon_{\mathrm{BC}} \rightarrow -\epsilon_{\mathrm{BC}}$, and $\beta(\epsilon_{\mathrm{BC}})$ is symmetric to $\epsilon_{\mathrm{BC}} = 0$.

3. Applications: compensation of image rotation, image scanning

A coudé telescope rotates the image uniformly by $360°$ per day. This can be compensated by a Dove prism or, in order to avoid chromatic aberrations, by a mirror system such as sketched in Fig. 3. This system has to be counter-rotated with half the angular velocity, i.e. with $180°$ per day. The device is mounted in front of the image plane. It may also be used to rotate the image into any desired orientation with respect to the post-focal instrumentation (spectrograph etc.). At the GCT we employ a very similar setup also for image scanning. The scanner consists again

of three mirrors with exactly the same orientation as in Fig. 3. But instead of rotating the device it is shifted in the direction perpendicular to the light beam and parallel to the plane of incidence. It may easily be seen that a displacement of this three-mirror system by a certain amount Δx shifts the image by $2\Delta x$, while the focal length of the telescope remains unchanged.
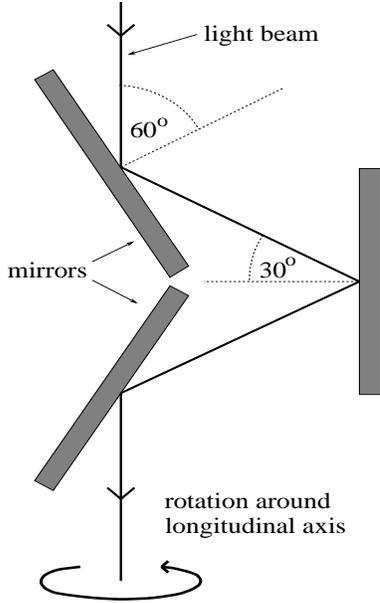

Fig. 3. Three-mirror system to compensate image rotation or to rotate the image in the focal plane

The essential point here is that the reflections introduce substantial polarization, mainly $U \leftrightarrow V$ crosstalk and some small amount of linear polarization. Since the planes of incidence of all three mirrors coincide the combined Mueller matrix of the image rotator IR is the product of the matrices of each mirror. The result is that of a single mirror which is (Capitani et al. 1989; Martínez Pillet & Sánchez Almeida 1991)

$$M_{\mathrm{IR}} = M_1 \cdot M_2 \cdot M_3$$
$$= \frac{R_\perp^2}{2} \begin{pmatrix} 1+X^2 & 1-X^2 & 0 & 0 \\ 1-X^2 & 1+X^2 & 0 & 0 \\ 0 & 0 & 2X\cos\epsilon & 2X\sin\epsilon \\ 0 & 0 & -2X\sin\epsilon & 2X\cos\epsilon \end{pmatrix} . \tag{13}$$

Here $X = R_\parallel / R_\perp$ with Frenel's amplitude reflection coefficients $R_\parallel$ and $R_\perp$ for the electric vector parallel and perpendicular to the plane of incidence, respectively, $R_\parallel = R_{\parallel,1} \cdot R_{\parallel,2} \cdot R_{\parallel,3}$, and $R_\perp = R_{\perp,1} \cdot R_{\perp,2} \cdot R_{\perp,3}$. ϵ is the sum of the phase retardations upon reflection, i.e.

$$\epsilon = \epsilon_{60°} + \epsilon_{30°} + \epsilon_{60°} = 2\epsilon_{60°} + \epsilon_{30°} . \tag{14}$$

