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Global astrometry with OSI

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Abstract. — The Orbiting Stellar Interferometer is a proposed space-based interferometer which makes very accurate wide-angle astrometric measurements on several thousand stars. As opposed to Hipparcos, OSI does not have a "basic angle" between two separate field-of-views but three separate interferometers and a highly accurate metrology system. The questions addressed in this paper are whether it is possible to perform global astrometry with such an instrument and what kind of results could then be expected. A time-independent observing scenario, leading to a solution for positions and ignoring effects of proper motion and parallax, as well as a global astrometric data analysis technique are presented and a covariance study is performed. We show that a gain of a factor of at least 4 on the final astrometric accuracy could be achieved if such a technique was used. With an initial single measurement precision of 10 micro-arcseconds, our solution leads to an astrometric accuracy of 2-3 micro-arcseconds.

Key words: astrometry — instrumentation: interferometers — techniques: interferometry — methods: data analysis

1. Introduction

The first astrometric mission from space was performed by Hipparcos between 1989 and 1992 (Perryman et al. 1992). The positions, proper motions and parallaxes of 120.000 stars were measured by means of accurate wide-angle measurements (58°) at the level of 1 milli-arcsecond (mas) for positions and parallaxes and 1 mas per year for proper-motions. The Orbiting Stellar Interferometer (OSI, Shao 1993) is a proposed space-based optical interferometer developed at JPL and dedicated to extremely accurate (3 to 30 micro-arcseconds (μas)) wide-angle astrometry on objects as faint as magnitude 20. In addition to astrometric measurement, OSI is also capable of performing synthesis imaging. OSI uses three collinear interferometers; two are used to lock on guide stars and provide the attitude reference, the third records high-precision data on science targets. For each interferometer, the raw data is the positions of the delay-lines used to equalize the optical path lengths between the two arms of each interferometer and the phase of dispersed fringes. OSI thus makes relative measurements between stars via the attitude determination of the spacecraft, as opposed to direct relative measurements in the case of Hipparcos. Since these measurements are being made over wide angles, the instrument has the intrinsic capability for global astrometry, as explained in Sect. 2. However, the global astrometric model used for Hipparcos is not readily adaptable to OSI, because of the latter's peculiar astrometric mode as described in Sect. 3. The mathematical model is presented in Sect. 4 and its results discussed in Sect. 5.

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2. Global astrometry

Global astrometry is a technique which combines an observation scenario and a data reduction method. Its purpose is the achievement of a global closure condition over the whole celestial sphere in order to provide a coherent reference frame of objects. A direct consequence of the construction of such a closure condition is the improvement of the astrometric accuracy of the parameters of the observed objects: indeed, the stars are consistently and redundantly tied together firstly by the observations and secondly by means of a least-squares solution for the observational unknowns. Building an inertial reference frame, i.e. a set of stars uniformly distributed over the entire celestial sphere and linked to extra-galactic sources, has further dramatic astrophysical consequences, such as the possibility of writing the equations of dynamics with no inertial terms. Moreover, the computation of absolute parallaxes is a feature specific to the global astrometric data reduction technique. It could seem surprising that one could derive absolute quantities from relative measurements but it is indeed the case when large angles can be bridged accurately and observations can be made several times over a long period of time. The reader is advised to refer to the paper by Lindegren et al. (1994) for a more detailed explanation.

A key objective in global astrometry is the avoidance of regional biases and systematic errors. Wide-angle measurements must then be performed with the same high precision as narrow-angle ones. Moreover, it is much better if they are made over the 4π steradians of the celestial sphere, hence the advantage of bringing the instrument up to space, which is also a solution to the limitation of the astrometric accuracy by atmospheric effects (Lindegren 1980). The theory of global astrometry was successfully demonstrated for the first time by Hipparcos which, after 37 months of observations, led to a self-coherent solution for positions, proper motions and absolute parallaxes for about 120.000 stars, down to magnitude 12. In the case of Hipparcos, the coherent optical frame was built without direct links to extra-galactic sources. In contrast, OSI can observe objects down to magnitude 20 so that its reference grid incorporates distant objects such as quasars and can thus tie the optical and radio frames.

Interferometers have intrinsic advantages over traditional telescopes for astrometry such as the easier control of systematic errors in the instrument and the decoupling between sensitivity and resolution. Several space-based interferometers are now being proposed for global astrometry (Shao 1993; Reasenberg et al. 1994; Lindegren & Perryman 1995). Although each of these interferometers has a unique observing method, OSI is the only one among them which does not directly measure relative angular positions for pairs of stars. This peculiar astrometric mode is discussed in the next section.

3. OSI's astrometric mode

OSI (Rayman et al. 1992; Shao 1993) uses three collinear Michelson interferometers, each composed of two siderostats, one delay-line and a beam combiner (Fig. 1). The siderostats which collect starlight are articulated in azimuth and elevation so that they can observe over a 30 by 30 degrees field and make wide-angle measurements possible. The arrangement of the siderostats is such that all baseline lengths from 0.5 to 7 meters in increments of 0.5 m can be achieved. The instrument is completed by two metrology systems, external and internal. In OSI's astrometric mode, all three interferometers do not have the same role: two of them are guide interferometers dedicated to the attitude determination of the spacecraft and the third one is the science interferometer which records data on faint stars. Guide interferometers lock up on reference bright stars by positioning the moving delay lines so that the lock be maintained on the centre of the white light fringe given by the interferometer at the beam combiner. The guide interferometers thus determine the orientation of the spacecraft in inertial space, provided that both reference stars do not lie parallel to the baseline. The external metrology system gives an extremely accurate measure of

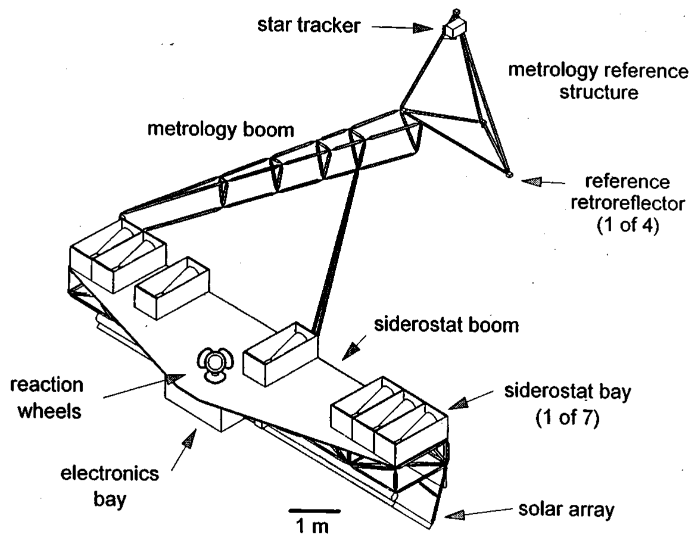

Fig. 1. The Orbiting Stellar Interferometer. The 7 siderostats lie on the same boom. The metrology reference triangle is located at the end of the metrology boom

the lengths of the three baselines and their relative orientation. The internal metrology is used to measure the position of the white light fringe and to control the delay line. Non-collinearity of the baselines and effects due to the non-rigidity of the spacecraft at the nanometer level are taken into account by both the internal and external metrology. The metrology systems are heterodyne systems using relative laser gauges for monitoring either the distance between two retro-reflectors or the pathlength differences between two arms of the interferometer. Once the attitude of the satellite and the length of each baseline are known, the third interferometer can be used to observe preselected targets and record data. In the case of the science interferometer, the interference fringes are dispersed by a spectrometer, which gives the information necessary to the positioning of the science delay line, i.e. the distance of the delay line from the centre of the white light fringe.

Hipparcos performed differential angular measurements between stars at a given time. By means of its complex mirror, stars from regions of the sky separated by approximately 58 degrees were brought together on the focal grid. The operating mode for OSI is rather different. Data is recorded for each star independently and later bound together by the attitude of the spacecraft at the time of observations. OSI's astrometric measurement for each interferometer is the optical path difference between its two arms:

$$d = \boldsymbol{B} \cdot \boldsymbol{S} + c \tag{1}$$

where $\boldsymbol{B}$ is the baseline vector, of length B, c the offset due to optics misalignment or thermal variations and $\boldsymbol{S}$ is the unit vector defining the direction of the star. Figure 2 shows the principle of this measurement. The delay-line position d can be directly related to the direction of the observed star by Eq. (1). However, this direction is measured only in the plane defined by the baseline and the star. At least one more observation with another baseline orientation should be made so as to fully determine the true position of the star.

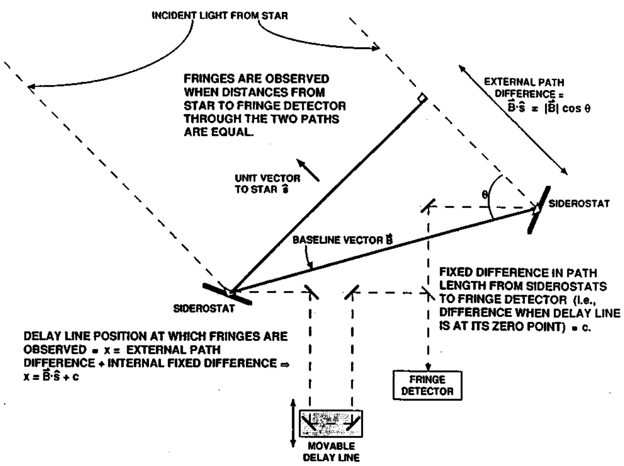

Fig. 2. Principle of an astrometric measurement with OSI

OSI's main astrometric goal is to reach a single measurement accuracy of 10 μas on stars as faint as magnitude 20. Equation (1) shows that it corresponds to an accuracy on the measurement of the delay-line position of 340 picometers with a 7 m baseline. Relative laser gauge metrology has been demonstrated at the picometer level at JPL (Gürsel 1993, 1994) and should not be an obstacle to such a challenge.