We use mirrors produced with a silver plating technique (Tafelmaier Dünnschicht Technik) for which we measure $X \approx 0.90-1.03$ for the total image rotator at wavelengths of $480-770$ nm, somewhat dependent on wavelength, while the total reflectivity is $(R_\perp^2 + R_\parallel^2)/2 \approx 0.87-0.98$. With this X close to unity Eq. (13) may be approximated as

$$M_{\mathrm{IR}}$$
$$\approx R_\perp^2 X \begin{pmatrix} 1 & 1-X & 0 & 0 \\ 1-X & 1 & 0 & 0 \\ 0 & 0 & \cos\epsilon & \sin\epsilon \\ 0 & 0 & -\sin\epsilon & \cos\epsilon \end{pmatrix} . \tag{15}$$

For $X = 0.90$ the matrix elements $(1+X^2)/(2X)$ in (13) amount to 1.0056 and the elements $(1-X^2)/(2X)$ are 0.1056 instead of 0.100 in (15). The errors are thus small. With the measured X, the device introduces $2-10$ percent linear polarization through the $I \rightarrow Q$ crosstalk. But this is constant within a certain wavelength band, e.g. across a spectral line, and can thus be corrected for during the data reduction.

The phase retardation ϵ depends on wavelength. It can be measured in the laboratory by inserting the device between rotatable linear polarizers into a preferentially parallel light beam and by measuring the transmitted intensity which depends on the orientation of the polarizers with respect to the device (for the detailed procedure of the determination of ϵ see Koschinsky 1995). The result for the image rotator (and for the image scanner with identically mounted mirrors) of the GCT on Tenerife is shown in Fig. 4.
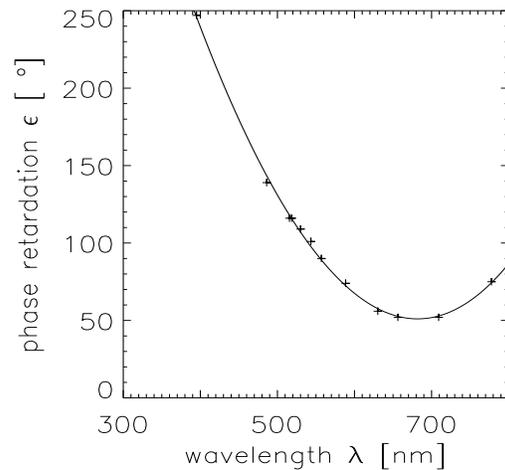

Fig. 4. Phase retardation produced by the three mirrors of the image rotator of the GCT. The crosses are the measurements, the solid curve is a third order polynomial fit to the crosses

A word about the use of mirrors with high reflectivity, $1 - R_\parallel^2 \ll 1$ and $1 - R_\perp^2 \ll 1$ is appropriate here. We
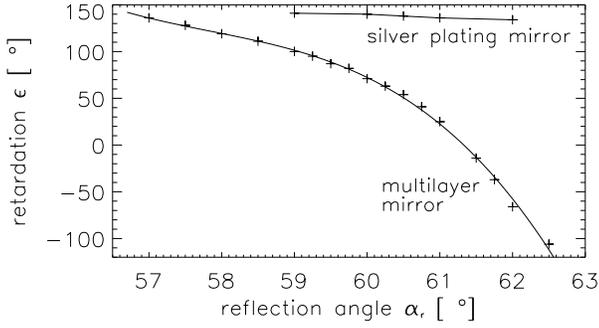
Fig. 5. Phase retardation and its dependence on the reflection angle for a single multi-layer mirror and for a silver plating mirror, measured with a He-Ne laser at 632.8 nm

have tested mirrors coated with multi-layers to give reflectivity close to 1.0 in the wavelength range 500−700 nm. Their retardation ϵ turned out to be extremely sensitive on the reflection angle α_r as is demonstrated in Fig. 5. The GCT has an f-ratio of 1:56, thus twice the aperture angle amounts to approximately 1°. From Fig. 5 one sees that for the multi-layer mirrors ϵ changes from about 90° at $\alpha_r = 59°\!.5$ to about 50° at $\alpha_r = 60°\!.5$. The device (image rotator or image scanner) would thus act as a depolarizer if mounted in an (only weakly) convergent light beam.