4. Model and method

4.1. *Hypotheses*

The hypotheses of the model presented here are as follows. First, we consider the metrology measurements as sufficiently accurate so that the baseline lengths B_i and the offsets c_i could be assumed constant, i.e. the error made on their determination is much smaller than the astrometric precision we are considering. Second, the baselines are assumed to be strictly collinear at all time. For the latter reason and because OSI's interferometers lie on a linear siderostat boom (the main body of the spacecraft, see Fig. 1), the representation of the satellite in space is given by the right ascension and the declination of the point towards which the main body of the satellite, i.e. the siderostat boom, is pointed. Third, time dependent terms are removed from our equations. In other words, our model is focussed on a set of instantaneous and equally weighted observations. However, we show the time-dependent formalism which will be referenced to in a future paper that will take time into account and provide a complete global astrometric solution. We do not derive in this paper a solution for proper motions and parallaxes so that problems due to resolved binary systems for instance are not present here. However, we show that it is possible to perform a covariance study with two astrometric parameters for each star and two attitude parameters for the spacecraft. The predicted final accuracy on these parameters can then be derived and plotted against the number of observations.

4.2. *Star distribution and observation scenario*

Among the three or four thousand targets pointed by OSI, approximately 400 will compose a reference grid, dedicated to the attitude determination of the satellite. These relatively bright stars (magnitude < 12) are uniformly distributed on the sky so that at least 5 of them are present at all time in each 30×30 degree cell. Whatever direction OSI is pointed at, it can then quickly acquire two reference stars before switching among science targets with the third interferometer. If time-dependent effects were considered each observation would be associated with a precise instant, consistent with the full observing scenario. We had to define an instantaneous observation scenario anyway since OSI needs at least three stars in its "field-of-view".

We define an "observation" as the measurement of three stars lying within a 30×30 degree patch. The theoretical number of stars necessary to achieve the grid lock-up, that is to close the sky with triplets of observed stars can be calculated. The most optimistic case is when all stars are located at the three vertices of equilateral triangles of side 30 degrees. The surface of these triangles is 390 deg^2 so there are $\sim$ 105 such triangles on the sky, if we assume that a sphere can be patched by plane triangles. Each star must be assigned a weight of 1/6 since it is part of 6 different triangles. With 3 stars per triangle, $\sim$ 55 stars should be sufficient to cover the whole sky. In practice, it is shown later that many more stars are needed so as to increase the number of observations per star for instance. Furthermore, a perfect patching of the sky as described above is not possible and more stars are to be used.

The key parameter in global astrometric measurement is the ratio M of number of observations to number of stars, not the number of stars itself. A minimum number of stars is needed to achieve sky closure but then, a large number of observations is needed so that each star is observed several times and is part of different triplets. However, four hundred stars would lead to a typical design matrix of more than 10 million elements, a size which would cause memory overflows with our present algorithm. For a future paper which would present a solution for parallaxes, we are considering a new algorithm that would not suffer from this problem. The optimization of this patching of the sky is important for the derivation of the minimum value of the ratio M.

A typical optimized sky coverage simulation is shown in Fig. 3 where 110 stars uniformly distributed on the sky are part of 311 observations (triplets).

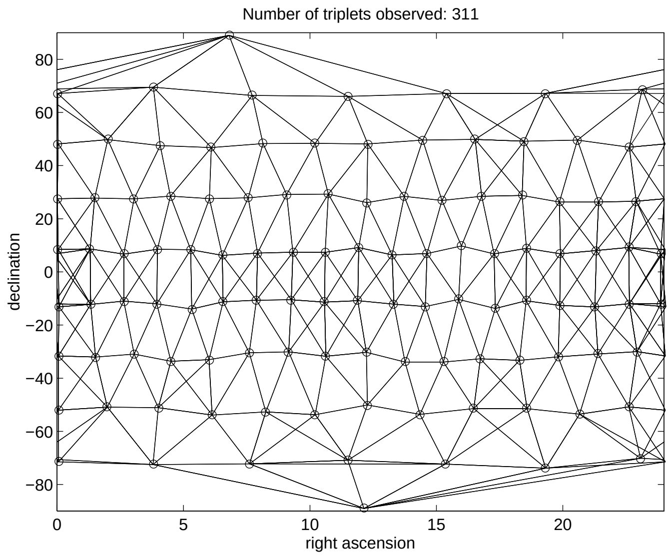

Fig. 3. Simulated sky coverage with 110 stars uniformly distributed on the celestial sphere, leading to 311 observations. The right ascension is in hours, the declination in degrees. This unconventional representation, instead of a classic projection was chosen for sake of clarity. One must keep in mind that stars around a pole are very close to each other whatever their right ascensions

4.3. *Linearized elementary observation equation*

Equation (1) is the elementary observation equation for a single interferometer: three such equations thus define the observations of a triplet. In the global astrometric process, astrometric quantities are known a priori by means of an input catalogue which has an accuracy generally much lower than the expected final one. In our case, these quantities are the coordinates of stars and the attitude parameters of the satellite which will also be solved for. In reality, the input catalogue is the most up-to-date compilation of stellar astrometric parameters. In our model, we generate an input catalogue from the simulated sky (cf. previous section) by adding a normally distributed error to each star parameter, whose standard deviation corresponds to the nominal accuracy of the catalogue, e.g. 1 mas for the Hipparcos catalogue. the input catalogue is used to estimate the value of the delay d. However, we are only interested in the difference between the observed value of this delay and the estimated one: the precision on this difference is given by the accuracy of the instrument. For this reason, a linearized observation equation is used, in which the unknowns for a star are given by small corrections (differences between observed and estimated values) to the preliminary astrometric parameters. These corrections correspond to the small change $\Delta\boldsymbol{S}$ of the coordinate direction and to the small change $\Delta\boldsymbol{n}$ of the direction (attitude) of the spacecraft at the given time, viz.

$$\Delta d = B(\boldsymbol{n}\cdot\Delta\boldsymbol{S} + \Delta\boldsymbol{n}\cdot\boldsymbol{S}) \qquad (2)$$

Each observation provides three delay line position measurements $d_{1,2,3}$, so that if $m_{\rm obs}$ is the number of observations, we have $3m_{\rm obs}$ measurements. As for unknown quantities, we have two attitude unknowns for each observation, i.e. $3m_{\rm obs}$ attitude unknowns and, if $m_{\rm s}$ is the total number of stars observed, we have $2m_{\rm s}$ astrometric parameters to be determined, corresponding to the position of each star. The condition for the over-determination of the system, which is a *sine qua non* condition for the obtaining of a meaningful solution, is $3m_{\rm obs} > 2m_{\rm obs} + 2m_{\rm s}$, i.e. $m_{\rm obs} > 2m_{\rm s}$. It would be $m_{\rm obs} > 5m_{\rm s}$ if proper motions and parallaxes were being solved for. In our case, we need at least a number of observations (of triplet of stars) greater than twice the number of observed stars.

A justification for the use of three interferometer can be consequently given: with only two interferometers, the left-hand side of Eq. (2) would be $2m_{\rm obs}$ and the over-determination condition, $0 > 2m_{\rm s}$, could then never be fulfilled.

4.4. *Astrometric parameters for stars*

Among all the parameters describing a star, five of them are purely astrometric ones. These are, for star i: α_i, δ_i= barycentric coordinates at the reference epoch T; $\mu_{\alpha i}$, $\mu_{\delta i}$ = proper motion components (where $\mu_{\alpha i}$ implicity includes the cos δ_i factor to make it a true arc); and ϖ_i = the parallax. The two parameters giving the position of the star are the only ones which do not depend on time. Although we will not use the other parameters in the following sections of this paper, we present here the complete equation giving the satellitocentric coordinate direction of a star, corrected for stellar aberration and gravitational deflection. The adopted units are: AU, year and radian. To first order in the small angles, the satellitocentric coordinate direction of a star i, $\boldsymbol{S}(t)$, can be written:

$$\boldsymbol{S} = \boldsymbol{r} + \boldsymbol{p}\mu_\alpha\tau + \boldsymbol{q}\mu_\delta\tau + \boldsymbol{s}\varpi \qquad (3)$$

(dropping the index i), where $\tau = t - T$ and we have introduced the four vectors

$$\boldsymbol{r} = \begin{pmatrix}\cos\delta\cos\alpha\\ \cos\delta\sin\alpha\\ \sin\delta\end{pmatrix}, \qquad (4)$$

$$\boldsymbol{p} = \begin{pmatrix}-\sin\alpha\\ \cos\alpha\\ 0\end{pmatrix}, \qquad (5)$$

$$\boldsymbol{q} = \begin{pmatrix}-\sin\delta\cos\alpha\\ -\sin\delta\sin\alpha\\ \cos\delta\end{pmatrix}, \qquad (6)$$

$$\boldsymbol{s} \;=\; \boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{b}) - \boldsymbol{b}\,. \tag{7}$$

$\boldsymbol{b}(t)$ is the position of the satellite relative to the solar system barycentre, expressed in AU and in the same (equatorial) system as used for the other vectors.