The actual, wavelength dependent settings of the $\lambda/8$ plates of a Bowen compensator in Fig. 4 are depicted in Fig. 6. We remind from the previous section that $-bc1$ and $-bc2$ (mod 360°) are also possible orientations. Not only the optimum values of $bc1$ and $bc2$ are given but also those orientations which would produce a crosstalk of 2 percent, or $(m_{42}^2 + m_{43}^2)^{1/2} = 0.02$ in Eq. (3), i.e. if ϵ_{BC} is too small or too large. It is seen that the adjustment of the $\lambda/8$ plates is not critical within $\pm 5°$ if one allows a small amount of crosstalk.

Actual retarder plates will not be fully achromatic and will not have the exact desired phase retardation, in the present case of 45°. The settings of such non-perfect plates has to be calculated from Eqs. (3) and (8) with use of the correct retardances.

The transmission of the image rotator and of the image rotator plus Bowen compensator is given in Table 1 for few wavelengths. The light loss of the compensated device is thus 22−32 percent.

4. Conclusion

It is sometimes inevitable to introduce into the optical path of a telescope additional optical elements to enlarge its capabilities and cope with the observational needs. A typical case is a mirror device as above in Fig. (3) to compensate image rotation. The instrumental polarization produced by such additional elements, especially the
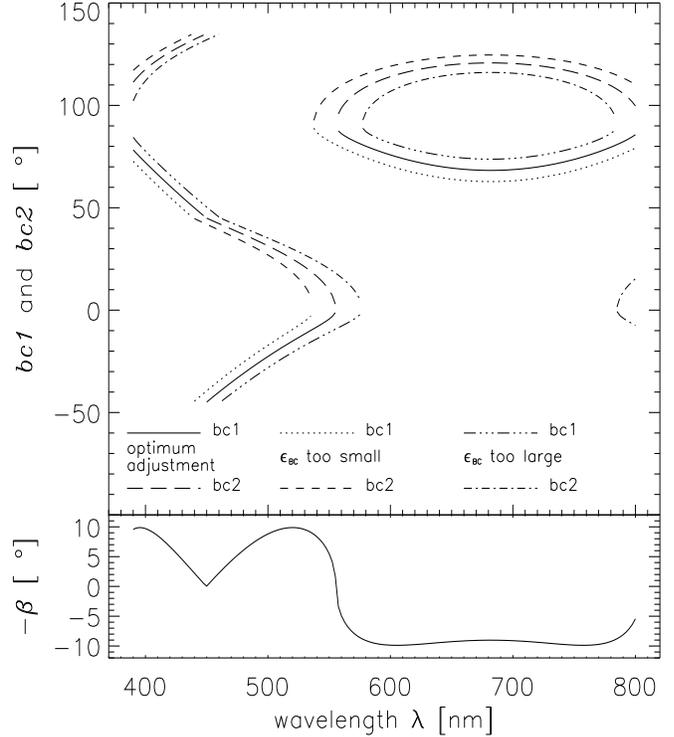

Fig. 6. Upper panel: orientation of the retarder plates with respect to the vertical vs. wavelength for the retardations of Fig. 4. Also shown are curves for which a $(Q,U) \leftrightarrow V$ crosstalk of 2 percent occurs. Lower panel: Rotation angle $-\beta$ vs. wavelength for the same retardances of Fig. 4

Table 1. Transmission of image rotator IR and image rotator plus Bowen compensator BC

λ (nm)	IR	IR + BC
525	0.88	0.68
630	0.88	0.78
656	0.85	0.75

$(Q,U) \leftrightarrow V$ crosstalk due to the phase changes upon reflection, can often be neutralized by a Bowen compensator. We have shown that a Bowen compensator is easy to handle, once it became clear how it acts. We have successfully implemented it in the Gregory Coudé Telescope on Tenerife.

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