Let $\boldsymbol{x}$ be a vector containing the corrections to the five astrometric parameters, with elements $x^{(1)} = \Delta\alpha\cos\delta$, $x^{(2)} = \Delta\delta$, $x^{(3)} = \Delta\mu_\alpha$, $x^{(4)} = \Delta\mu_\delta$, $x^{(5)} = \Delta\varpi$. Then, the linearized equation for the satellitocentric coordinate of the considered star is:

$$\Delta\boldsymbol{S} = \boldsymbol{p}(x^{(1)} + x^{(3)}\tau) + \boldsymbol{q}(x^{(2)} + x^{(4)}\tau) + \boldsymbol{s}x^{(5)} \tag{8}$$

4.5. *Attitude parameters*

We need to add the terms relative to the attitude determination of the satellite to Eq. (8). The direction of OSI is determined by the right ascension and declination of the point it aims at, as explained before. This direction is chosen so that it minimizes the errors on the determination of the attitude. This is achieved when the direction of the stellar beam is perpendicular to the spacecraft direction. Therefore, for each triplet of star S_1, S_2, S_3, OSI points in a direction orthogonal to both the line joining the two guide stars (chosen as S_1 and S_2) and to the direction of the barycentre of these stars. This unit vector defining the orientation of the spacecraft is then:

$$\boldsymbol{n} = \frac{(\boldsymbol{S}_1 + \boldsymbol{S}_2)}{|\boldsymbol{S}_1 + \boldsymbol{S}_2|} \times \frac{(\boldsymbol{S}_1 - \boldsymbol{S}_2)}{|\boldsymbol{S}_1 - \boldsymbol{S}_2|} = \frac{\boldsymbol{S}_2 \times \boldsymbol{S}_1}{|\boldsymbol{S}_2 \times \boldsymbol{S}_1|} \tag{9}$$

where $\times$ denotes the cross product of two vectors. This vector can be rewritten in the convenient form:

$$\boldsymbol{n} = \begin{pmatrix} \cos\delta_n\cos\alpha_n \\ \cos\delta_n\sin\alpha_n \\ \sin\delta_n \end{pmatrix} \tag{10}$$

The variations of the attitude are then modeled by those of δ_n and α_n. We thus have:

$$\Delta\boldsymbol{n} = \boldsymbol{p}_n(x_n^{(1)}) + \boldsymbol{q}_n(x_n^{(2)}) \tag{11}$$

where

$$\boldsymbol{p}_n = \begin{pmatrix} -\sin\alpha_n \\ \cos\alpha_n \\ 0 \end{pmatrix}, \boldsymbol{q}_n = \begin{pmatrix} -\sin\delta_n\cos\alpha_n \\ -\sin\delta_n\sin\alpha_n \\ \cos\delta_n \end{pmatrix}, \tag{12}$$

and x_n is a vector containing the corrections of the attitude, which components are $x_n^{(1)} = \Delta\alpha_n\cos\delta_n$, $x_n^{(2)} = \Delta\delta_n$.

4.6. *Observation matrix*

Keeping only the time independent terms in the above equations (ignoring proper motion and parallax), we obtain the observation equation for our model. At the first order, $\boldsymbol{r}$ is a good approximation of $\boldsymbol{S}$. We thus have:

$$\Delta d = B[\boldsymbol{n}\cdot\boldsymbol{p}(x^{(1)}) + \boldsymbol{n}\cdot\boldsymbol{q}(x^{(2)}) + \boldsymbol{r}\cdot\boldsymbol{p}_n(x_n^{(1)}) + \boldsymbol{r}\cdot\boldsymbol{q}_n(x_n^{(2)})] \tag{13}$$

Each triplet of stars leads to three such equations, for which the attitude terms are the same, since, in our model, OSI performs three observations without changing its attitude. This define a sparse matrix A of dimensions $(3m_{\rm obs}, 2m_{\rm obs} + 2m_{\rm s})$. For the time being, all the observations are equally weighted; in consequence A is normalized by the standard deviation of the observations. To derive the accuracy by which the unknown parameters can be obtained, we first compute the design matrix (the matrix of the normal equations) A^TA. Because no reference direction on the sky has been chosen at this time, the design matrix is necessarily singular. In that case, it is either possible to choose a distant object for which parallax and proper motions are zero and its positions accurately known, or to find the pseudo-inverse of the design matrix by means of a singular value decomposition (SVD). We chose the second solution for our study. In that case, the pseudo-inverse of the design matrix is then the relevant covariance matrix of the system. Let D be the matrix of the measurements Δd_i normalized by the same factor as the observation matrix. The corrections to the attitude and to the astrometric parameters are the elements of the matrix $(A^TA)^+A^TD$, where the $^+$ denotes the pseudo-inverse of the considered matrix.

For the reason given above, namely the lack of reference directions, the design matrix presents a rank deficiency, which value can be more or less intuitively deduced. Betti & Sansó (1983) have carried out a mathematical study of this deficiency and have shown that it should be 6 if all 5 astrometric parameters are taken into account or 3 if only positions are being solved for. Let us briefly apply their method to our case. We have one single observation equation (Eq. (13)) per interferometer. When looking at this equation, it can be seen that only one independent observation can be made to determine the corresponding Δd_i. In the case of Hipparcos, the observable is an angle between a pair of stars: two fixed stars, on the equator for instance, form a reference frame from which the position of any star can be determined by two angle measurement. In the case of OSI, the observable is, once again, a delay line position. If two reference stars are fixed, it is also possible, with several measurements made in different planes, to determine the position of the third star. By fixing two stars on our net of stars, we fix 4 degrees of freedom when only positions are considered. But we have only one observation equation and therefore a rank deficiency of 3.

It is then convenient to check the rank of the design matrix, once the SVD is performed, since it is equal to the rank of the diagonal matrix which contains the singular values. If the rank is different from $2m_{\rm obs} + 2m_{\rm s} - 3$ the sky coverage is not satisfying, i.e. one or more stars are isolated. The expected result is then quite meaningless in terms of derived values but interesting in terms of number of stars vs number of observations. By this method, we try to define a statistical condition on the ratio M defined previously.

5. Results and discussion

The results consist of sets of standard deviations of astrometric and attitude parameters, and of small corrections to these parameters. The latter part of our results is used mostly for consistency check of the solutions. We present the results in the form of histograms of the distribution of the standard deviations obtained for the astrometric parameters of the stars and for the attitude parameters of the spacecraft. The computed corrections to the system parameters, i.e. what we would be theoretically solving for in the real data reduction process, are also presented. These runs consist in generating different skies with various numbers of stars and observations.

5.1. *Astrometric and attitude parameters*

Recall that the nominal measurement accuracy is $10\,\mu$as, i.e. 340 picometers for the delay line positions with a 7-m baseline. Our results enable us to evaluate statistically the factor by which the original accuracy is divided. Depending on the nature of the coverage of the sky, we obtain different results. For instance, with 119 stars bound by 372 observations ($M = 2.94$), the global treatment leads to a result for which the mean of the astrometric parameters accuracies distribution is $\sim$ 100 pm and its standard deviation $\sim$ 30 pm. This histogram can be seen in Fig. 4. Such an example shows that a significant factor can be gained for the final accuracy, greater than 3 in the present case.

The results we obtain can be subdivided into different categories. The first one corresponds to the case where there is a rank deficiency in the design matrix greater than the expected one. This is due to a defect in the coverage of the sky by our simulated observations: a star could be isolated or maybe bound to only one pair of stars. In such cases, there is usually a convergent solution for the SVD algorithm but distribution of the astrometric accuracies is widely spread.

For cases falling in the second category, the rank of the design matrix is indeed the expected one but the histogram of the distribution is well spread. These cases are marginal and their reasons could be many: isolated star impossible to trace inside the dense observation net or non-convergence of the SVD, for instance. A simple case

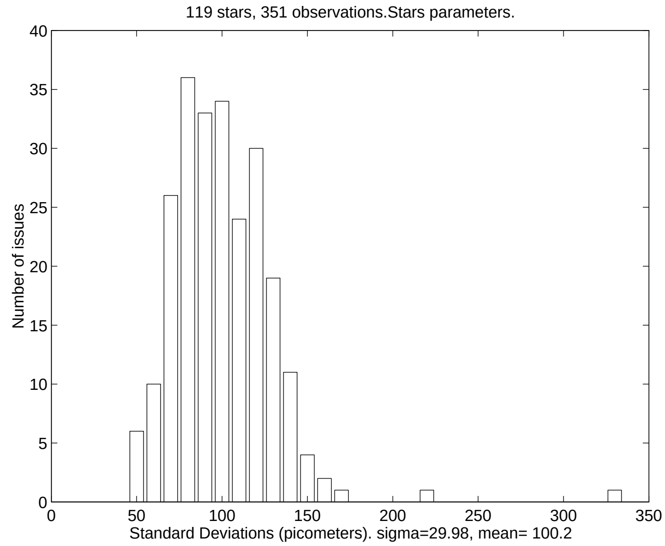

Fig. 4. Histogram of the accuracies by which the astrometric parameters are obtained for a ratio M equal to 2.94. The ordinate represents the number of astrometric parameters for which the standard deviation has a value sigma (given in units of picometers)

falling in this category is when we restrain the distribution of the stars to a strip around the equator: the rank of the design matrix has the expected value but the final astrometric accuracy is multiplied by a significant factor instead of being improved. This result shows that one should be careful in setting the grid of stars. Global astrometry is efficient when performed over the whole celestial sphere.

The third category is the one for which the distribution of astrometric accuracies obtained is narrow and centred on a value which corresponds to a gain in the nominal astrometric accuracy. As an example, for a value $M = 2.39$ obtained with 111 stars and 265 observations, the histogram is centred on 130 picometers, less than half of the nominal astrometric accuracy but its standard deviation is 114 picometers. Another example, at the other extreme of the range of M, is obtained for 132 stars and 457 observations, i.e. $M = 3.46$. The distribution is then centred on 74 picometers and its standard deviation is 15 picometers.

For this category of results, it is also interesting to study the variations of the mean of the distribution of the astrometric accuracies with the ratio M. These results are summarized in Table 1 and displayed, along with following results, in Fig. 6, where the mean μ of the distribution is plotted against the ratio M with a logarithmic scale. The relevant part of this graphic corresponds to the range $0.3 - 0.6$ for $\log(M)$. Higher values, as explained in the next section, correspond to a larger field-of-view and are used to study the shape of the function relating the mean of the distribution to M. The corrections to the initial astrometric parameters, what we would be eventually solving for,

Table 1. Summary of the results obtained for various values of M and plotted in Fig. 6

M	$m_{\rm s}$	$m_{\rm obs}$	σ	μ
2.50	125	313	114.1	128.5
2.57	109	280	131.1	143.3
2.65	113	315	89.04	116.7
2.70	120	325	39.71	106.8
2.83	110	311	32.31	102.4
2.84	111	316	26.11	104.4
2.88	109	314	31.44	115.0
2.94	119	351	29.98	100.2
2.95	121	357	20.00	87.66
3.05	119	364	28.18	94.45
3.12	119	372	24.73	93.40
3.26	114	372	19.06	84.58
3.46	132	457	15.42	73.58
3.57	135	482	15.26	73.28
3.69	120	443	24.48	83.22
3.87	133	514	15.90	72.03

are computed once the SVD is performed. The standard deviations presented previously represent the accuracies by which these corrections are obtained. As expected, their order of magnitude is equal to the precision of the input catalogue, 1 mas. An example of the corrections obtained is shown in Fig. 5.

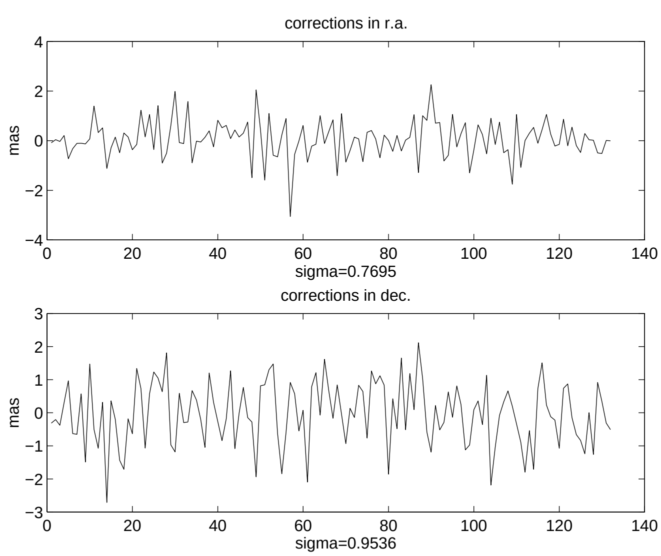

Fig. 5. Corrections to the astrometric parameters: top, right ascension; bottom, declination. The abscissa represents the star number and the standard deviation sigma is given in units of mas

Along with the astrometric parameters, the attitude parameters of the spacecraft are being solved for during the global reduction of the data, which is one of the key point to this method: internal and external parameters can be added to the model and solved for simultaneously with the others. In our model, there is an intrinsic uncertainty on the attitude due to the precision of the simulated input catalogue. The attitude of the satellite is pre-calculated with the initial positions of the stars and improved by the global reduction. The final accuracy by which the attitude parameters can be determined after the global treatment described above is typically of the order of 5 μas, which represents a gain of a factor 2 from the inital accuracy. This number is limited by the factor of redundancy M. In the following section, we present a method which makes it possible to simulate much higher values for M.

5.2. *Enlargement of OSI's field-of-view*

As explained previously, it is quite difficult to increase the number of stars of our samples beyond 140 for trivial computing questions. In order to test our simulations for much higher M ratios without modifying the structure of our code, we expanded OSI's field-of-view to 50 or 60 degrees, which makes it possible to get a large number of observations for a few dozens of stars. For example, with a 50° field-of-view, 60 stars uniformly distributed on the sky are tied together by 441 observations, i.e. a ratio M equal to 7.35. The distribution of astrometric accuracies has a mean of 43.4 pm and a standard deviation of 11.2 pm, representing a gain of a factor ~ 8 on the initial accuracy. Likewise, with a 60° field-of-view, we tie 51 stars by 584 observations ($M = 11.45$) and obtain a narrow distribution which has a mean of 31.3 pm and a standard deviation of 6.4 pm: a factor of almost 11 is gained. The results presented in this section are summarized in Table 2. Including these new results in the diagram giving the mean of the distribution of the astrometric accuracies as a function of the ratio M, we obtain an ensemble of points that we can try to fit by a power law. We fitted the logarithmic plot we got (Fig. 6) via linear least-squares. The best fit is a line of slope very close to -1 (-0.99). The goodness-of-fit of this linear regression, calculated with an incomplete Gamma function for the χ^2 we considered is excellent (~ 1). The mean of the distribution of the astrometric accuracies is

Table 2. Results obtained with an extended field-of-view

M	$m_{\rm s}$	$m_{\rm obs}$	σ	μ
4.58	55	252	12.60	58.73
5.98	55	329	12.12	48.08
6.03	56	338	13.39	49.30
7.35	60	441	11.18	43.40
7.93	55	436	9.25	39.19
11.45	51	584	6.36	31.33

thus found to vary as the inverse of the mean number of observations per star, M.

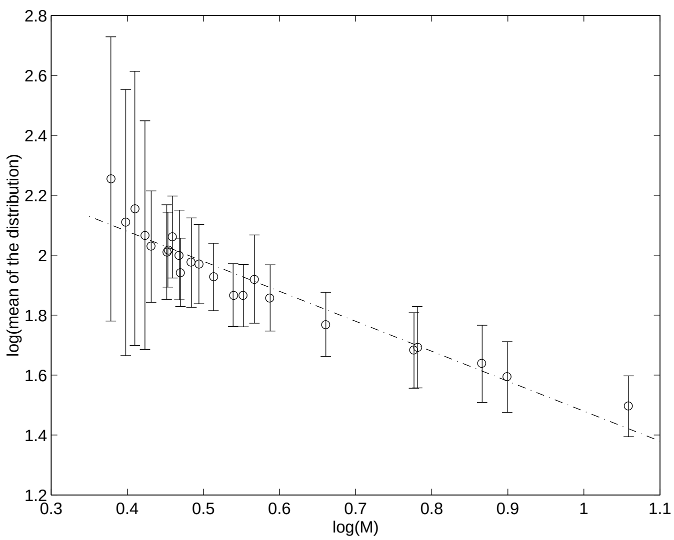

Fig. 6. Logarithmic plot of the variations of the mean of the astrometric accuracies distribution with M and a fitting linear regression of slope ~ -1. The error bars have a width equal to the standard deviation σ of the the distribution divided by the corresponding value

5.3. *Use of three different baselines*

In the previous calculations, a single value for the three baselines was assumed. It is easy to introduce three different values for these baselines, e.g. 6, 6.5 and 7 meters and run the simulation programme on the same sample of stars as for a single baseline value. According to the theory described in Sect. 3, the astrometric accuracy that can be reached with an interferometer is directly proportional to its baseline length. We therefore notice a slight decrease of the final accuracy directly proportional to the change in the baseline length. For instance, a run on a sample of 114 stars with 309 observations produces a mean astrometric accuracy of 107.2 pm with 7 meter baselines and 119.5 pm with baselines equal to 7, 6.5 and 6 meters respectively.

6. Conclusion

Following the great success of the Hipparcos mission, several space-based interferometers have been proposed mainly in response to the need for greater astrometric accuracy. Among these instruments, OSI is being designed to make very accurate wide-angle astrometric measurements and reaches the 10 μas level on several thousand stars as faint as magnitude 20. In this paper, we show that the implementation of a global data reduction process would significantly enhance the instrument's performances without any modification on its structure, configuration or operating mode. Our results show that with a nominal accuracy of 10 μas and a uniform coverage of the sky, a factor of the order of 4 could be gained, leading to a final astrometric accuracy of 2−3 μas. Furthermore, we show that this accuracy scales as the inverse of the number of independent observations per star M and would be affected by gaps in the sky coverage.

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